On the Performance Limits of Joint Source-Channel Coding Using Subband Decomposition and OFDM Transmission

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Intro

Goal: To make a **joint source-channel coding** (JSSC) system

Why? JSSC to make efficient and robust system with low delay

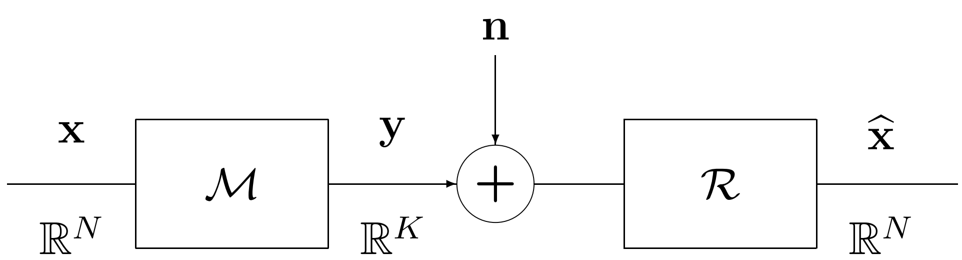
How?

- Signal decomposition by subband filtering
- Channel representation by OFDM
- In between: Dimension changing mappings

More or less symmetric system

How does the structure imposed degrade compared to the optimal system?

One channel discrete time JSSC system



Transmit vector $\mathbf{x}$ with N components using channel representation $\mathbf{y}$ with K components.

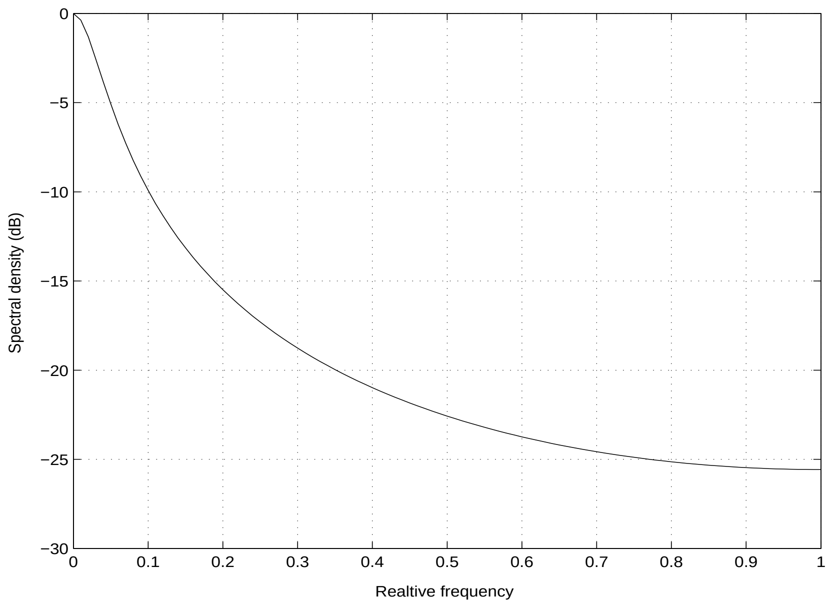
- Bandwidth compression: $K < N$
- Bandwidth expansion: $K > N$

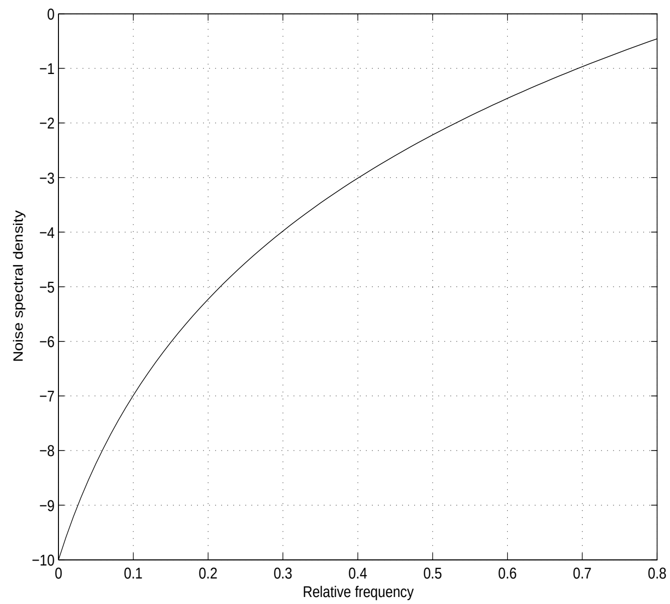
Bandwidth compression generally introduces approximation noise

For expansion approximation noise can be avoided

Received and decoded signal: $\hat{\mathbf{x}} = \mathcal{R} \circ (\mathcal{M} \circ \mathbf{x} + \mathbf{n})$, $\quad (\mathcal{R} \approx \mathcal{M}^{-1})$

