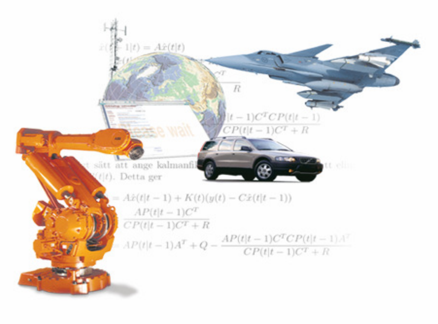
Localization in Networks Based on Log Range Observations



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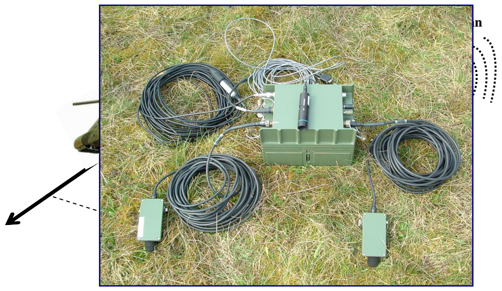
Outline

- FOCUS Area Control
- Sensor Measurement Modeling
- Nuisance Parameter Elimination
- Simulations

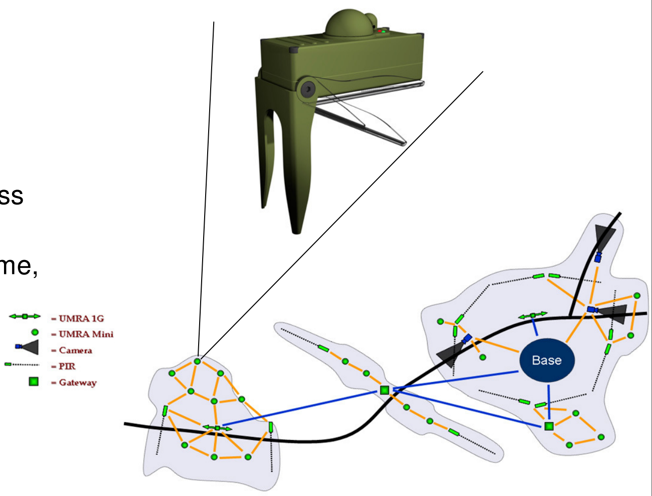
FOCUS Area Control

- VINNOVA Excellence Center 2007-2009
- Area Control: FOI, LiU, Exensor
- Baseline: Passage control
- UMRA – Underrättelse Multisensor RAdio
- Target: Area Control



UMRA Evolution – UMRA Mini

- UMRA Mini
  - Microphone
  - Geophone
  - GPS
  - Self-Healing wireless mesh network
  - 2 months battery time, normal operation

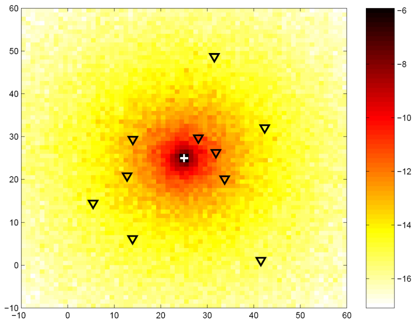


Sensor Network Modeling

Problem definition and notation:

- One target with unknown position $x(t)$ (2D).
- N *sensor nodes* with known positions p_k, $k = 1, \dots, N$.
- Each sensor node consists of M *sensor types*.
- Observations denoted $y_{k,i}(t)$, $k = 1, \dots, N$ and $i = 1, \dots, M$.
- Problem: Localization (from one snapshot $y_{k,i}(t)$) and tracking of $x(t)$.
- Assumption: target speed times sampling interval small compared to network dimensions.
- Restriction: Communication, sensor calibration and multi-target localization with data association are not considered here.

Example scenario with $M = 1$ and $N = 10$.



