Broadband Interference Suppression in a Spread-spectrum Communication System

C. Carlemalm, H.V. Poor, A. Logothetis
Department of Electrical Engineering
Princeton University
Princeton, NJ 08544
cath@ee.princeton.edu

Abstract

We consider the problem of estimating and suppressing many unknown independent and time-varying interferers in a spread-spectrum communication system. The interferers are assumed to be present in a wide frequency range. In order to detect, estimate and track the broadband interference, we use a bank of hidden Markov model filters operating in the frequency domain. The hidden Markov model filters' outputs are then used to suppress the existing interference. The computational complexity of our scheme is only linear in the number of interferers. The simulation studies show that our proposed novel schemes adapt quickly in tracking the time-varying nature of the broadband interference.

Keywords: wideband CDMA, interference suppression, spread spectrum communications, hidden Markov model.

1 Introduction

In this paper, we consider the problem of detecting, tracking and suppressing broadband interference in a spread-spectrum communication system. The broadband interference is assumed to consist of many unknown, independent and time-varying narrowband interferers.

There are several advantages with spreading the spectrum of the signal that you wish to transmit; see *e.g.* [1, 2]. One of the major advantages is the inherent ability to reject interfering signals whose bandwidths are small compared to that of the spread spectrum. However, the interference may be powerful enough so that communication becomes effectively impossible. In order to improve the performance of a spread-spectrum communication link in an interference corrupted environment without increasing transmit bandwidth (or reducing throughput), some additional means of interference removal must be used. The problem of interference suppression has been studied extensively over the last decades [1, 2, 3]. Previous work in this area can be classified into frequency domain and time domain approaches.

Time domain techniques for narrowband interference can be split into linear and non-linear methods. [1] provides a discussion of parametric linear techniques for narrowband interference suppression. In these linear techniques, also known as estimator/subtracter methods [7], a transversal filter is used to obtain estimates of the received signal based on previous samples and model assumptions. The filter is implemented using single-sided taps (linear prediction filter) or double-sided taps (linear interpolation filter). Interpolation linear filters were found to give greater interference suppression. Adaptive schemes based on the least mean squares (LMS) algorithm and lattice filters have been developed [8, 4, 9]. Different models for narrowband interference exist in the literature; *e.g.* in [9, 10], the interference is modeled as a Gaussian AR process, in [4] as a sum of sinusoids and in [8] as a pulsed RF tone. In 1991, Vijayan and Poor [11] suggested a non-linear technique for prediction of the narrowband interference signal that took into account the non-Gaussian distribution of the observation noise. This led to various nonlinear techniques to combat narrowband interference suppression in spread spectrum systems [7, 11, 21]. The interference in [7, 11, 21] is modeled as a Gaussian AR process. For known interference statistics the interference is estimated using an approximate condition mean (ACM) filter [22]. The ACM filter is a modification of the Kalman filter that deals with non-Gaussian distributions in the observation or the state process. For the specified assumptions on the observation process in [7, 11, 21], the ACM filter for interference estimation turns out to be a Kalman-type recursive filter which includes some nonlinearities. In [23], a new (suboptimal) nonlinear filter and parameter estimator for narrowband interference suppression in spread spectrum systems is presented. A cross-coupled hidden Markov model and a Kalman filter was used to compute the desired state estimates. In [19], no assumptions on the statistics of the involved noise processes are made. An iterative scheme based on the coordinate descent method is proposed where the spread-spectrum signal is estimated using the Viterbi algorithm while the interference is estimated in an l_1 sense using an interior point based scheme.

Frequency domain techniques are usually non-parametric and require no prior knowledge of the characteristics of the interference. These algorithms are based on transform domain filtering; see *e.g.* [4, 5, 6]. The key idea is that the received signal is estimated and used to design filters that attenuate the signal in the frequency range where the interference is dominant. [4] employs the fast Fourier transform (FFT) algorithm for performing spectral analysis on the received signal, and on the basis of the spectral estimate, a transversal filter is designed for suppressing the interference. In [6, 5], real-time Fourier transformation using a surface acoustic wave (SAW) with a chirp impulse response built into the taps is used. In [24], the problem of suppressing many independent narrowband interferers that are present in different frequency bins. Adaptive filtering in the frequency domain via the LMS algorithm is performed in order to track and estimate the interference.

In this paper, we consider a similar problem to the one posed in [24], thus, the problem of detection and suppression of *many* unknown independent narrowband interferers that are present in a wide frequency range, *i.e.* forming broadband interference. Contrary to [24], though, it is here assumed unknown for which frequencies these interferers exist. Furthermore, the presence and statistics of the interferers are highly time-varying, *i.e.*, an interferer is allowed to appear, disappear and possibly be replaced by another interferer with different statistics. This model is physically more realistic and flexible than a model assuming fixed broadband interference. By modeling the amplitude of each narrowband interferer with an independent Markov chain as indicator functions, the possible presence of the interferers is described efficiently.

We use a bank of independent hidden Markov model (HMM) estimators operating in the frequency domain to detect and estimate the interference. Each of the HMM estimator yields the conditional mean estimate of a narrowband interferer. The estimated narrowband signals are then removed prior to transforming back to time domain. By detecting, estimating and suppressing the existing broadband interference in the frequency domain instead of in the time domain, we make use of the fact that the interferers are independent in the frequency domain. We can therefore apply an adaptive estimator to each of the interferers independently and thereby, the computational complexity of detecting the narrowband interferers is linear in the number of interferers instead of exponentially growing which would have been the case if the estimation of the interference would have been performed in the time domain. Keeping the computational complexity as a function of the number of interferers as low as possible is crucial since the number of interferers creating the broadband interference can be very high.

The outline of this paper is as follows. In Section 2, we describe the signal model and state the objectives. Next, in Section 3, we propose and analyze off-line and on-line schemes for wide-band interference suppression. Section 4 gives some experimental results and finally, Section 5 provides concluding remarks.

2 Signal Model and Objectives

In this section, we describe the model for the received noise corrupted signal and we then state our estimation objectives.

Continuous-time Signal Model

We assume that the continuous-time received signal, $y(t)$, consists of the sum of the spread spectrum signal $s(t)$, the broadband interference $x(t)$, and the observation noise $w(t)$, *i.e.*,

$$y(t) = s(t) + x(t) + w(t). \tag{1}$$

The interference $x(t)$, is assumed to consist of at the most M superimposed independent narrowband interferers, $x^m(t)$, $m = 1, 2, \dots M$, where M is a known integer, *i.e.*

$$x(t) = \sum_{m=1}^{M} x^m(t). \tag{2}$$

[8] models the interference as a sum of sinusoids. Here, we extend this model by considering $x(t)$ to be a superposition of amplitude modulated sinusoids. Each amplitude is modulated by a two state Markov chain serving as an indicator function in order to model the interferer's possible presence. In particular,

$$x^m(t) = A^m(t)\exp\{j2\pi mFt\},\ m = 1, 2, \dots M, \tag{3}$$

where $A^m(t)$ is an unknown stochastic random variable indicating the signal strength and presence of the mth interferer at the known frequency mF.