Example where bandwidths of signal and channel are different





Performance limit: OPTA

Calculating the source *rate-distortion function* at the *channel capacity* renders OPTA (optimal performance theoretically attainable)

Assumptions:

- The signal is Gaussian with a known non-white spectrum
- The channel is additive, Gaussian with a known noise spectrum and attenuation (assumption: constant attenuation)
- The signals and channels are time discrete (no loss in performance due to A/D and D/A processes)

Rate distortion function

Rate (in bits per second):

$$R(\lambda) = \int_{-W_s}^{W_s} \max \left\{ 0, \frac{1}{2} \log_2 \frac{S_{XX}(F)}{\lambda} \right\} dF$$

Corresponding distortion:

$$D(\lambda) = \int_{-W_s}^{W_s} \min \left\{ \lambda, S_{XX}(F) \right\} dF$$

By selecting a value of λ a rate and the corresponding distortion result

Channel capacity

The capacity in bits per second is given by

$$C(\theta) = \int_{-W_c}^{W_c} \frac{1}{2} \log_2 \left\{ 1 + \frac{(\theta - S_{NN}(F))^+}{S_{NN}(F)} \right\} dF,$$

where the parameter θ is found from the power constraint

$$\int_{-W_c}^{W_c} (\theta - S_{NN}(F))^+ \, dF = P$$

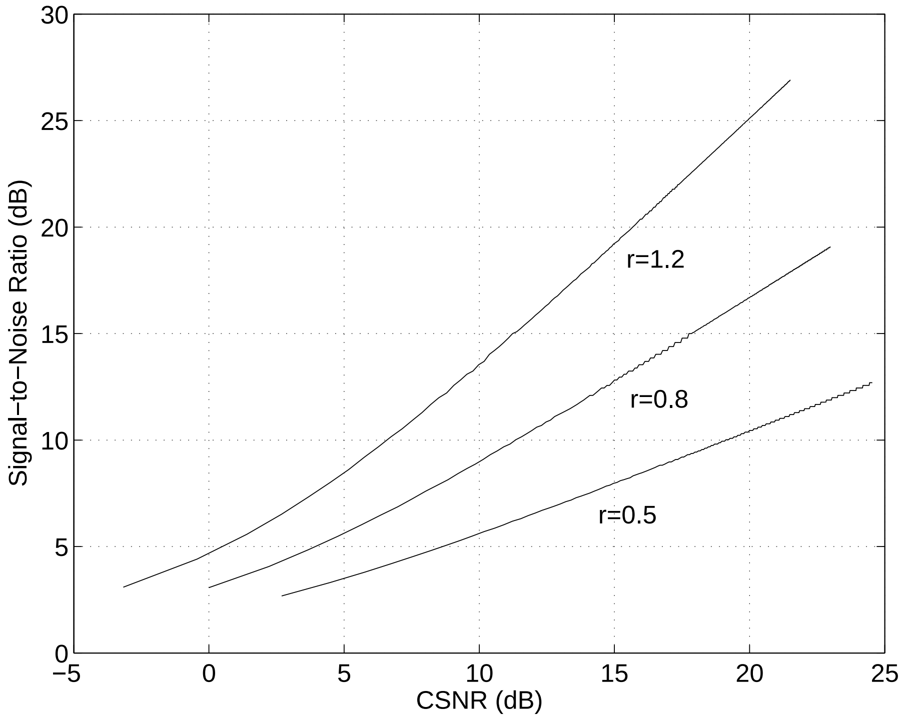
The following definition has been used:

$$(x)^+ = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ otherwise} \end{cases}$$

OPTA curves (example)

Signal: AR(1) process with $R_{XX}(k) = \sigma_X^2 \rho^{-|k|}$, $\rho = 0.9$ (bandwidth=1)

Channel noise: $S_{NN}(f) = a * (0.1 + f)$, $f \in [0, r]$ (r: channel bandwidth)



Special case

White signal and noise:

If the signal has bandwidth B and variance (σ_X^2) higher than the signal noise variance (σ_D^2), then

$$R = B \log_2 \left(\frac{\sigma_X^2}{\sigma_D^2} \right),$$

and the channel power density (S) is higher than the channel noise density (N), then

$$C = W \log_2 \left(1 + \frac{WS}{WN} \right)$$

Equating rate with capacity and solving with respect to the distortion, we obtain

