Adaptive Trellis-Coded Modulation with Imperfect CSI at Receiver and Transmitter, and Receive Antenna Diversity

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This presentation and the paper can also be found at http://www.tele.ntnu.no/projects/beats/

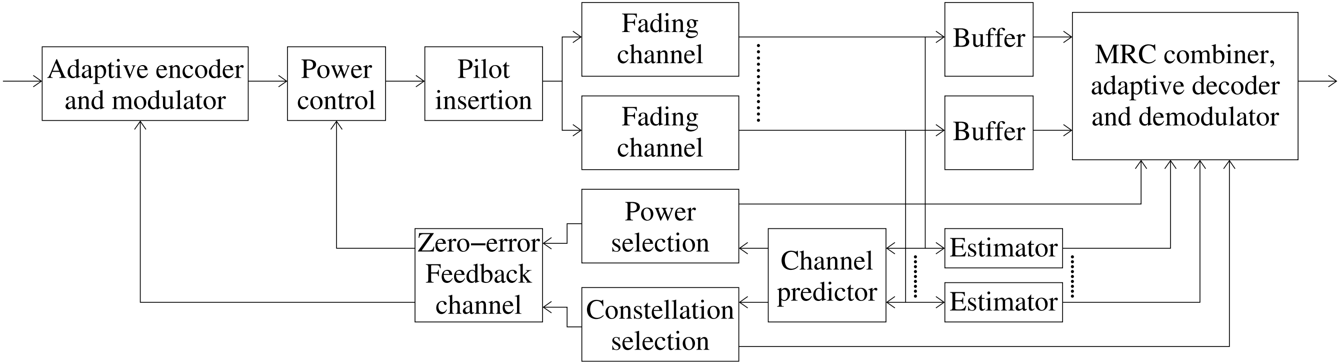
Outline

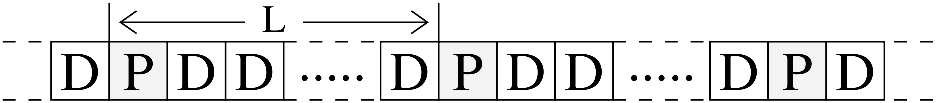
1. Motivation.
2. System model description.
3. BER analysis for both estimation and prediction cases.
4. ASE analysis and the optimization process.
5. Results and concluding remarks.

Motivation

- Adaptive coded modulation (ACM) is a promising technique to enhance *average spectral efficiency* (ASE) of wireless transmission over fading channels.
- Diversity mitigates fading and makes the channel *look* Gaussian-like.
- Constellation size, transmit power are adjusted according to the channel quality, which increases the ASE without wasting or sacrificing error probability performance.
- Non-zero estimation error.
- **Goal**: Maximize ASE while keeping instantaneous BER $\leq$ BER_0 when both estimation and prediction error are considered.

System Overview





$$y_{pl,b}(k) = \sqrt{\mathcal{E}_{pl}} h_b(k;l) s(k;l) + \eta_b(k;l) \qquad l = 0 \qquad (1)$$

$$y_{d,b}(k;l) = \sqrt{\mathcal{E}_d} h_b(k;l) s(k;l) + \eta_b(k;l) \qquad l \in [1, L-1] \qquad (2)$$

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Assumptions

- $E[|s(k;l)|^2] = |s(k;0)|^2 = 1$.
- $h_b(k;l)$ is stationary complex Gaussian RP with zero mean, and variance $\sigma_h^2 = 1$.
- Subchannels are mutually uncorrelated.
- $\eta_b(k;l)$ is complex AWGN with zero mean and variance $N_0/2$ per component, with the components being uncorrelated.
- The channel statistics are known.