The spread spectrum signal $s(t)$ is assumed to be transmitted over the entire frequency range $[0, f_{max}]$, where $f_{max} = MF$, while the mth narrowband interferer, $x^m(t)$, operates at frequency mF, for $m = 1, 2, \dots M$. Thus, the interferers are assumed to be equispaced in frequency.

Discrete-time Signal Model

The discrete-time model is the result of sampling the continuous-time model in (1) at sampling rate $1/T_s$. The discrete-time observations are given as follows

$$y_k = s_k + \sum_{m=1}^{M} A_k^m \exp\{j2\pi mFkT_s\} + w_k, \tag{4}$$

where w_k is a zero mean white circular Gaussian process with variance σ_w^2. The sampling period, T_s, is assumed to be equal to the chip period, T_c, of the spread spectrum signal. The condition $T_s = T_c$ ensures the sampled spread spectrum signal s_k to be an independent identically distributed (iid) process. $A_k^m,\ m = 1, 2, \dots M$ is a discrete-time, homogeneous, first-order, 2-state Markov chain with states $q^m = (q_1^m, q_2^m)$, where $q_1^m = 0$ and q_2^m is an unknown non-zero complex number, *i.e.* A^m serves as an indicator function for the presence of the mth interferer. Thus, when the mth Markov chain A_k^m is at state q_1^m, it implies that there is no interferer present at frequency mF. On the other hand, when A_k^m is equal to q_2^m, then the interfering amplitude is q_2^m. The transition probability matrix of A^m is denoted by $\Pi^m = (\pi_{np}^m)$, where π_{np}^m denotes the probability that A_{k+1}^m is in state q_p^m given that A_k^m was in state q_n^m. That is,

$$\pi_{np}^m = P(A_{k+1}^m = q_p^m | A_k^m = q_n^m),\ n, p \in \{1, 2\}. \tag{5}$$

Of course, $\pi_{np}^m \geq 0$ and $\sum_{p=1}^{2} \pi_{np}^m = 1$, for each $n, p \in \{1, 2\}$ and $m \in \{1, 2, \dots M\}$. Let $\bar{\pi}^m,\ m = 1, \dots M$ denote the initial state probability vector that is the vector containing the probabilities of the states of A_1^m.

$$\bar{\pi}^m = (\bar{\pi}_n^m),\ \forall m = 1, 2, \dots M, \tag{6}$$

where

$$\bar{\pi}_n^m = P(A_1^m = q_n^m)\ \forall m = 1, 2, \dots M. \tag{7}$$

Objectives

Given the discrete-time observations y, the objective is to propose both off-line and on-line algorithms that

- are of low complexity. The computational complexity should increase linearly with the number of possible interferers.

- detect accurately which frequencies mF for $m = 1, 2, \ldots M$ are corrupted with interference. Using the model description in Section 2, this is equivalent to estimating the state of the Markov chain A^m for $m = 1, 2, \ldots M$.
- estimate accurately with what probability the mth interferer x^m, $m = 1, 2, \ldots M$, is present.
- estimate accurately the interferers x^m, $\forall m = 1, 2, \ldots M$. Using the model description in Section 2, this is equivalent to determining q_2^m, $\forall m = 1, 2, \ldots M$.
- estimate accurately the transition probabilities from absence to presence of the interferers, *i.e* determine Π^m, $\forall m = 1, 2, \ldots M$ in our model description above.
- suppress the broadband interference, x.

3 Algorithms for Broadband Interference Estimation and Suppression

In this section, we propose off-line and on-line schemes for detection, tracking, estimation and suppression of broadband interference in a spread-spectrum communication system.

The key idea of the suggested algorithms is to perform adaptive filtering in the frequency domain using a bank of hidden Markov model (HMM) estimators (filters for the on-line scheme and smoothers for the off-line scheme) to detect and estimate the interference. The outputs of the HMM filters/smoothers are then used to determine the parameters of our proposed suppression scheme which is also run in the frequency domain. We choose to perform the adaptive estimation in the frequency domain and not in the time domain in order to make use of the fact that the interferers are independent in the frequency domain. Therefore, the computational complexity of detecting the narrowband interferers is linear in the number of interferers instead of exponentially growing which would have been the case if the estimation of the interference would have been performed in the time domain. Keeping the computational complexity as a function of the number of interferers as low as possible is crucial since the number of interferers creating the broadband interference can be very high.

In Fig. 1, our proposed scheme is depicted. The output data are grouped into batches of T consecutive blocks. Each block contains N data points. The discrete Fourier transform (DFT) of the bth batch is given by

$$Y(kF_s, b) = T_s \sum_{n=bN-N}^{bN-1} y_n \exp\{-j\frac{2\pi n}{N}k\},\ k = 0, \ldots N-1,\ b = 1, \ldots T, \tag{8}$$

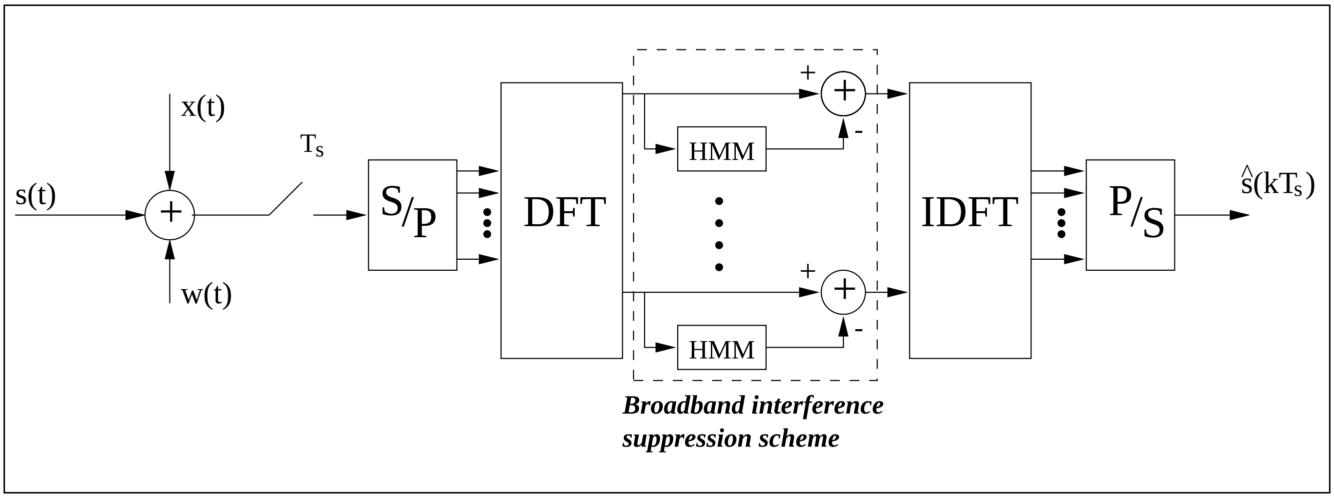

Figure 1: Schematics of the proposed broadband interference suppression schemes using a bank of HMM estimators.