Sensor Model

Assumption: the received power for each sensor type follows an exponential decay

$$\bar{P}_{k,i} = \bar{P}_{0,i} ||x - p_k||^{n_{p,i}}.$$

where

- the transmitted energy is denoted $P_{0,i}$,
- the path loss constant is denoted $n_{p,i}$,

are different for each sensor type i but the same at each node k.

The log range model where the power is measured in decibels with additive noise $e_{k,i}$ with variance $\mathrm{Var}(e_{k,i}) = \sigma^2_{p,i}$:

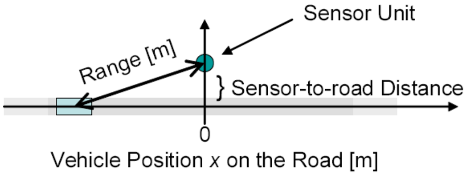
$$P_{k,i} = P_{0,i} + n_{p,i} \underbrace{\log\left(||x - p_k||\right)}_{\triangleq c_k(x)},$$

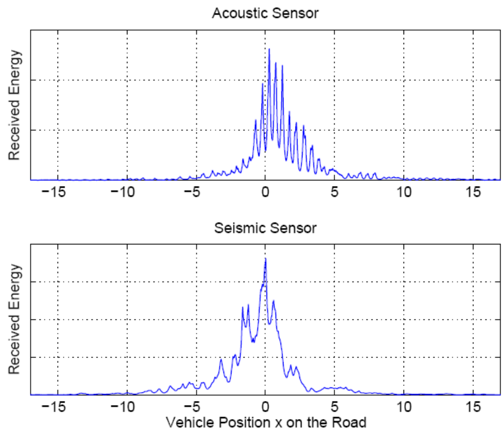
$$y_{k,i} = P_{k,i} + e_{k,i}.$$

The fundamental log range (LR) term $c_k(x)$ is introduced here.

Sensor Experiment

- One vehicle passage and one sensor node
- Received power as a function of time for two sensor types.



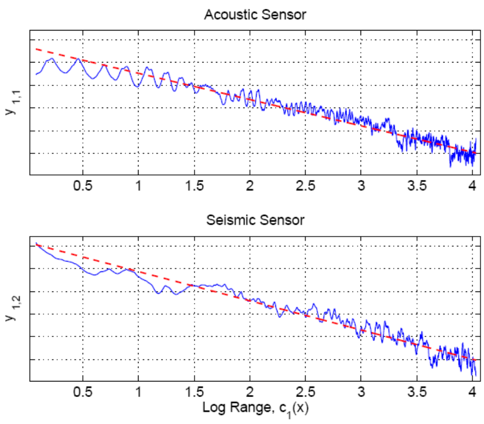


Sensor Model Validation

$$P_{k,i} = P_{0,i} + n_{p,i} \underbrace{\log\left(\|x - p_k\|\right)}_{\triangleq c_k(x)}$$

$$y_{k,i} = P_{k,i} + e_{k,i}$$

- Log range versus log of received signal power.
- Good fit to the log range model for all sensor types
- Non-trivial path loss constants -2.3 and -2.6.



Sensor Network Measurement Model

$$\mathbf{y} = \mathbf{h}(x,\theta) + \mathbf{e},$$

$$\mathbf{y} = \begin{pmatrix} y_{1,1} \\ y_{2,1} \\ y_{3,1} \\ \vdots \\ y_{N,M} \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} e_{1,1} \\ e_{2,1} \\ e_{3,1} \\ \vdots \\ e_{N,M} \end{pmatrix}, \quad \mathbf{h}(x,\theta) = \begin{pmatrix} P_{0,1} + n_{p,1}c_1(x) \\ P_{0,2} + n_{p,1}c_2(x) \\ P_{0,2} + n_{p,1}c_3(x) \\ \vdots \\ P_{0,M} + n_{p,M}c_N(x) \end{pmatrix},$$

$$\theta_i = (n_{p,i}, P_{0,i})^T,$$

$$\mathrm{Cov}(e_{k,i}) = \sigma^2_{p,i}.$$

Target location x unknown, θ unknown nuisance parameters and σ_i may or may not be known.
Both $n_{p,i}$ and $\sigma_{p,i}$ may depend on the local environment.
Note: $\mathbf{h}(x,\theta)$ is linear in θ.