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Channel Estimation

- The linear channel estimate of the bth branch:

$$h_{e,b}(k;l) = \mathbf{w}_e^{\mathrm{H}} \mathbf{y}(k), \tag{3}$$

- MSE of the estimation error:

$$\begin{aligned} \sigma^2_{\epsilon_{e,b}} &= 1 - \sqrt{\mathcal{E}_{pl}}\, \mathbf{r}_e^{\mathrm{H}} \mathcal{D}^*(\mathbf{s}) \mathbf{w}_e & (4) \\ &= 1 - \sum_{k=1}^{K_e} \frac{\left| \mathbf{u}_k^{\mathrm{H}} \mathbf{r}_e \right|^2 (1-\alpha) L \bar{\gamma}_b}{(1-\alpha) L \bar{\gamma}_b \lambda_k + 1} & (5) \end{aligned}$$

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Channel Estimation cont'd

- Given $h_{e,b}(k;l)$, we do symbol-by-symbol ML detection. The CSNR of a single branch [1] and after MRC are respectively given by

$$\gamma_b(k;l) = \frac{\mathcal{E}_d \left| h_{e,b}(k;l) \right|^2}{N_0 + g\mathcal{E}_d \sigma^2_{\epsilon_{e,b}}(l)}, \tag{6}$$

$$\gamma(k;l) = \sum_{b=1}^{B} \gamma_b(k;l) = \frac{\mathcal{E}_d}{N_0 + g\mathcal{E}_d \sigma^2_{\epsilon_e}(l)} \sum_{b=1}^{B} \left| h_{e,b}(k;l) \right|^2 \tag{7}$$

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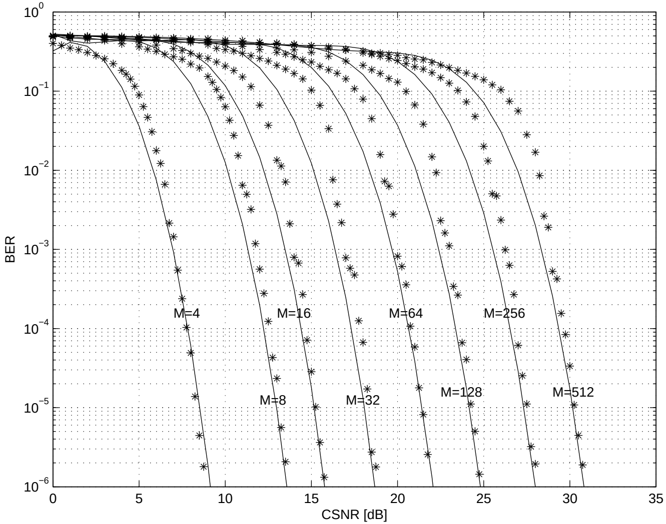
BER Accounts for Estimation Error

- Approximated BER performance for TCM on AWGN channels

$$\mathrm{BER}(e_n|\gamma) = \sum_{i=1}^{I} a_n(i) \exp\left(-\frac{b_n(i)}{M_n}\gamma\right) \tag{8}$$



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BER Accounts for Estimation Error cont'd

- Replacing (7) into (8) we have

$$\mathrm{BER}(e_n|\left\{h_{e,b}\right\}) = \sum_{i=1}^{I} a_n(i) \prod_{b=1}^{B} \exp\left(-A_n \mathcal{E}_d \left|h_{e,b}(k;l)\right|^2\right) \qquad (9)$$

where $A_n = b_n(i)/(M_n(N_0 + g\mathcal{E}_d \sigma^2_{\epsilon_e}(l)))$.

Channel Prediction

- The linear predicted channel of a single branch:

$$h_{p,b}(k;l) = \mathbf{w}_p^{\mathrm{H}} \mathbf{y}(k), \tag{10}$$

- MMSE of the prediction error

$$\sigma_{\epsilon_{p,b}}^2 = 1 - \sum_{k=1}^{K_p} \frac{\left|\mathbf{u}_k^{\mathrm{H}} \mathbf{r}_p\right|^2 (1-\alpha) L \bar{\gamma}_b}{(1-\alpha) L \bar{\gamma}_b \lambda_k + 1} \tag{11}$$

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BER Accounts for both Prediction and Estimation Errors

- Consider the relationship

$$\begin{aligned} h_{e,b}(k;l) &= h_b(k;l) - \epsilon_{e,b}(k;l) \\ &= h_{p,b}(k;l) + \epsilon_{p,b}(k;l) - \epsilon_e(k;l). \end{aligned} \quad (12)$$

- For single antenna sysems, given $h_{p,b}(k;l) \Longrightarrow p\left(\left|h_{e,b}\right| \,\middle|\, h_{p,b}\right) \sim$ Rice distribution with

$$K = \frac{|(1-\rho)h_{p,b}(k;l)|^2}{\tilde{\sigma}^2_{h_{e,b}}},$$

where $\rho =$ correlation between estimation error $\epsilon_{e,b}(k;l)$ and the predicted channel $h_{p,b}(k;l)$ (very small $\Rightarrow$ set equal to 0).