$$\sigma_D^2 = \sigma_X^2 \left(1 + \frac{S}{N} \right)^{-r}, \; (r = W/B)$$

Comments

OPTA does not indicate a structure for making systems with good quality

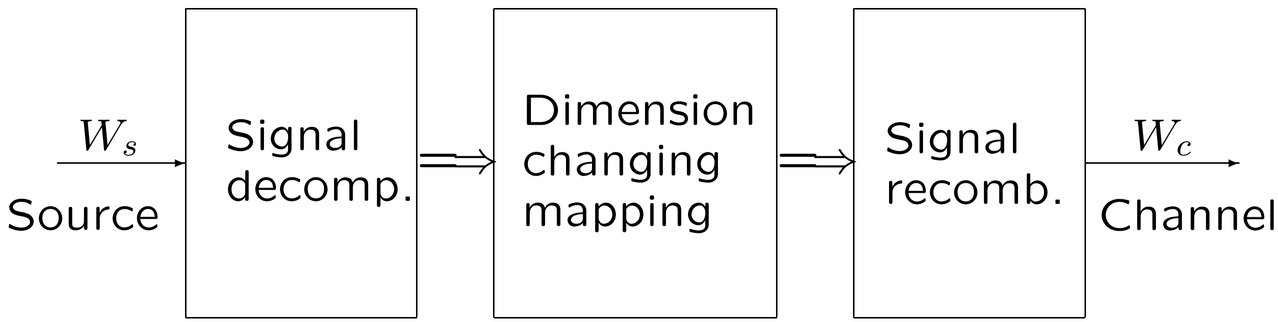
Shannon's separation theorem states that source coding and channel coding can be done separately without loss of optimality

But:

- Optimal source coding requires infinite dimensional Vector Quantizer
- Optimal channel coding requires e.g. a turbo-like coder with infinite delay and complexity

We will look at alternative structures for getting close to OPTA

Structured system



- Signal decomposition by transform or filter bank
- Dimension change by nonlinear mapping
- Signal recombination by OFDM

Comments

Source decomposition results in several sub-sources with different statistics (mainly different variances)

Each sub-source is to be transmitted on sub-channel where each channel has different statistics

Goal: By constraining the overall bandwidth and the total power, how do we allocate the available resources to the sub-channels?

Result obtained is relevant for a single user, but can be applied to multiuser system if average SNR is the optimization criterion

Towards the SBC-OFDM system: Assumptions

- Split signal by ideal uniform filter bank into M subbands
  - The split is so fine that the spectrum is constant within each subband
- Map each subband individually to a part of the channel using optimal bandwidth and power in order to fill total channel bandwidth and exactly use allowed power
  - Each part of the channel has constant noise spectrum and attenuation
  - Assume OPTA performance for each mapping

Mathematical formulation

Assume that we know the channel noise level experienced by each signal component

Total channel power:

$$\sum_{m=1}^{M} \sigma_{C_m}^2 = \sum_{m=1}^{M} S_m W_m = \sum_{m=1}^{M} S_m B r_m \leq P_{tot}$$

Total bandwidth:

$$\sum_{m=1}^{M} W_m = \sum_{m=1}^{M} B r_m = M B r_{avg} = W_{tot}$$

Optimization using the following object function:

$$O = \sum_{m=1}^{M} \sigma_{D_m}^2 + \lambda_1 \sum_{m=1}^{M} S_m r_m + \lambda_2 \sum_{m=1}^{M} r_m$$

Optimal values

The optimization leads to:

Power density relations:

$$S_j + (S_j + N_j)\log\left(1 + \frac{S_j}{N_j}\right) = \quad S_k + (S_k + N_k)\log\left(1 + \frac{S_k}{N_k}\right)$$

Rates:

$$r_j = \frac{c_a}{c_j}\left[r_{avg} + \frac{1}{2M}\sum_{k=1}^{M}\frac{1}{c_k}\log_2\left(\frac{\sigma^2_{X_k}}{\sigma^2_{X_j}}\frac{N_j + S_j}{N_k + S_k}\right)\right].$$

where

$$c_m = \frac{1}{2}\log_2\left(1 + \frac{S_m}{N_m}\right) \text{ and } \frac{1}{c_a} = \left(\frac{1}{M}\sum_{j=1}^{M}\frac{1}{c_j}\right)$$

Water filling

How does water filling relate to this result?

Optimal when:

- all the channels have equal bandwidths
- all the sources have equal variances

Single user system

The present model assumes a constant noise level in each channel

If we have a continuous channel which is split into sub-channels, we do not a priori know what noise densities to use

Strategy:

- Assume a uniform distribution of bandwidths and use the noise density at the midpoint to represent each channel band
- Use the formulas to find the corresponding bandwidths if the assumption were true
- Split the channel into bands according to the obtained bandwidths and find midpoint noise densities
- Calculate new densities and bandwidths
- Continue process until convergence