where $F_s = 1/(NT_s)$. The received signal is, as stated in Section 2, a sum of the transmitted spread spectrum signal, s, the observation noise, w, and the interference, x. The DFTs of each of these components are as follows.

$$\begin{aligned} X(kF_s, b) &= T_s \sum_{n=bN-N}^{bN-1} x_n \exp\{-j\frac{2\pi n}{N}k\} \\ &= T_s \sum_{n=bN-N}^{bN-1} \sum_{m=1}^{M} A_n^m \exp\{j2\pi mFnT_s\} \exp\{-j\frac{2\pi n}{N}k\}, \\ & k = 0, \dots N-1, \ b = 1, \dots T. \end{aligned} \tag{9}$$

A^m is assumed to be constant in the bth data batch. It is denoted as A_b^m. Furthermore, the interferers x^m, $m = 1, 2, \dots M$ are, as previously stated, assumed to be mutually independent in the frequency domain. Assuming the interference is constant during one batch we have

$$\begin{aligned} X^m(kF_s, b) &= T_s A_b^m \sum_{n=bN-N}^{bN-1} \sum_{m=1}^{M} \exp\{j2\pi mFnT_s\} \exp\{-j\frac{2\pi n}{N}k\}, \\ &= T_s A_b^m \sum_{n=bN-N}^{bN-1} \sum_{m=1}^{M} \exp\{j\frac{2\pi n}{N}(\frac{mF}{F_s} - k)\} \\ & k = 0, \dots N-1, \ b = 1, \dots T. \end{aligned} \tag{10}$$

By choosing the sampling period T_s and the number of data points N per batch, such that $F_s = F$ and

$N = M$, we have the following result

$$X^m(kF_s, b) = \begin{cases} NT_s A_b^m & \text{if } m = k \\ 0 & \text{otherwise} \end{cases} \quad k = 0, \ldots N-1,\ b = 1, \ldots T.$$

The DFT of the observation noise is given by

$$W(kF_s, b) = T_s \sum_{n=bN-N}^{bN-1} w_n \exp\{-j\frac{2\pi n}{N}k\},\ k = 0, \ldots N-1,\ b = 1, \ldots T. \tag{11}$$

and finally, the DFT of the spread spectrum signal is

$$S(kF_s, b) = T_s \sum_{n=bN-N}^{bN-1} s_n \exp\{-j\frac{2\pi n}{N}k\},\ k = 0, \ldots N-1,\ b = 1, \ldots T. \tag{12}$$

Theorem 3.1 *Let w_k be a sequence of independent and identically distributed (iid), zero mean white circular Gaussian random variables with finite variance σ_w^2. Let s_k be a sequence of iid random variables with zero mean and finite variance σ_s^2. If the sequences $W(kF_s, b)$ and $S(k, F_s, b)$ are computed as follows*

$$W(kF_s, b) = T_s \sum_{n=bN-N}^{bN-1} w_n \exp\{-j\frac{2\pi n}{N}k\} \tag{13}$$

$$S(kF_s, b) = T_s \sum_{n=bN-N}^{bN-1} s_n \exp\{-j\frac{2\pi n}{N}k\} \tag{14}$$

for $k = 0, \ldots N-1$ and $b = 1, \ldots T$, then

- *$W(kF_s, b)$ and $S(kF_s, b)$ are zero mean iid random variables with variance $T_s^2 N\sigma_w^2$ and $T_s^2 N\sigma_s^2$, respectively.*

- *For large N, $W(kF_s, b)$ and $S(kF_s, b)$ are approximately Gaussian distributed.*

PROOF. See Appendix A. □

From the Theorem above, we observe that the signals at each frequency bin k, consist of 1) a finite-state Markov chain $X(kF_s, b)$ and 2) for large N, additive white Gaussian noise. Hidden Markov theory can thus be used to extract the Markov chains impeded in the additive background noise. In the following two subsections, we use off-line and on-line Maximum likelihood techniques for estimating and subtracting the interference.

Notation: For notational convenience, we choose to denote $Y(mF_s, b)$ at the frequency mF as Y_b^m. We denote the set $\{Y_b^m,\ b=1,\dots T\}$ as $\mathcal{Y}^m$ and $\{Y_b^m,\ b=b_1,\dots b_2\}$ as $\mathcal{Y}^m_{b_1:b_2}$. Similar notation holds for $S(mF_s,b), X(mF_s,b)$ and $W(mF_s,b)$. Furthermore, we let $\mathcal{A}^m$ denote $\{A_b^m,\ b=1,\dots T\}$. Let $\theta^m \triangleq \{q_2^m, \Pi^m\}$, $m=1,\dots M$.

3.1 Off-line Broadband Interference Suppression Scheme

We use a bank of M *independent* HMM smoothers in the frequency domain, that estimate, detect and subtract the interference present at each frequency bin. The estimation and detection of the mth interferer is presented here.

Interference Parameter Estimation

The EM algorithm proposed in [20] is used to obtain the maximum likelihood (ML) estimate of θ^m which is denoted here as $(\theta^m)^{ML}$. As a by-product of the E-step, conditional mean estimates of the state A^m of the mth interference is obtained. It is shown in [12] that under mild regularity conditions, the sequence $\left\{\theta^{m,l}\right\}$ for $l=1,2,\dots$, of the EM algorithm converges to a stationary value of the likelihood function. Some of the results presented in this section are derived in [14]. The EM algorithm is outlined in Fig. 2.

Algorithm 1

0. *Determine the initial estimate* $\theta^{m,1}$.

1. Expectation step:
Evaluate

$$Q^m\left(\theta^m, \theta^{m,l}\right) \triangleq E\left\{\ln f(\mathcal{Y}^m, \mathcal{A}^m \mid \theta^m) | \mathcal{Y}^m, \theta^{m,l}\right\}. \tag{15}$$

2. Maximization step:
Compute

$$\theta^{m,l+1} = \arg\max_{\theta^m} Q^m\left(\theta^m, \theta^{m,l}\right). \tag{16}$$

3. $l := l+1$. *Iterate steps 1–3 until* $\|\theta^{m,l+1} - \theta^{m,l}\| < \varepsilon$, *where* ε *is some specified constant.*

Figure 2: The EM algorithm

First, the lth iteration of the E-step in the above mentioned EM algorithm is performed.

The Expectation Step:
The aim in the E-step is to evaluate $Q^m\left(\theta^m, \theta^{m,l}\right)$, defined in (15).