$\tilde{\sigma}^2_{h_{e,b}} \approx \sigma^2_{\epsilon_{e,b}} + \sigma^2_{\epsilon_{p,b}}$ ($\rho = 0, \epsilon_{e,b}(k;l)$ and $\epsilon_{p,b}(k;l)$ uncorrelated).

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BER Accounts for both Prediction and Estimation Errors cont'd

- For multiple uncorrelated receive antenna systems $p\left(\{|h_{e,b}|\} \,\Big|\, \{h_{p,b}\}\right) = \prod_{b=1}^{B} p\left(|h_{e,b}| \,\Big|\, h_{p,b}\right)$
- BER given the set of predicted channel can be obtained as:

$$\mathrm{BER}\left(e_n | \{h_{p,b}\}\right) = \int_0^\infty \cdots \int_0^\infty \mathrm{BER}\left(e_n \Big| \{|h_{e,b}|\}\right) \times p\left(\{|h_{e,b}|\} \,\Big|\, \{h_{p,b}\}\right) d\,|h_{e,1}| \cdots d\,|h_{e,B}| \quad (13)$$

- Let the predicted CSNR on each subchannel be $\hat{\gamma}_b = \bar{\mathcal{E}}_d \left|h_{p,b}(k;l)\right|^2 / N_0$ [2, 3] then the combined CSNR using MRC is

$$\hat{\gamma} = \frac{\bar{\mathcal{E}}_d}{N_0} \sum_{b=1}^{B} \left|h_{p,b}(k;l)\right|^2 \Longrightarrow \bar{\hat{\gamma}} = r\bar{\gamma}_b B \quad \text{where } r = \bar{\mathcal{E}}_d(1-\sigma^2_{\epsilon_p})/\mathcal{E}$$

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BER Accounts for both Prediction and Estimation Errors cont'd

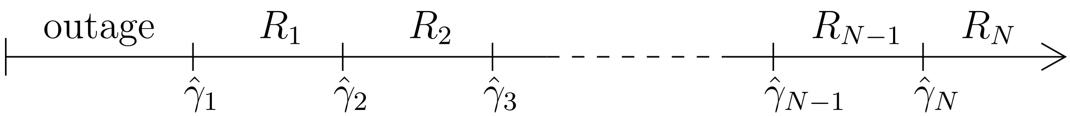
- Assuming $\tilde{\sigma}^2_{h_{e,b}} = \tilde{\sigma}^2_{h_e} \forall b$, and letting $d_n = 1/(A_n \mathcal{E}_d \tilde{\sigma}^2_{h_e} + 1)$, the BER given predicted CSNR is:

$$\mathrm{BER}(e_n|\hat{\gamma}) = \sum_{i=1}^{I} a_n(i) d_n^B \exp\left(-\frac{\hat{\gamma} d_n A_n \mathcal{E}\mathcal{E}_d}{\bar{\gamma}_b \bar{\mathcal{E}}_d}\right) \tag{14}$$

- The overall BER (over all codes)

$$\mathrm{BER} = \frac{\sum_{n=1}^{N} R_n \int_{\hat{\gamma}_n}^{\hat{\gamma}_{n+1}} \mathrm{BER}(e_n|\hat{\gamma}) p(\hat{\gamma}) d\hat{\gamma}}{\sum_{n=1}^{N} R_n P_n}$$

ACM Concept



- Set of codes $\{M_n\}_{n=1}^{N}$ and the corresponding spectral efficiencies (SE) $\{R_n\}_{n=1}^{N}$.
- If the predicted CSNR $\hat{\gamma} \in [\hat{\gamma}_n, \hat{\gamma}_{n+1}\rangle$, the nth code (thus SE R_n) will be used. $\hat{\gamma}_0 = 0$ and $\hat{\gamma}_{N+1} = \infty$.
- $\mathrm{BER}(e_n|\hat{\gamma}) \leq \mathrm{BER}_0$ for all modes.

