Localization in Networks Based on Log Range Observations

Outline

- FOCUS Area Control
- Sensor Measurement Modeling
- Nuisance Parameter Elimination
- Simulations
*FOCUS Area Control*

- VINNOVA Excellence Center 2007-2009
- Area Control: FOI, LiU, Exensor
- Baseline: Passage control
- UMRA – Underrättelse Multisensor RAdio
- Target: Area Control

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**UMRA Evolution – UMRA Mini**

- UMRA Mini
  - Microphone
  - Geophone
  - GPS
  - Self-Healing wireless mesh network
  - 2 months battery time, normal operation
Sensor Network Modeling

Problem definition and notation:
- One target with unknown position $x(t)$ (2D).
- $N$ sensor nodes with known positions $p_k$, $k = 1, \ldots, N$.
- Each sensor node consists of $M$ sensor types.
- Observations denoted $y_{k,i}(t), k = 1, \ldots, N$ and $i = 1, \ldots, M$.
- Problem: Localization (from one snapshot $y_{k,i}(t)$) and tracking of $x(t)$.
- Assumption: target speed times sampling interval small compared to network dimensions.
- Restriction: Communication, sensor calibration and multi-target localization with data association are not considered here.

Sensor Model

Assumption: the received power for each sensor type follows an exponential decay

$$P_{k,i} = P_{0,i} ||x - p_k||^{n_{p,i}}.$$  

where

- the transmitted energy is denoted $P_{0,i}$,
- the path loss constant is denoted $n_{p,i}$,

are different for each sensor type $i$ but the same at each node $k$.

The log range model where the power is measured in decibels with additive noise $e_{k,i}$ with variance $\text{Var}(e_{k,i}) = \sigma^2_{p,i}$:

$$P_{k,i} = P_{0,i} + n_{p,i} \log (||x - p_k||) + \hat{e}_{k,i},$$

$$y_{k,i} = P_{k,i} + c_{k,i}.$$ 

The fundamental log range (LR) term $c_k(x)$ is introduced here.
Sensor Experiment

- One vehicle passage and one sensor node
- Received power as a function of time for two sensor types.

Sensor Model Validation

\[ P_{k,i} = P_{0,i} + n_{P,i} \log (\|x - p_k\|) \]
\[ y_{k,i} = P_{k,i} + \epsilon_{k,i} \triangleq c_i(x) \]

- Log range versus log of received signal power.
- Good fit to the log range model for all sensor types
- Non-trivial path loss constants -2.3 and -2.6.
Sensor Network Measurement Model

\[ y = h(x, \theta) + e, \]
\[ y = \begin{pmatrix} y_{1,1} \\ y_{2,1} \\ y_{3,1} \\ \vdots \\ y_{N,M} \end{pmatrix}, \quad e = \begin{pmatrix} e_{1,1} \\ e_{2,1} \\ e_{3,1} \\ \vdots \\ e_{N,M} \end{pmatrix}, \quad h(x, \theta) = \begin{pmatrix} P_{0,1} + n_{p,1}c_1(x) \\ P_{0,2} + n_{p,1}c_2(x) \\ P_{0,2} + n_{p,1}c_3(x) \\ \vdots \\ P_{0,M} + n_{p,M}c_N(x) \end{pmatrix}, \]
\[ \theta_i = (n_{p,i}, P_{0,i})^T, \]
\[ \text{Cov}(e_{k,i}) = \sigma_{p,i}^2. \]

Target location \( x \) unknown, \( \theta \) unknown nuisance parameters and \( \sigma_i \) may or may not be known.
Both \( n_{p,i} \) and \( \sigma_{p,i} \) may depend on the local environment.
Note: \( h(x, \theta) \) is linear in \( \theta \).