Example

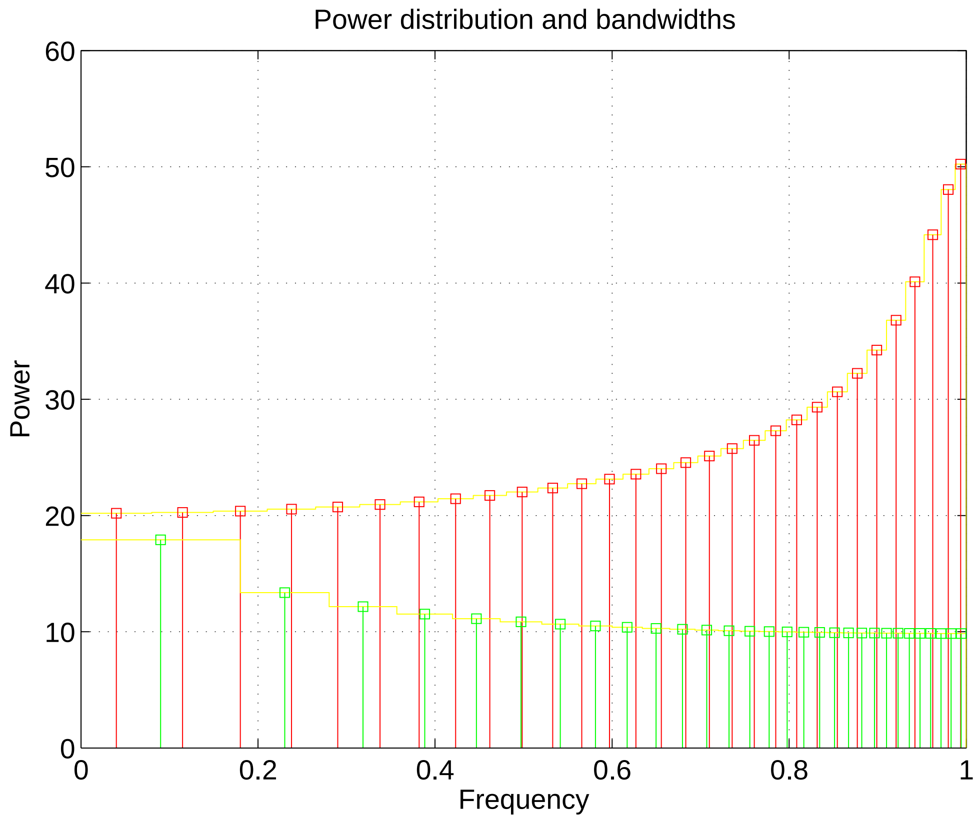
Signal: AR(1) process with $\rho = 0.9$

Noise: Same or reversed spectrum (Channel capacity equal for both)