ACM cont'd

- The average power per data symbol and pilot symbol:

$$\bar{\mathcal{E}}_d = \alpha L \mathcal{E}/(L-1) \quad \text{and} \quad \mathcal{E}_{pl} = (1-\alpha) L \mathcal{E}$$

- Since no transmission when $\hat{\gamma} < \hat{\gamma}_1$

$$\mathcal{E}_d = \frac{\bar{\mathcal{E}}_d}{\int_{\hat{\gamma}_1}^{\infty} p(\hat{\gamma}) d\hat{\gamma}} = \frac{\bar{\mathcal{E}}_d}{Q(B, \hat{\gamma}_1 / r\bar{\gamma})}$$

where $p(\hat{\gamma})$ is the pdf of the predicted CSNR, which is Gamma distributed, and $Q(\cdot,\cdot)$ is *normalized incomplete gamma function*.

ASE Performance

- Average spectral efficiency is given by

$$\begin{aligned} \text{ASE} &= \frac{L-1}{L}\sum_{n=1}^{N} R_n P_n \\ &= \frac{L-1}{L}\sum_{n=1}^{N}\left(\log_2(M_n) - \frac{1}{G}\right) \\ &\quad \times \left\{ Q\left(B, \hat{\gamma}_n / r\bar{\gamma}_b\right) - Q\left(B, \hat{\gamma}_{n+1}/r\bar{\gamma}_b\right) \right\} \end{aligned} \tag{15}$$

- For each value of $L \in \left[2, \lfloor 1/(2 f_d T_s) \rfloor\right]$ [4] we find

$$\begin{aligned} &\max_{\alpha} \ \ \text{ASE}(\alpha) \\ &\text{subject to} \ \ 0 < \alpha < 1 \end{aligned} \tag{16}$$

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Outage Probability

- No data is transmitted when $\hat{\gamma} < \hat{\gamma}_1$, the system suffers an outage of

$$P_{out} = \int_0^{\hat{\gamma}_1} p(\hat{\gamma}) d\hat{\gamma} = 1 - Q(B, \hat{\gamma}_1 / r\bar{\gamma})$$

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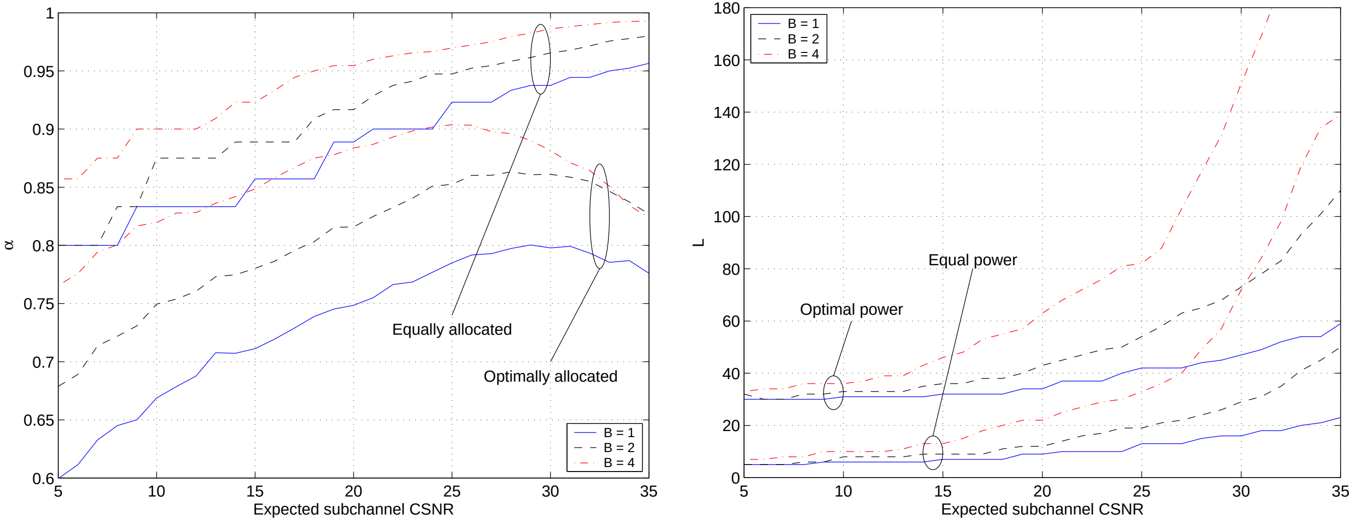
Simulation Parameters