The probability density function for the complete data $(\mathcal{Y}^m, \mathcal{A}^m)$ can be expressed as

$$
\begin{aligned}
\ln f^m(\mathcal{Y}^m, \mathcal{A}^m|\theta^m) &= \sum_{b=1}^{T} \ln f(Y_b^m \mid A_b^m, \theta^m) + \ln f(A_1^m \mid \theta^m) + \sum_{b=2}^{T} \ln f(A_b^m \mid A_{b-1}^m, \theta^m) \\
&= \sum_{b=1}^{T} \ln \left(\frac{1}{2\pi\sigma^2} \exp\left\{ -\frac{(Y_b^m - T_s N A_b^m)^2}{\sigma^2} \right\} \right) \\
&\quad + \ln f(A_1^m \mid \theta^m) + \sum_{b=2}^{T} \ln f(A_b^m \mid A_{b-1}^m, \theta^m) \qquad (17)
\end{aligned}
$$

where σ^2 is the sum of the variance of the spread-spectrum signal and the variance of the observation noise after the DFT. The value of σ^2 is derived in Appendix A. Using Eq. (17), we compute the function defined in (15) to be

$$
\begin{aligned}
Q^m(\theta^m, \theta^{m,l}) &= E\left\{ \ln f(\mathcal{Y}^m, \mathcal{A}^m) \mid \mathcal{Y}^m, \theta^{m,l} \right\} \\
&= -T2\pi\sigma^2 - \frac{1}{\sigma^2} \sum_{b=1}^{T} ((Y_b^m)^2 \gamma_b^{m,l}(1) + (Y_b^m - T_s N q_2^m)^2 \gamma_b^{m,l}(2)) \\
&\quad + \sum_{i=1}^{2} \ln(\bar{\pi}_i^m) \gamma_b^{m,l}(i) + \sum_{b=2}^{T} \sum_{i,j=1}^{2} \ln(\pi_{ij}^m) \gamma_b^{m,l}(i,j), \qquad (18)
\end{aligned}
$$

where

$$
\gamma_b^{m,l}(i) \triangleq P(A_b^m = i | \mathcal{Y}^m, \theta^{m,l}) \qquad (19)
$$

and

$$
\gamma_b^{m,l}(i,j) \triangleq P(A_b^m = j, A_{b-1}^m = i | \mathcal{Y}^m, \theta^{m,l}) \qquad (20)
$$

for $b = 1, 2, \ldots T,\ i,j \in \{1, 2\}$.

From [14] the forward un-normalized aposteriori state probabilities, α, are recursively computed as follows

$$
\begin{aligned}
\alpha_b^{m,l}(j) &\triangleq f^m(Y_b^m, A_b^m = j | \theta^{m,l}) = \\
&= \sum_{i=1}^{2} f^m(Y_b^m, A_b^m = j, A_{b-1}^m = i | \theta^{m,l}) \\
&= \sum_{i=1}^{2} f^m(Y_b^m | A_b^m = j, \theta^{m,l}) \pi_{ij}^{m,l} \alpha_{b-1}^{m,l}(i) \qquad (21)
\end{aligned}
$$

and the backwards un-normalized probabilities, β, are given by

$$
\beta_b^{m,l}(i) \triangleq f^m(\mathcal{Y}_{b+1:T}^m | A_b^m = i, Y_b^m, \theta^{m,l})
$$

$$= \sum_{j=1}^{2} f^m(\mathcal{Y}^m_{b+2:T}, Y^m_{b+1}, A^m_{b+1} = j | A^m_b = i, Y^m_b, \theta^{m,l})$$
$$= \sum_{j=1}^{2} f^m(\mathcal{Y}^m_{b+2:T} | Y^m_{b+1}, A^m_{b+1} = j, \theta^{m,l}) f^m(Y^m_{b+1} | A^m_{b+1} = j, \theta^{m,l}) \pi^{m,l}_{ij}$$
$$= \sum_{j=1}^{2} \beta^{m,l}_{b+1}(j) f^m(Y^m_{b+1} | A^m_{b+1} = j, \theta^{m,l}) \pi^{m,l}_{ij} \tag{22}$$

for $b = 1, 2, \dots T,\ i, j \in \{1, 2\}$.

Initialization for the backward probabilities is $\beta^{m,l}_T(i) = 1,\ i \in \{1, 2\}$. The initial forward probabilities are given by $\alpha^{m,l}_1(i) = \bar{\pi}^{m,l} f^m(Y^m_1 \mid A^m_b = i, \theta^{m,l})$, for $i \in \{1, 2\}$.

Using the relations above the following relationships are achieved

$$\gamma^{m,l}_b(i) = \frac{\alpha^{m,l}_b(i)\beta^{m,l}_b(i)}{\sum_{i=1}^2 \alpha^{m,l}_b(i)\beta^{m,l}_b(i)}. \tag{23}$$

and

$$\gamma^{m,l}_b(i, j) = \frac{\alpha^{m,l}_{b-1}(i) f(Y^m_b | A^m_b = q^m_j, \theta^{m,l}) \pi^{m,l}_{ij} \beta^{m,l}_b(j)}{\sum_{i,j=1}^2 \alpha^{m,l}_{b-1}(i) \pi^{m,l}_{ij} f(Y^m_b | A^m_b = q^m_j, \theta^{m,l}) \beta^{m,l}_b(j)} \tag{24}$$

for $b = 1, 2, \dots T,\ i, j \in \{1, 2\}$.

Note: When the model parameters are known, the aposteriori probabilities are computed via Equations (21), (22) and (23) by replacing the lth parameter estimates $\theta^{m,l}$ by the true parameters θ^m.

The Maximization Step:

The aim in the M-step is to update the parameter estimates $\theta^{m,l+1}$ from the previous estimates $\theta^{m,l}$ as follows

$$\theta^{m,l+1} = \arg\max_{\theta^m} Q^m(\theta^m, \theta^{m,l}). \tag{25}$$

I. The Markov chain parameters.

The following problem is to be solved

$$\begin{cases} \arg\max_{\pi^m_{ij}} Q^m(\theta^m, \theta^{m,l}) \\ \text{subject to} : \begin{cases} \sum_{j=1}^2 \pi^m_{ij} = 1 \\ \pi^m_{ij} \geq 0 \qquad i, j = 1, 2 \end{cases} \end{cases}$$

The solution is as follows

$$\pi^{m,l}_{ij} = \frac{\sum_{b=1}^T \gamma^{m,l}_{b-1}(i, j)}{\sum_{b=1}^T \gamma^{m,l}_b(i)}, \qquad i, j \in \{1, 2\}. \tag{26}$$

II. The coefficient, q^m_2.

The state level q^m_2 on the $l+1$th iteration is computed by solving the following problem

$$\arg\max_{q^m_2} Q^m(\theta^m, \theta^{m,l}). \tag{27}$$

The solution is given by

$$q_2^{m,l} \quad = \quad \frac{\sum_{b=1}^{T} Y_b^m \gamma_b^{m,l}(2)}{\sum_{b=1}^{T} T_s N \gamma_b^{m,l}(2)}. \tag{28}$$

Detection (State Estimation)

Here, the goal is to detect the interferer's possible presence. Each interferer's amplitude is, as discussed previously, modeled as a 2-state Markov chain with one state-level being equal to zero and the other a non-zero complex number. When the state of the Markov chain is zero, the corresponding interferer is absent. Thus, by estimating the state of the mth interferer's Markov chain, we can conclude whether the mth interferer is present or not. The minimum probability of error estimate of detection of the interferer is given as a by-product of the E-step of the HMM smoother as follows

$$(\hat{A}_b^m)^{MAP} \quad = \quad \arg\max_{A_b^m} f\left(A_b^m \mid \mathcal{Y}^m, \theta^{m,l}\right) = \arg\max_{j} \gamma_b^{m,l}(j) \tag{29}$$