Number of channels: 30

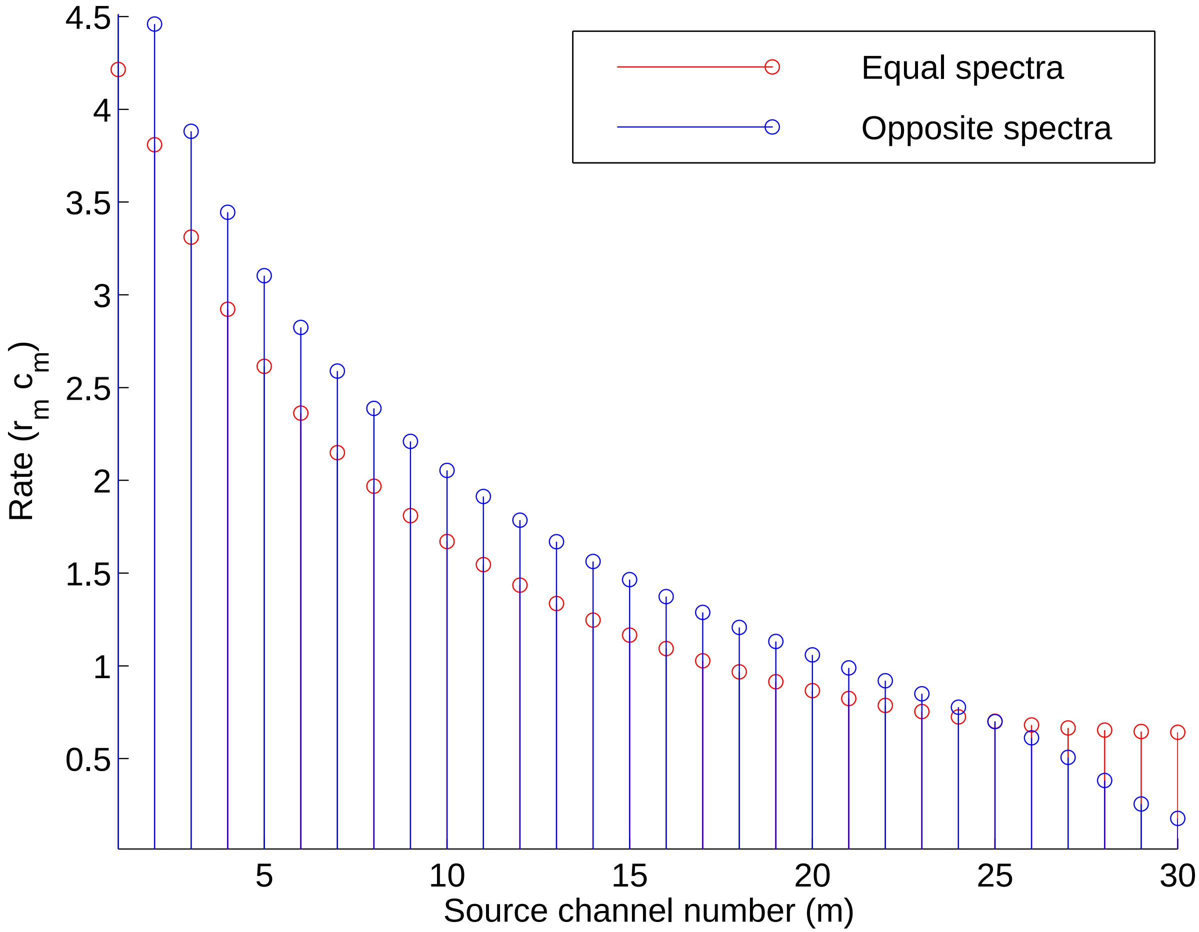
Rate (bandwidth) change: r=0.5

Power and bandwidth distributions



Red: Opposite spectra
Green: Equal spectra

Corresponding rates



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Constant power

Relatively small power variations

By inspecting the Lagrange multipliers, the one related to bandwidth dominates

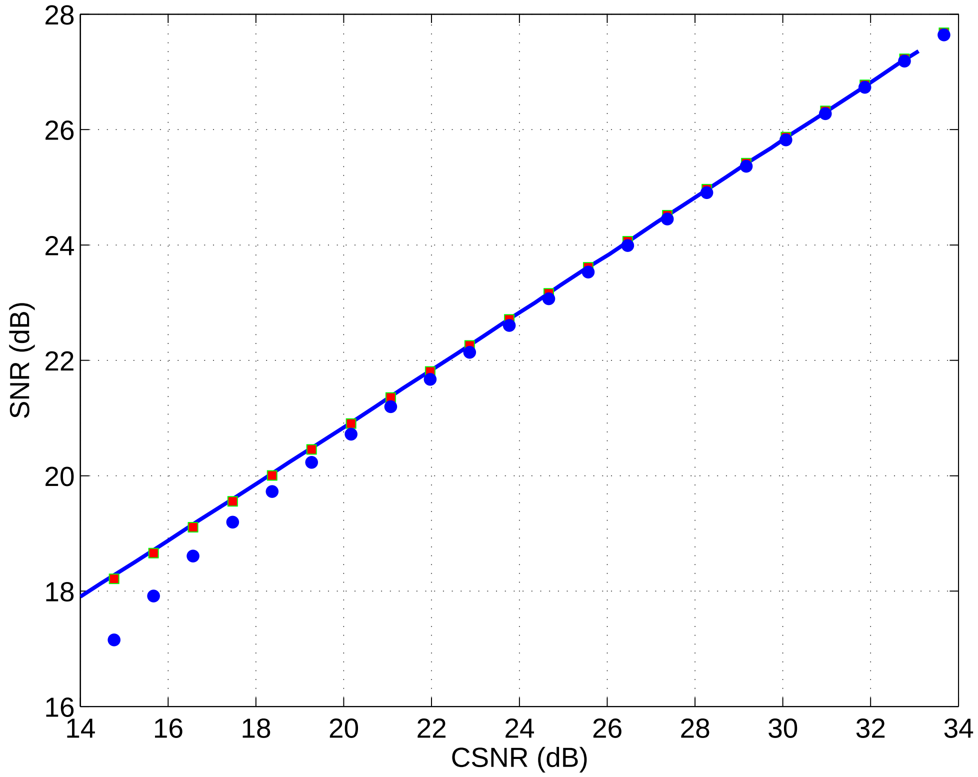
From these observations it is tempting to optimize without the power constraint (but still adjust the power)

Resulting rate formula:

$$r_k = \frac{c_a}{c_k}\left[r_{avg} + \frac{1}{2M}\sum_{j=1}^{M}\frac{1}{c_j}\log_2\left(\frac{\sigma^2_{X_k}}{\sigma^2_{X_j}}\frac{c_k}{c_j}\right)\right]$$