Non-linear Least Squares (NLS)

Straightforward application of non-linear least squares (NLS).
Assume first that σ_p is known from off-line experiments.

$$\widehat{(x,\theta)} = \arg\min_{x,\theta} V(x,\theta,\sigma_p),$$

$$V(x,\theta,\sigma_p) = \sum_{i=1}^{M}\sum_{k=1}^{N} \frac{\big(y_{k,i} - h(c_k(x),\theta_i)\big)^2}{\sigma^2_{p,i}},$$

$$h(c_k(x),\theta_i) = \theta_{i,1} + \theta_{i,2}c_k(x),$$

$$c_k(x) = \log\big(\|x - p_k\|\big).$$

$2M+2$ unknowns, MN non-linear equations.
Solvable only if $N \geq 3$.

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The nuisance parameters θ appear linearly in all equations.
Estimate these first with standard weighted least squares (WLS).

$$\hat{\theta}_i(x) = \underbrace{\left[\begin{pmatrix} N & \sum_{k=1}^N c_k(x) \\ \sum_{k=1}^N c_k(x) & \left(\sum_{k=1}^N c_k(x)\right)^2 \end{pmatrix}\right]^{-1}}_{R(x)} \underbrace{\begin{pmatrix} \sum_{k=1}^N y_{k,i} \\ \sum_{k=1}^N c_k(x) y_{k,i} \end{pmatrix}}_{f_i(x)}$$

Parameter estimate depends on target position x only via $c_k(x)$.
Explicit matrix inverse:

$$R(x) = \frac{1}{N\sum_{k=1}^N c_k^2(x) - \left(\sum_{k=1}^N c_k(x)\right)^2} \begin{pmatrix} \sum_{k=1}^N c_k^2(x) & -\sum_{k=1}^N c_k(x) \\ -\sum_{k=1}^N c_k(x) & N \end{pmatrix}$$

NLS in *x* Only

Plugging in the nuisance parameter estimate gives

$$\hat{x} = \arg\min_x \min_\theta V(x, \theta, \sigma_p) = \arg\min_x V(x, \hat{\theta}(x), \sigma_p),$$

$$V(x, \hat{\theta}(x), \sigma_p) = \sum_{i=1}^M \sum_{k=1}^N \frac{\left(y_{k,i} - h(c_k(x), \hat{\theta}_i(x))\right)^2}{\sigma_{p,i}^2},$$

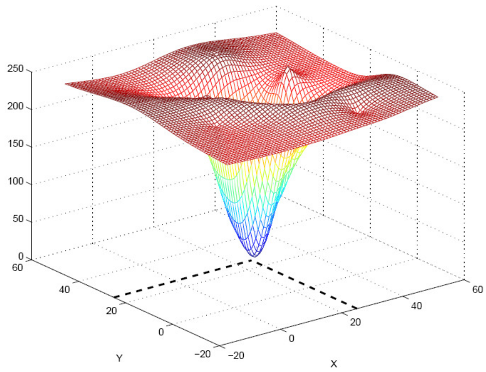
$$= \sum_{i=1}^M \frac{\sum_{k=1}^N y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x)}{\sigma_{p,i}^2},$$

2 unknowns, $M(N-2)$ degrees of freedom in the non-linear equations.
Solvable only if $M \geq 2, N = 3$ or $N \geq 4$.

Grid Method to Solve for *x*

$$V(x, \hat{\theta}(x), \sigma_p)$$
$$= \sum_{i=1}^{M} \frac{\sum_{k=1}^{N} y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x)}{\sigma_{p,i}^2}$$

- Tedious to compute the gradient ...
- Grid method for the lazy ...