Interference Suppression

The outputs of the HMM smoother, $\gamma^{m,l}, q_2^{m,l}$, are used to estimate the mth interferer. The estimate of the narrowband interference at frequency mF_s on the bth data batch is given by the conditional mean estimate

$$\hat{X}_b^{m,l} \triangleq E\{A_b^m | \mathcal{Y}^m, \theta^{m,l}\}, \tag{30}$$

where $\hat{X}_b^{m,l}$ denotes the estimate of X_b^m on the lth iteration. (30) reduces to

$$\hat{X}_b^{m,l} = \gamma_b^{m,l}(2) q_2^{m,l}, \; b = 1, \ldots T. \tag{31}$$

Equation (31) represents the estimate of the narrowband interfere at the mth frequency bin, which is computed by weighting the estimate of the interferer's signal strength by the probability that the interferer is active. The broadband interference suppression is performed by subtracting the estimated interferer $\hat{X}_b^m$ from the data Y_b^m of the mth frequency bin, prior to taking the inverse discrete Fourier transform (IDFT). The spread spectrum signal after interference suppression is given by

$$\hat{s}_k = \frac{1}{NT_s} \sum_{m=0}^{N-1} (Y_{\mathrm{int}(k/N)+1}^m - \gamma_b^{m,l}(2) q_2^{m,l}) \exp\{j \frac{2\pi}{N} mk\}, \; k = 0, 1, \ldots TN - 1, \tag{32}$$

where $\mathrm{int}(k/N)$ denotes the integer part of the division k/N.

3.2 On-line Broadband Interference Suppression Scheme

We use a bank of M *independent* HMM filters in the frequency domain, that estimate, detect and subtract the interference present at each frequency bin. The estimation and detection of the mth interferer is presented below.

Interference Parameter Estimation

The on-line stochastic gradient algorithm studied in [15, 17] is applied to the posed problem. The model parameters are updated according to the following iterative scheme

$$\theta_b^m = \theta_{b-1}^m + \frac{1-\rho}{1-\rho^{b-1}} K_b^m I_b^m, \tag{33}$$

where ρ is a predetermined real constant and

$$I_b^m = \rho I_{b-1}^m + V_b^m\left(\theta_{b-1}^m\right),\ I_1^m = V_1^m\left(\theta_0^m\right),\ I_0 = 0. \tag{34}$$

and θ_0^m is the initial model parameter estimate. The incremental score vector, V_b^m, is defined by

$$V_b^m\left(\theta_{b-1}^m\right) \triangleq \left.\frac{\partial E\left\{\ln f(Y_b^m, A_b^m|\mathcal{Y}_{b-1}^m, \mathcal{A}_{b-1}^m, \theta^m\right\}}{\partial\theta^m}\right|_{\theta^m=\theta_{b-1}^m}. \tag{35}$$

K_b^m denotes the step size at the bth time instant and is chosen as

$$K_b^m = \frac{1}{b}. \tag{36}$$

In order to speed the convergence, K_b^m could instead be chosen as b^{-l}, where l is a positive, real constant [16].

I. The Markov chain parameters.

By using a differential geometric approach suggested in [13], the formulae for updating the transition probabilities are given by the following set of equations

$$\pi_{ij,b}^m = \frac{g_{ij,b}^m}{\sum_{i,j=1}^2\left(g_{ij,b}^m\right)^2}, \tag{37}$$

where $\pi_{ij,b}^m$ denotes the transition probability estimate on the bth iteration, and

$$g_{ij,b}^m = g_{ij,b-1}^m + \frac{1-\rho}{1-\rho^{b-1}} K_b^m I_b^m, \tag{38}$$

$$I_b^m = \rho I_{b-1} + V_b^m\left(g_{ij,b-1}^m\right), \tag{39}$$

and

$$V_b^m\left(g_{ij,b-1}^m\right) = 2\left(\frac{\alpha_{b-1}^m(i,j)}{g_{ij,b-1}^m} - \alpha_{b-1}^m(i)g_{ij,b-1}^m\right). \tag{40}$$

II. Update of the coefficient, q_2^m.

The element of the incremental score vector corresponding to the state level q_2^m, is given by

$$V_b^m\left(q_{2,b-1}^m\right) = (Y_b^m - NT_s q_{2,b-1}^m)\frac{NT_s}{\sigma^2}\alpha_b^m(2), \tag{41}$$

where $q_{2,b}^m$ denotes the signal strength estimate of the mth interferer on the bth iteration. $\alpha_b^m(2)$ denotes the un-normalized aposteriori probability that the interferer is present on the bth iteration. $\alpha_b^m(2)$ is iteratively computed from the iteration in (21), by replacing the model parameters on the bth iteration by the model parameter estimates θ_b^m.

Detection (State Estimation)

The minimum probability of error in detecting the mth interferer using data up to time index b is given by

$$A_b^m = \arg\max_{A_b^m} f(A_b^m \mid \mathcal{Y}_b^m, \theta_b^m) = \arg\max_j \alpha_b^m(j). \tag{42}$$

Interference Suppression

The outputs of the HMM filter, α^m, q_2^m, are used to estimate the mth interferer. The estimate of the narrowband interference at frequency mF_s on the bth data batch is given by the conditional mean estimate

$$\hat{X}_b^m \triangleq E\{A_b^m|\mathcal{Y}_b^m, \theta_b^m\} = q_{2,b}^m\alpha_b^m(2), \tag{43}$$

where $\hat{X}_b^m$ denotes the estimate of X_b^m.

Finally, prior to taking the IDFT, the data Y_b^m in each frequency bin is reduced by the interfering estimate $\hat{X}_b^m$, that is

$$Y_b^m - q_{2,b}^m\alpha_b^m(2) \tag{44}$$

Computational Complexity

As stated previously, there is a major advantage to perform the detection, estimation and suppression of the broadband interference in the frequency domain instead of in the time domain. This is so, since the interferers are assumed to be independent in the frequency domain. Our schemes have computational complexities that are linear in the number of interferers instead of exponentially growing as would have been the case if we would have worked in the time domain.

In Table 1, the computational complexities of our off-line and on-line broadband interference schemes are given. Here, the complexity requirements for the Fourier transformation is not included. We choose to perform the transformation using the fast Fourier transform (FFT) which has a computational complexity of $N \log_2(N)$ per batch. Furthermore, the complexity requirements for subtracting the interference for both the on-line and off-line case is $O(M)$ per batch.

	Off-line complexity per pass	*On-line complexity per batch*
γ_b^m	$O(M2^2T)$	$O(M2^2)$
Π^m	$O(M2^2T)$	$O(M2^2)$
q_2^m	$O(MT)$	$O(M)$

Table 1: Computational complexity requirements of the algorithms.