Loss compared to OPTA

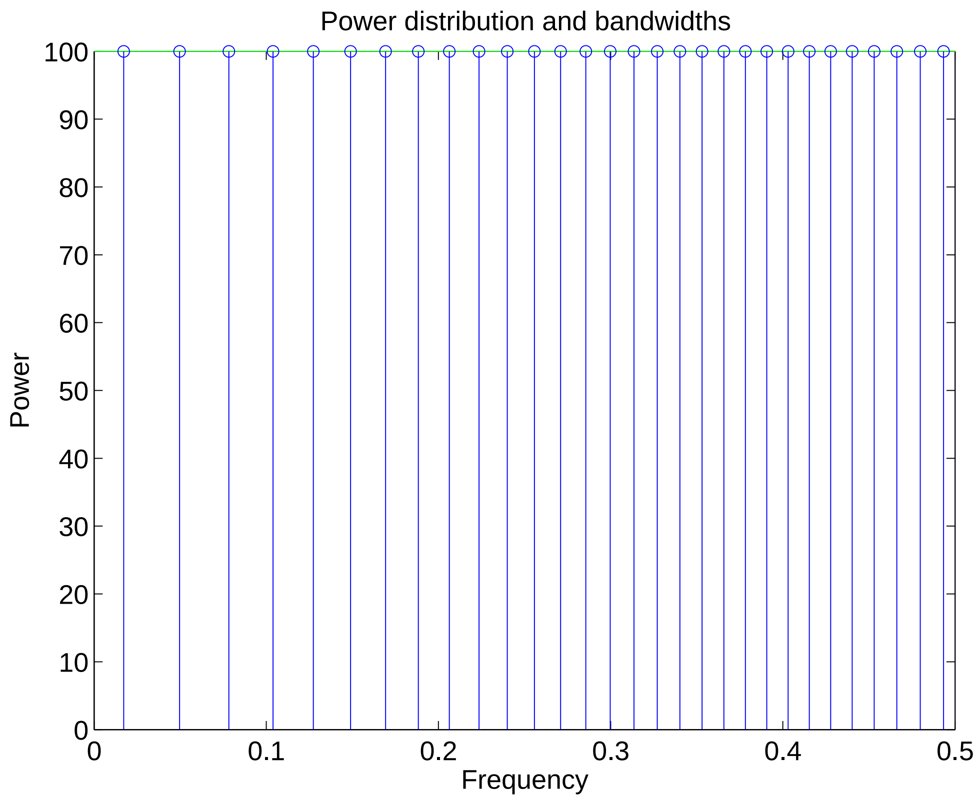
1000 channels (close to optimal)



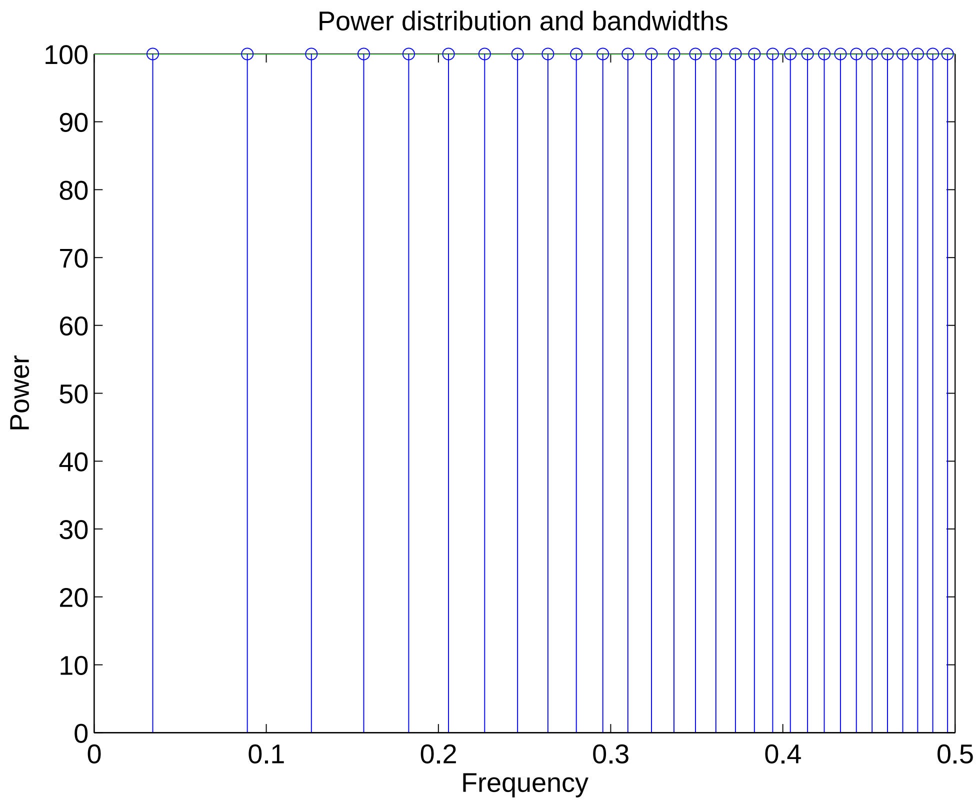
Red squares: Equal signal and noise spectra
Blue dots: Noise spectrum turned around