$V(x, \hat{\theta}(x), \sigma_P)$

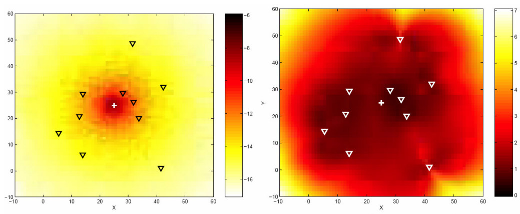
Cramer-Rao Lower Bound

Position RMSE bounded by the Cramér-Rao lower bound (CRLB)

$$\mathrm{RMSE} = \sqrt{\mathrm{E}\left((x_1^o - \hat{x}_1)^2 + (x_2^o - \hat{x}_2)^2\right)} = \sqrt{\mathrm{tr}\,\mathrm{Cov}(\hat{x})} \geq \sqrt{\mathrm{tr}\, J^{-1}(x^o)}$$

In case of Gaussian measurement errors, the Fisher Information Matrix (FIM) equals

$$J(x) = \sum_{i=1}^{M}\sum_{k=1}^{N} \frac{\nabla_x h(c_k(x), \theta_i)\nabla_x^T h(c_k(x), \theta_i)}{\sigma_{p,i}^2}$$
$$= \sum_{i=1}^{M}\sum_{k=1}^{N} \frac{\theta_{i,1}^2}{\sigma_{p,i}^2 \|x - p_k\|^2}(x - p_k)(x - p_k)^T$$



Network

CRLB $\sqrt{\mathrm{tr}\, J^{-1}(x^o)}$

Handling Unknown Noise

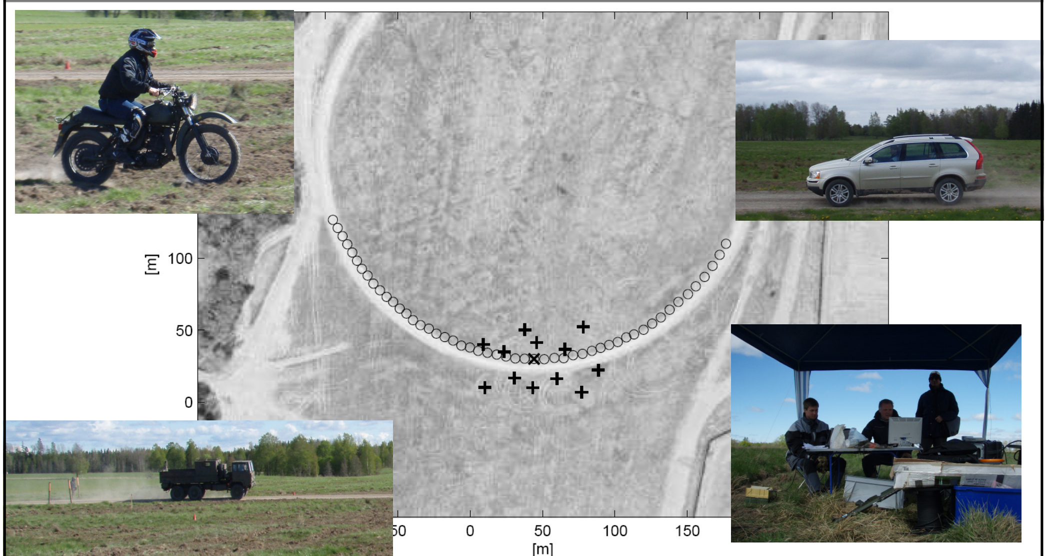
Marginalize linear parameters as above.