Non-linear Least Squares (NLS)

Straightforward application of non-linear least squares (NLS).
Assume first that \( \sigma_p \) is known from off-line experiments.

\[ \widehat{(x, \theta)} = \arg \min_{x,\theta} V(x, \theta, \sigma_p), \]
\[ V(x, \theta, \sigma_p) = \sum_{i=1}^{M} \sum_{k=1}^{N} \frac{(y_{k,i} - h(c_k(x), \theta_i))^2}{\sigma_{p,i}^2}, \]
\[ h(c_k(x), \theta_i) = \theta_{i,1} + \theta_{i,2}c_k(x), \]
\[ c_k(x) = \log \left( \|x - p_k\| \right). \]

2M + 2 unknowns, MN non-linear equations.
Solvable only if \( N \geq 3 \).
The nuisance parameters \( \theta \) appear linearly in all equations. Estimate these first with standard weighted least squares (WLS).

\[
\hat{\theta}_i(x) = \left[ \frac{\sum_{k=1}^{N} c_k(x)}{\sum_{k=1}^{N} c_k(x)^2} \right] R(x) \left[ \sum_{k=1}^{N} \frac{y_{k,i}}{c_k(x)} \right] f_i(x)
\]

Parameter estimate depends on target position \( x \) only via \( c_k(x) \).

Explicit matrix inverse:

\[
R(x) = \frac{1}{N \sum_{k=1}^{N} c_k^2(x) - \left( \sum_{k=1}^{N} c_k(x) \right)^2} \begin{pmatrix}
\sum_{k=1}^{N} c_k^2(x) & -\sum_{k=1}^{N} c_k(x) \\
-\sum_{k=1}^{N} c_k(x) & N \end{pmatrix}
\]

NLS in \( x \) Only

Plugging in the nuisance parameter estimate gives

\[
\hat{x} = \arg\min_{\theta} \min_{\sigma_p} V(x, \theta, \sigma_p) = \arg\min_{\hat{\theta}(x), \sigma_p} V(x, \hat{\theta}(x), \sigma_p),
\]

\[
V(x, \hat{\theta}(x), \sigma_p) = \sum_{i=1}^{M} \sum_{k=1}^{N} \frac{(y_{k,i} - h(c_k(x), \hat{\theta}_i(x)))^2}{\sigma_{p,i}^2}
\]

\[
= \sum_{i=1}^{M} \sum_{k=1}^{N} \frac{y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x)}{\sigma_{p,i}^2}
\]

2 unknowns, \( M(N - 2) \) degrees of freedom in the non-linear equations.

Solvable only if \( M \geq 2, N = 3 \) or \( N \geq 4 \).
Grid Method to Solve for $x$

$$V(x, \hat{\theta}(x), \sigma_p) = \sum_{i=1}^{M} \sum_{k=1}^{N} \frac{y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x)}{\sigma_{p,i}^2}$$

- Tedious to compute the gradient ...
- Grid method for the lazy ...

Cramer-Rao Lower Bound

Position RMSE bounded by the Cramér-Rao lower bound (CRLB)

$$\text{RMSE} = \sqrt{\text{E} \left( (x_1 - \hat{x}_1)^2 + (x_2 - \hat{x}_2)^2 \right)} = \sqrt{\text{tr Cov}(\hat{x})} \geq \sqrt{\text{tr } J^{-1}(x^*)}$$

In case of Gaussian measurement errors, the Fisher Information Matrix (FIM) equals

$$J(x) = \sum_{i=1}^{M} \sum_{k=1}^{N} \nabla_x h(c_k(x), \theta_i) \nabla_x^T h(c_k(x), \theta_i) \sigma_{p,i}^2$$

$$= \sum_{i=1}^{M} \sum_{k=1}^{N} \frac{\theta_{i,1}^2}{\sigma_{p,i}^4} \|x - p_k\|^2 (x - p_k)(x - p_k)^T$$
Handling Unknown Noise

Marginalize linear parameters as above.

The maximum likelihood method assuming Gaussian noise gives

$$\min_{\sigma_p, \theta} V^{GML}(x, \theta, \sigma_p) = \sum_{i=1}^{M} N \log \left( \sum_{k=1}^{N} y_{k,i}^2 - f_i^T(x) \theta_i(x) \right)$$

which again can be optimized over $x$. 
Point Estimate Example

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Estimation Performance, 2-Stroke MC
Estimation Performance, XC90

Some Issues

- Multi-target tracking
- Pre-filtering
- Filtering and marginalization
- Spatial dependency
- Signature correlation