Bandwidths

Opposite spectra, 30 channels

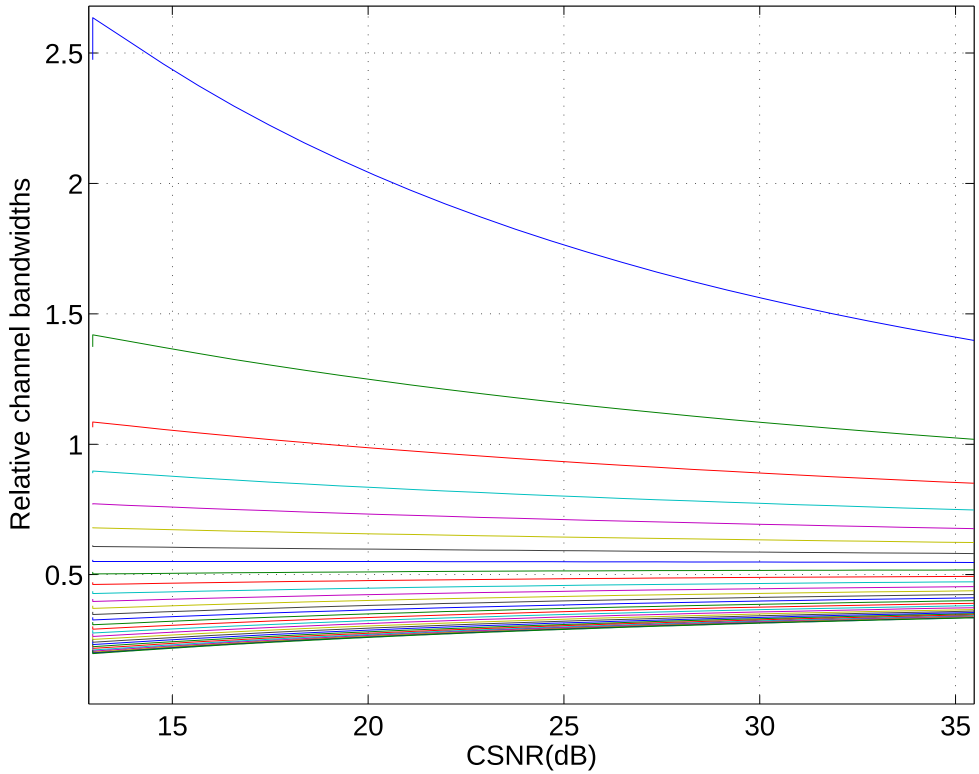


Equal spectra, 30 channels



Bandwidths as function of CSNR

Equal spectra, 30 channels



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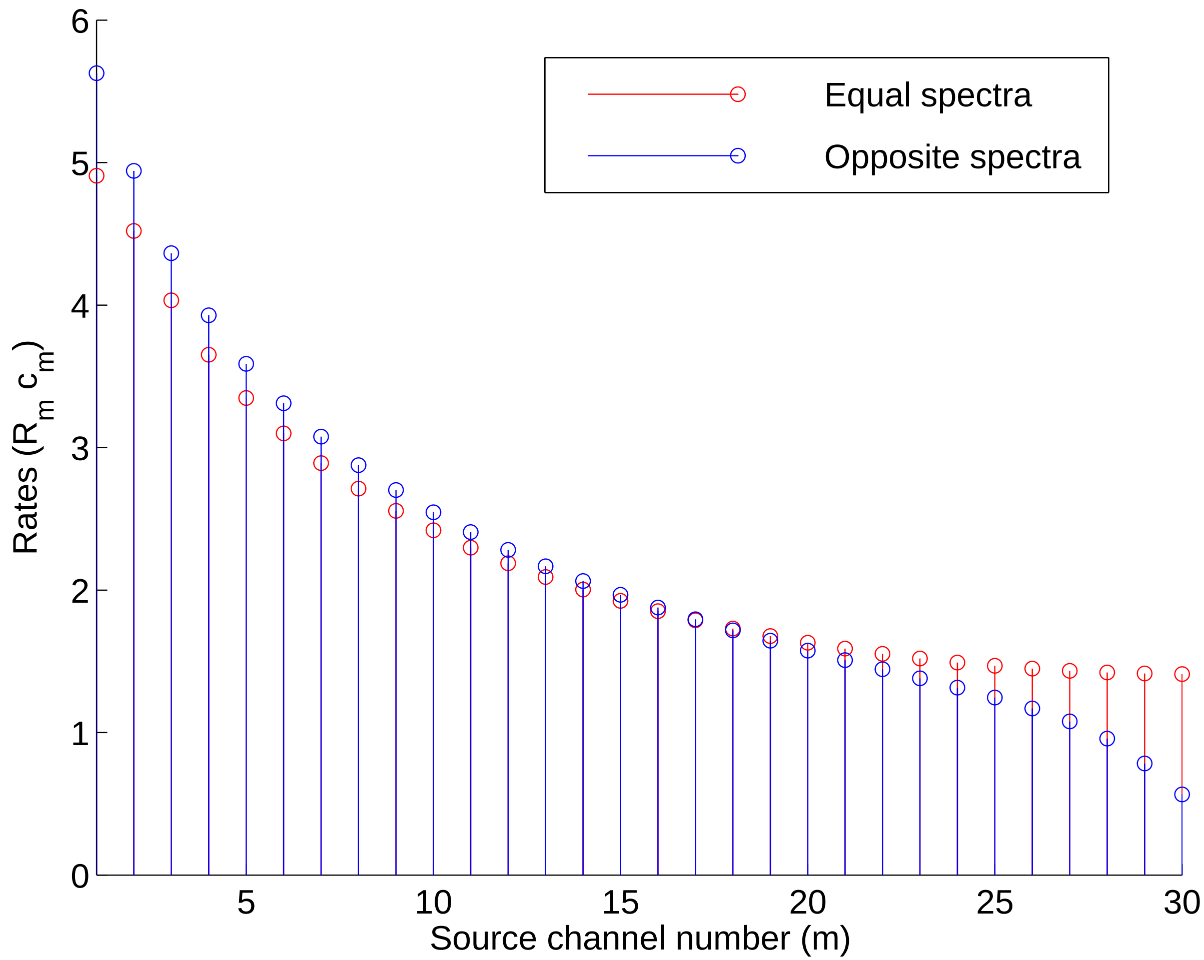
Corresponding rates