- $\{M_n\} = \{4, 8, 16, 32, 64, 128, 256, 512\}$.
- $f_c = 2$ GHz.
- Symbol period $T_s = 5\ \mu s$.
- Mobile velocity $v = 30$ m/s.
- System delay $\tau = DLT_s = 1$ ms (or $\tau = 0.2/f_d$).
- Estimator order $K_e = 20$, and predictor order $K_p = 250$.
- $\text{BER}_0 = 10^{-5}$.
- $B = \{1, 2, 4\}$

Results: Optimum α and L



(a) Optimum power allocation

(b) Optimum pilot symbol spacing

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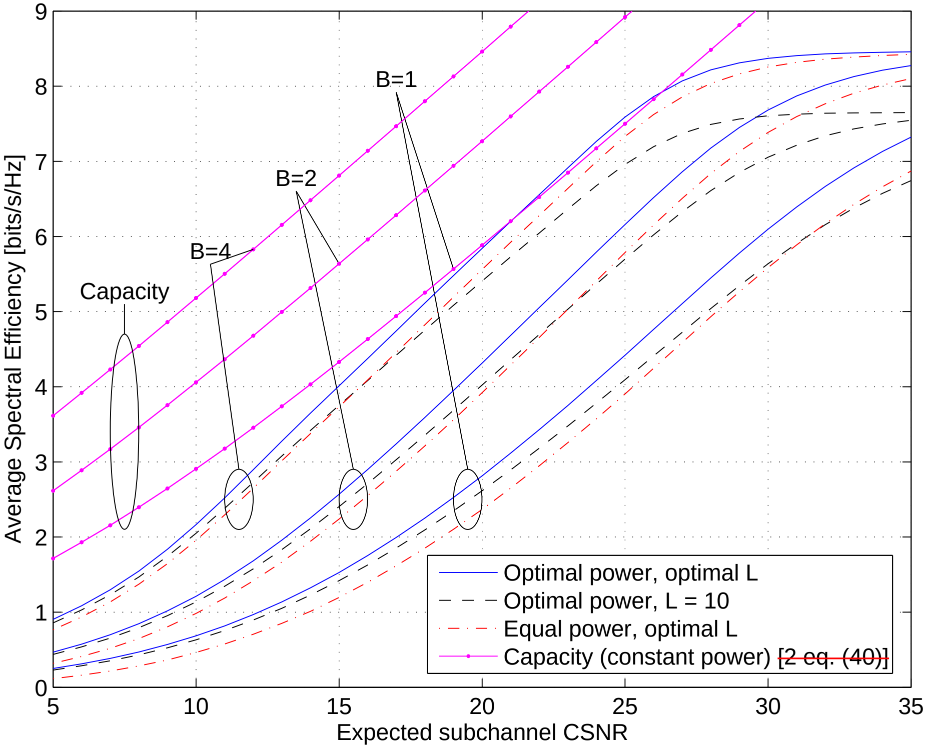
Results: ASE