4 Experimental Results

In this section, we illustrate the performance of our proposed off-line and on-line schemes by way of simulation studies. We investigated the performance of our algorithms by varying the level of the amplitudes of the interfering signals, by varying the number of users transmitting spread spectrum signals, and finally by varying the number of possible narrowband interferers that the broadband interference contains. The off-line and on-line schemes were evaluated for the case when the state level q_2^m is known and the case when the interference amplitude is unknown and has to be estimated. As a comparison to our proposed schemes, we ran a bank of least mean squares (LMS) algorithms in the frequency domain, that replaces our bank of HMM smoothers/filters. The LMS algorithm for the mth interferer is given by

$$C_{b+1}^m = C_b^m + \mu(Y_b^m - C_b^m), \tag{45}$$

where $\mu \geq 0$. Suppression was computed by subtracting C_b^m from Y_b^m prior to taking the IDFT. Note that Eq. (45) reduces to our proposed online interference estimation scheme of (33) and (41) by setting $\rho = 0$, $K_b^m = \mu N T_s/\sigma^2$ and $\alpha_b^m(2) = 1$.

We evaluated the results in the time-domain and we used the SNR improvement as the performance measure. The SNR improvement is defined as

$$SNR_{imp} \triangleq \frac{E(|y_k - s_k|^2)}{E(|y_k - s_k - \hat{x}_k|^2)}, \tag{46}$$

where $\hat{x}_k$ denotes the estimate of the broad-band interference.

Common fixed parameters for all the simulations below were the following. We set the standard deviation of the additive observation noise to be $\sigma_w = 0.001$, the frequency $F = 30000$ and the transition probability matrix Π^m, was given by

$$\Pi^m = \begin{pmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{pmatrix}, \ \forall m = 1, \dots M. \tag{47}$$

For the off-line case, the number of data batches was $T = 500$ and for the on-line case $T = 1600$. The update constant μ was chosen as 0.05. Fine-tuning of the on-line parameters was not thoroughly investigated. For all the simulation studies below, we ran 100 Monte Carlo simulations. The last 100 data points of each Monte Carlo run were used to compute the SNR improvement.

- Varying the amplitudes, $q^m(2)$, of the interfering signals.

  The number of possible narrowband interferers were chosen as $M = 16$ and there was only one user transmitting a random sequence of ± 1. We varied the amplitudes $q^m(2),\ m = 1, \dots M$ of the interferers between 0.4 and 1.4. The amplitudes $q^m(2)$ for different $m \in \{1, \dots M\}$ were kept the same in each simulation. This corresponds to varying the SNR between -9.06 dB and -1.81 dB.

  Firstly, we ran the off-line scheme with known state-levels $q_2^m,\ m = 1, \dots M$, and thereafter, we ran our off-line schemes assuming unknown state-levels $q_2^m,\ m = 1, \dots M$. The EM algorithm was iterated five times. The initialization of q_2^m was then set to a random non-zero number.

  Next, the on-line scheme assuming known state-levels $q_2^m,\ m = 1, \dots M$, was studied. Thereafter, the on-line scheme assuming unknown state-level $q_2^m,\ m = 1, \dots M$ was run. The initialization of q_2^m was set to a random non-zero number.

  Finally, as a comparison a bank of LMS schemes was run under the same conditions.

  In Fig. 3, the results are presented.

- Varying the number of users transmitting spread spectrum signals

  The number of possible narrowband interferers were chosen as $M = 16$ and the amplitudes $q^m(2),\ m = 1, \dots M$ of the interferers were all chosen to $q^m(2) = 1,\ m = 1, \dots M$. The number of users transmitting random sequences of ± 1 was varied between 1 and 4.

Firstly, we ran the off-line scheme with known state-levels q_2^m, $m = 1, \ldots M$, and thereafter, we ran our off-line schemes assuming unknown state-levels q_2^m, $m = 1, \ldots M$. The EM algorithm was iterated five times. estimation. The initialization of q_2^m was then set to a random non-zero number.

Next, the on-line scheme assuming known state-levels q_2^m, $m = 1, \ldots M$, was studied. Thereafter, the on-line scheme assuming unknown state-level q_2^m, $m = 1, \ldots M$ was run. The initialization of q_2^m was set to a random non-zero number.

Finally, as a comparison a bank of LMS schemes was run under the same conditions.

In Fig. 4, the results are presented.

- Varying the number of possible narrowband interferers

  The number of users transmitting a random sequence of ± 1 was chosen to 1 and the amplitudes $q^m(2)$, $m = 1, \ldots M$ of the interferers were all chosen to $q^m(2) = 1$, $m = 1, \ldots M$. The number of possible narrowband interferers was varied between 2 and 32.

  Firstly, we ran the off-line scheme with known state-levels q_2^m, $m = 1, \ldots M$, and thereafter, we ran our off-line schemes assuming unknown state-levels q_2^m, $m = 1, \ldots M$. The EM algorithm was iterated five times. estimation. The initialization of q_2^m was then set to a random non-zero number.

  Next, the on-line scheme assuming known state-levels q_2^m, $m = 1, \ldots M$, was studied. Thereafter, the on-line scheme assuming unknown state-level q_2^m, $m = 1, \ldots M$ was run. The initialization of q_2^m was set to a random non-zero number.

  Finally, as a comparison a bank of LMS schemes was run under the same conditions.

  In Fig. 5, the results are presented.

As can be seen from Figs. 3–5, the off-line schemes perform better than the on-line schemes, which is not surprising since off-line schemes smooth the data while on-line schemes filter the data. To bridge the gap between the on-line and off-line schemes, fixed-lag smoothers can be used. The parameter estimation via the off-line schemes outperform the parameter estimation of the on-line schemes. We are currently investigating the efficiency of our parameter estimators in terms of Cramer Rao lower bounds (CRLB).

5 Conclusions

In this paper, both off-line and on-line schemes for broadband interference suppression in a spread-spectrum system have been proposed and verified experimentally in simulation studies. The interference is assumed to consist of many unknown narrowband interferers that are independent in the frequency domain. Our proposed algorithms detect, track and estimate these interferers by running a bank of hidden Markov model estimators in the frequency domain. By performing the detection and estimation of the interferers in the transform domain instead of in the time domain, we make use of the fact that the narrowband interferers are mutually independent in the frequency domain. Therefore, the bank of hidden Markov model estimators are mutually independent which reduces the computational complexity significantly.

Future work will investigate the performance of our broadband interference schemes when the frequencies of the interferers are not equispaced. Other statistics of the interferers will be studied.

Appendix A. Proof of Theorem 1.