The maximum likelihood method assuming Gaussian noise gives

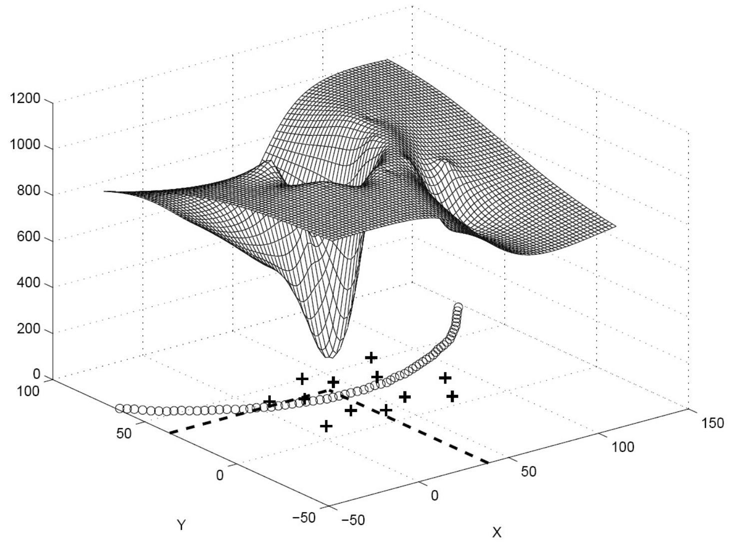
$$\min_{\sigma_p,\theta} V^{GML}(x,\theta,\sigma_p) = \sum_{i=1}^{M} N \log \left(\sum_{k=1}^{N} y_{k,i}^2 - f_i^T(x)\hat{\theta}_i(x) \right)$$

which again can be optimizied over *x*

Field Trials, Skövde



Point Estimate Example



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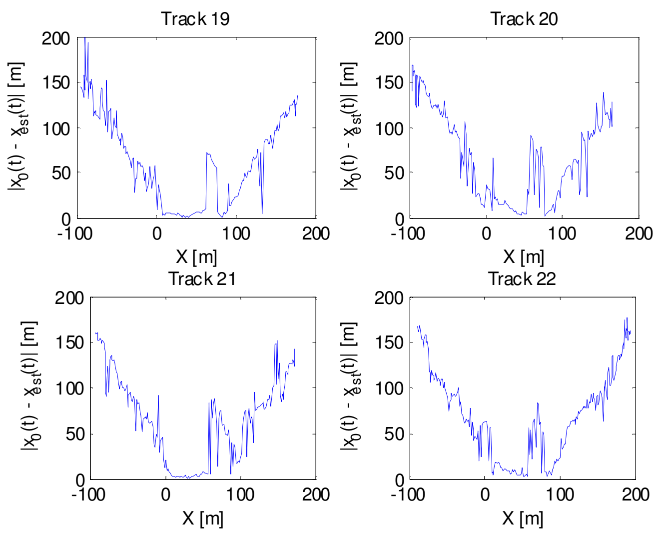
Estimation Performance, 2-Stroke MC



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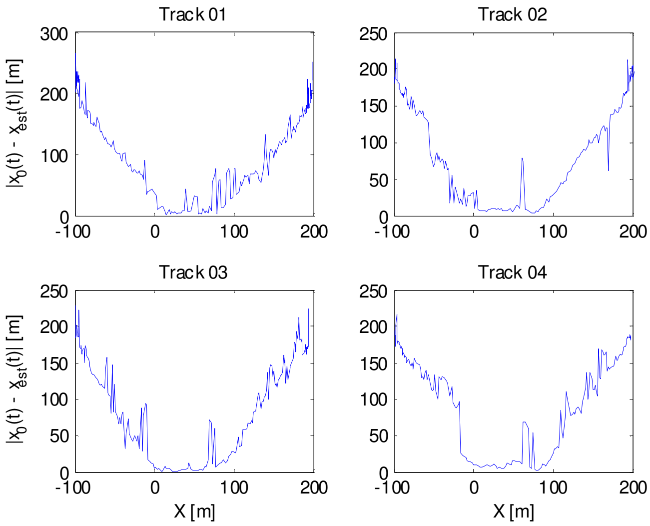
Estimation Perforamance, XC90



Some Issues

- Multi-target tracking
- Pre-filtering
- Filtering and marginalization
- Spatial dependency
- Signature correlation