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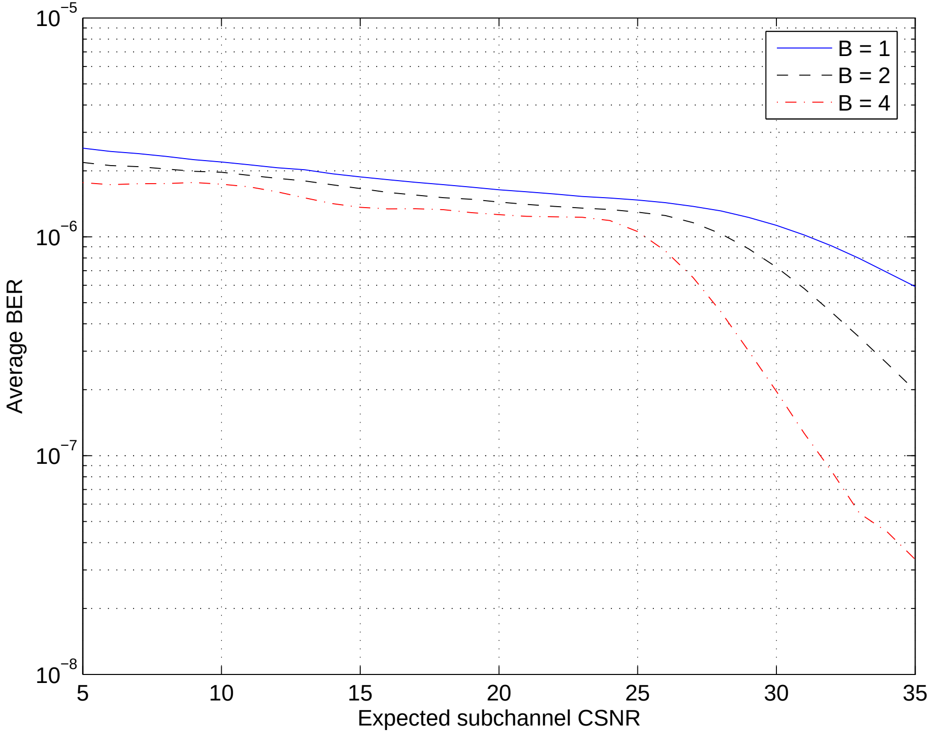
Results: BER



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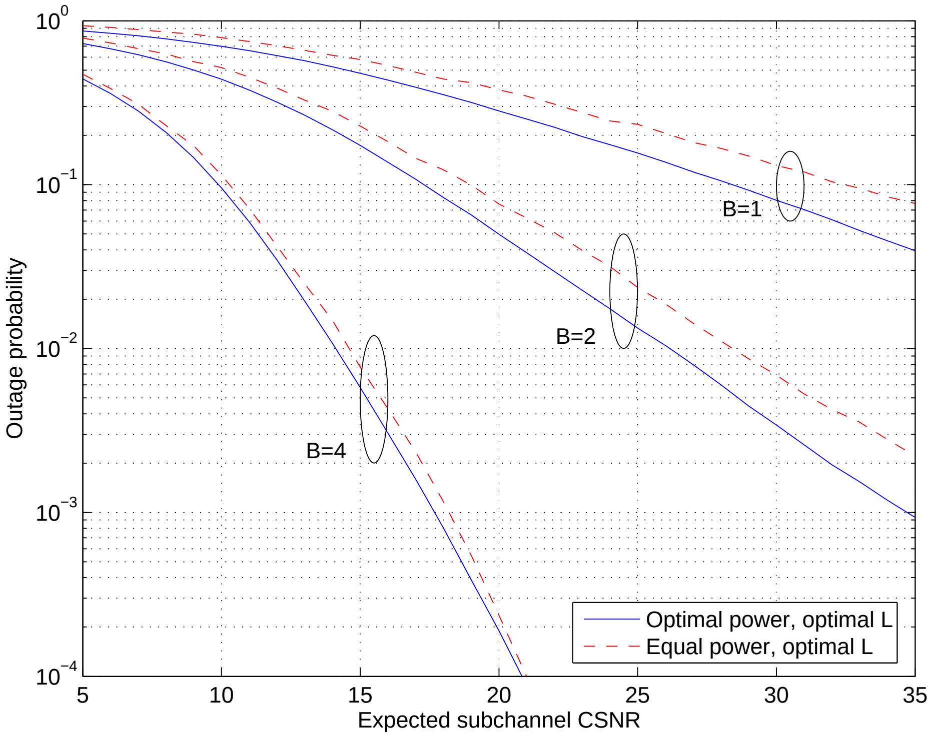
Results: Outage probability



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Conclusion

- Adaptive trellis-coded PSAM with receive antenna diversity is investigated.
- The pilot symbol period and power allocation between pilot and data symbols were optimized to maximize ASE.
- We achieved higher ASE and lower probability of outage without sacrificing BER performance.

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