In this appendix we prove Theorem 1, *i.e.* we show that $W(kF_s,b)$ and $S(kF_s,b)$ are approximately Gaussian for large N and independent for all $b=1,\dots T,\ k=0,\dots N-1$. It is clear that $W(kF_s,b)$ is zero mean since $E\{w_n\}=0$, for all n. Next, we show that $W(kF_s,b)$ is independent for different b and k.

$$\begin{aligned}
E(W(kF_s,b)W^*(k'F_s,b')) &= E\{(T_s\sum_{n=bN-N}^{bN-1} w_n\exp\{-j\frac{2\pi nkT_s}{N}\})(T_s\sum_{p=b'N-N}^{b'N-1} w_p\exp\{-j\frac{2\pi pk'T_s}{N}\})^*\} \\
&= T_s^2\sum_{n=bN-N}^{bN-1} E\{w_nw_n^*\}\exp\{-j\frac{2\pi n(k-k')T_s}{N}\} \\
&= \begin{cases} T_s^2N\sigma_w^2 & \text{for } k=k' \text{ and } b=b' \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{48}$$

where $W^*(kF_s,b)$ denotes the complex conjugate of $W(kF_s,b)$. We therefore conclude that $W(kF_s,b)$ has variance $T_s^2N\sigma_w^2$. Invoking the central limit theorem (CLT) for iid random variables of finite means and variances, we can conclude that $W(kF_s,b)$ is approximately a Gaussian random variable for large N.

The derivation of the variance and cross-correlation of $S(kF_s,b)$ for different $b=1,\dots T,\ k=0,\dots N-1$ is analogous to the derivation of the variance and cross-correlation of $W(kF_s,b)$ above. The result is as follows: the variance is given by $T_s^2N\sigma_s^2$ and the cross-correlation is zero. Therefore, the stochastic variable $S(kF_s, b)$ is independent for different $b = 1, \ldots T$, $k = 0, \ldots N-1$. Since $E\{S(kF_s, b)\} = 0$, we conclude that $S(kF_s, b)$ is approximately a Gaussian random variable for large N.

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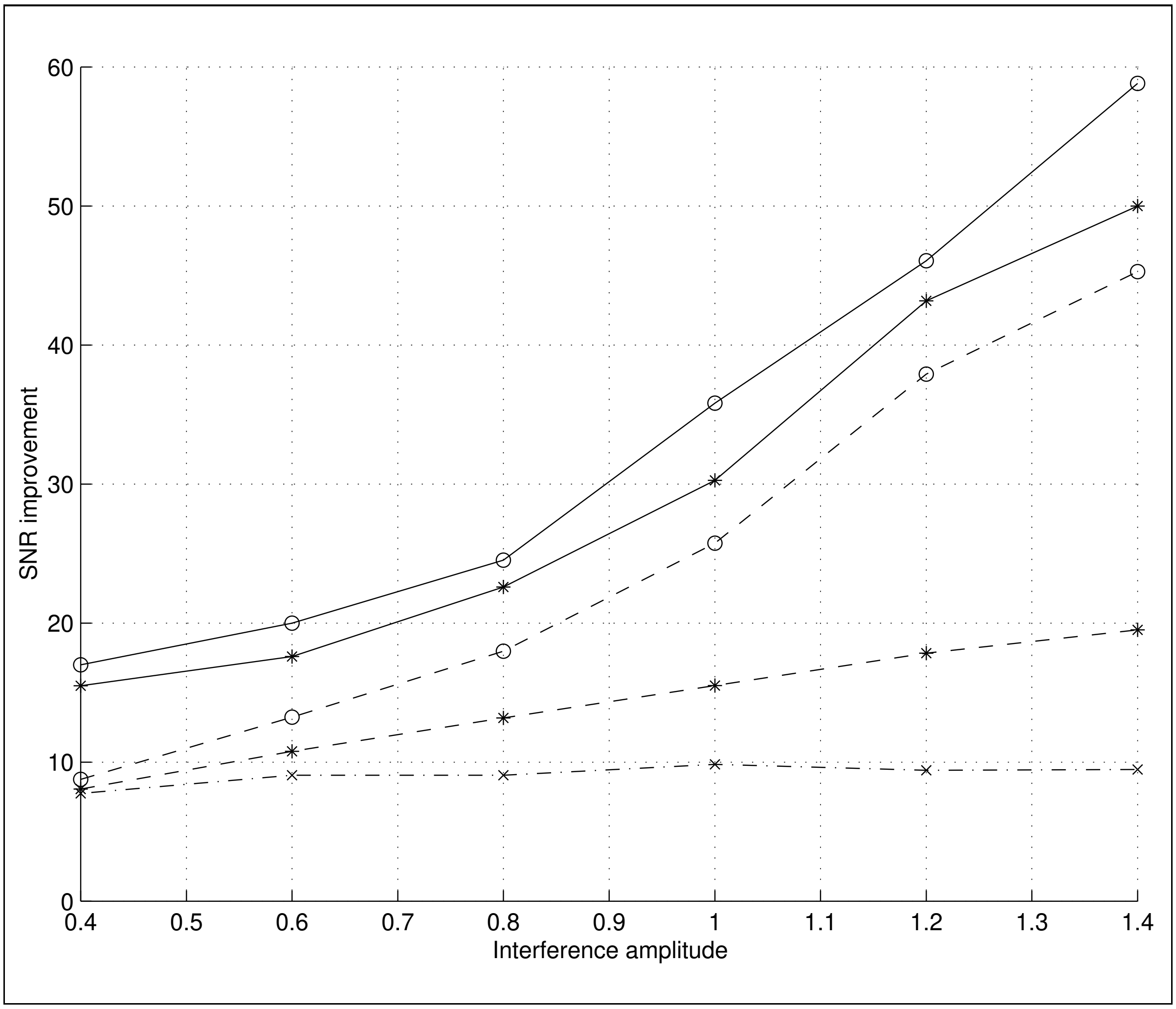

Figure 3: Performance in terms of SNR improvement for varying number of possible interferers. Off-line scheme results are denoted by solid lines '–', on-line scheme results by dashed lines '- -' and finally LMS algorithm results by dashed-dotted line '-.-'. We use 'o' and '*' to show the performance when the parameters are known and unknown, respectively.

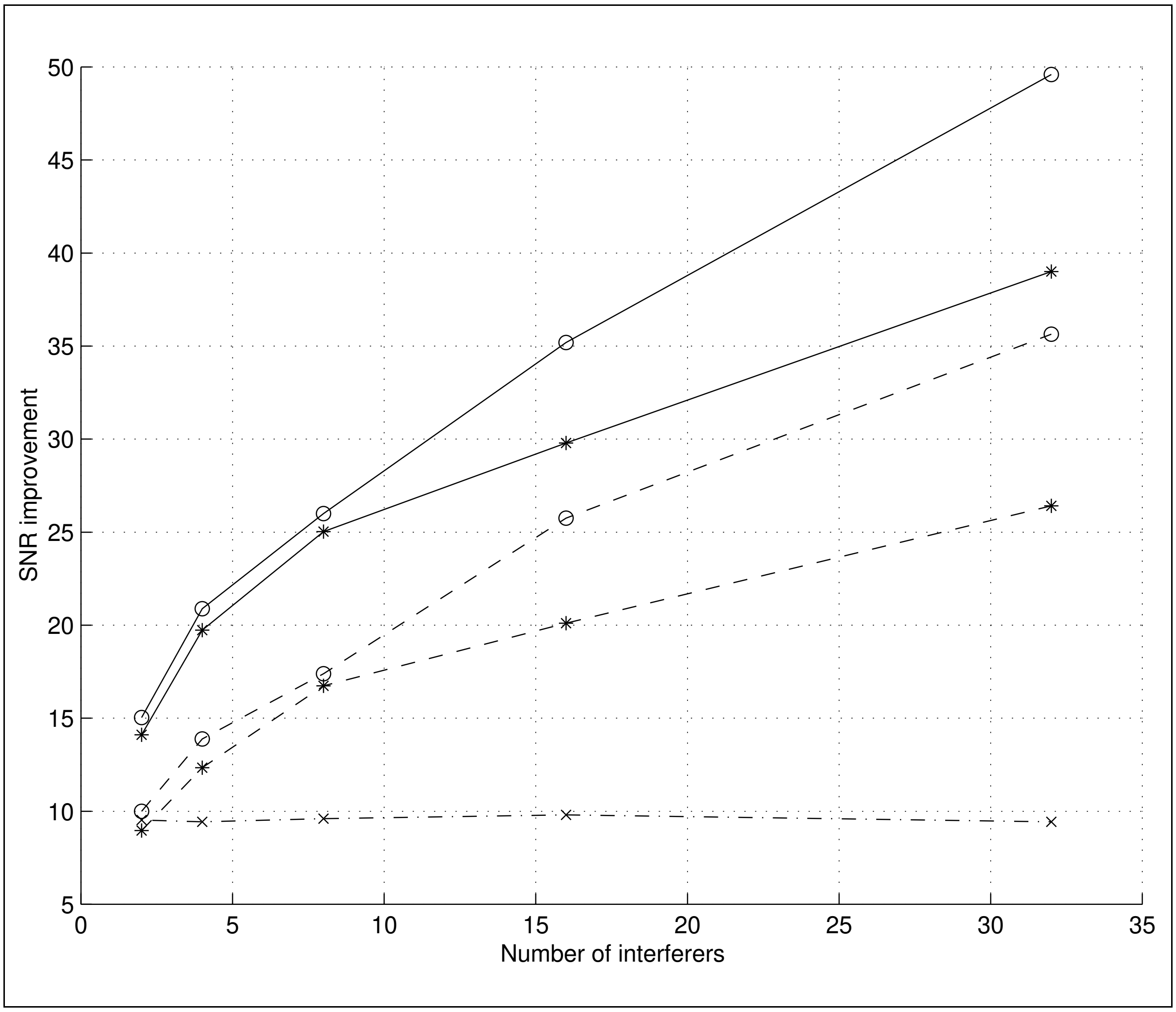

Figure 4: Performance in terms of SNR improvement for varying the amplitudes, $q^m(2)$, of the interfering signals. Off-line scheme results are denoted by solid lines '–', on-line scheme results by dashed lines '- -' and finally LMS algorithm results by dashed-dotted line '-.-'. We use 'o' and '*' to show the performance when the parameters are known and unknown, respectively.

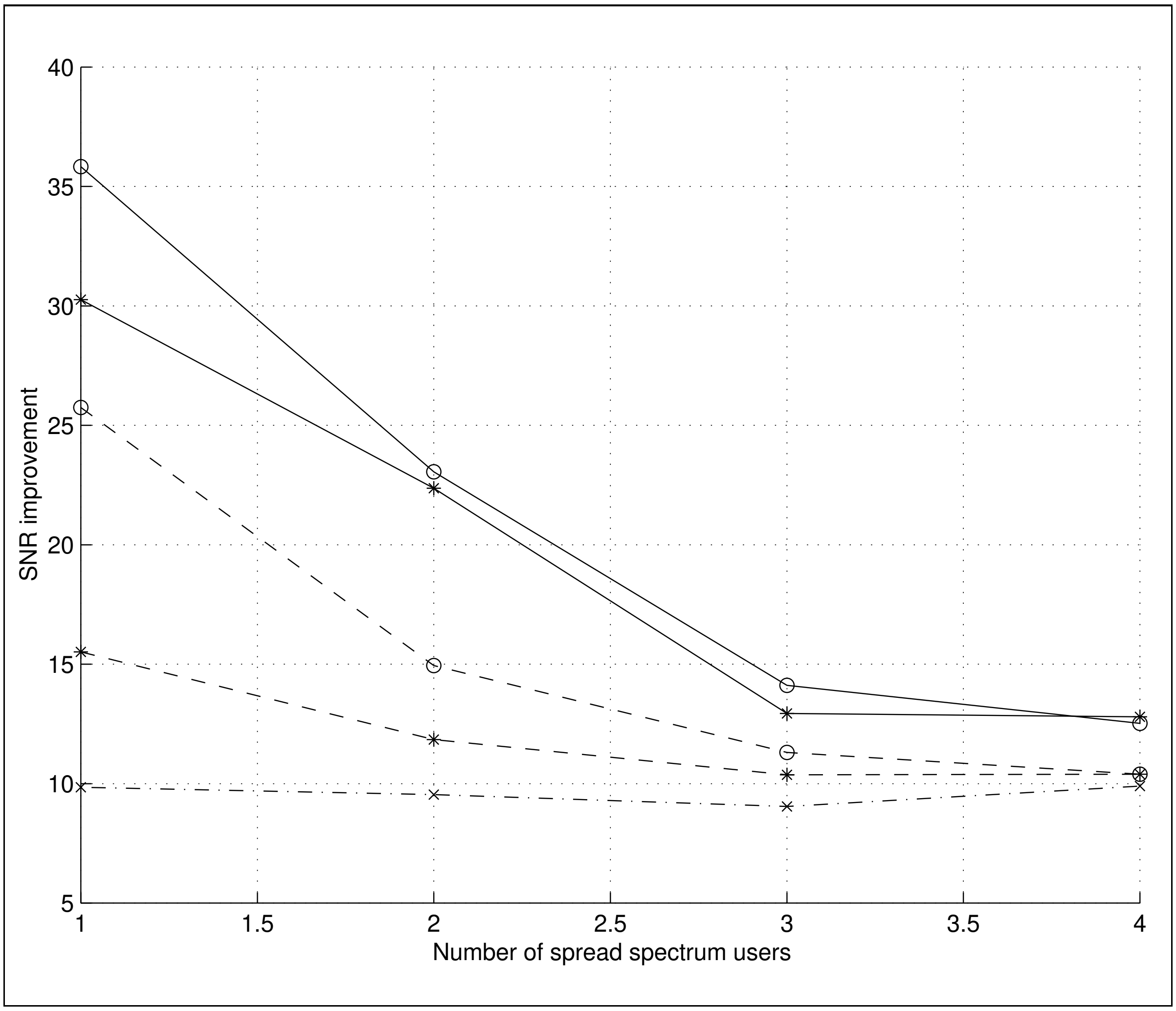

Figure 5: Performance in terms of SNR improvement for varying number of users transmitting spread spectrum signals. Off-line scheme results are denoted by solid lines '–', on-line scheme results by dashed lines'- -' and finally LMS algorithm results by dashed-dotted line '-.-'. We use 'o' and '*' to show the performance when the parameters are known and unknown, respectively.