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Further practical considerations

The above theory could be used as an approximation if the OFDM channels are quite narrow, that is each OFDM channel is much smaller than W_m, $m = 1, 2, \ldots, M$ to get approximately correct bandwidths

On the other hand, the noise must be the same in all the OFDM channels for the theory to be exact

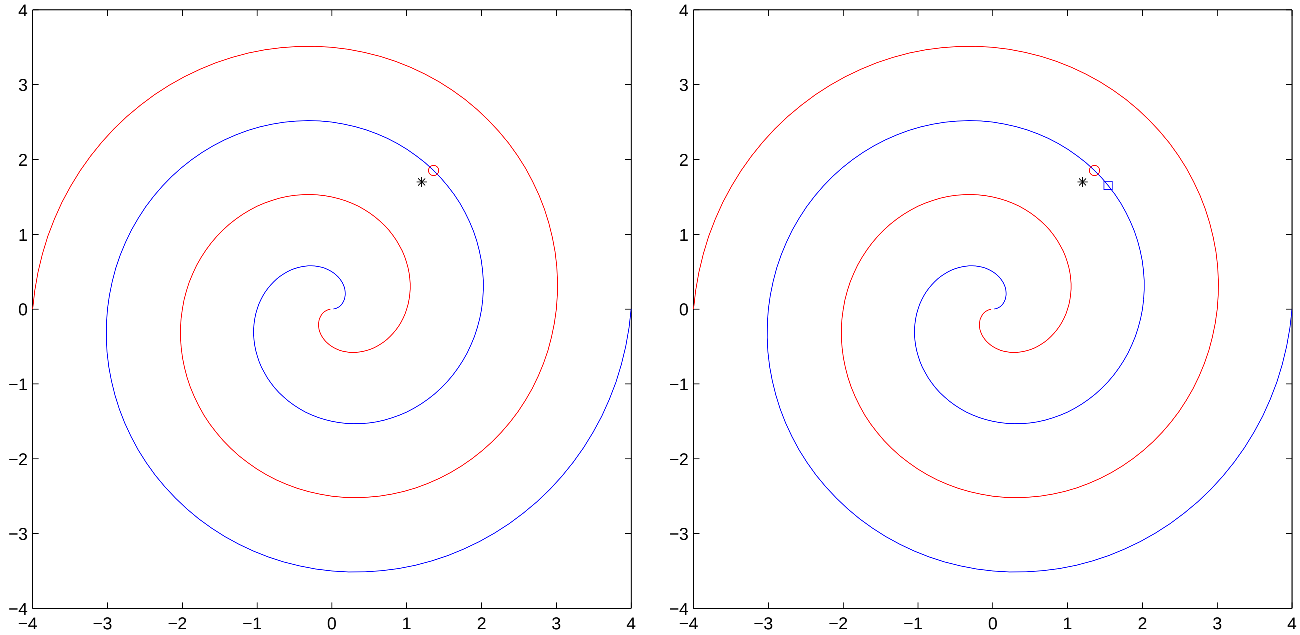
If not, the bandwidths must be quantized while preserving total bandwidth

Only a small set of dimension changing mappings will be available

Practical non-linear mapping will lead to further degradation

Mapping example: 2:1

2$\times$ spiral of Archimedes



Possible channel repr.: Distance from origin used as PAM channel samples:

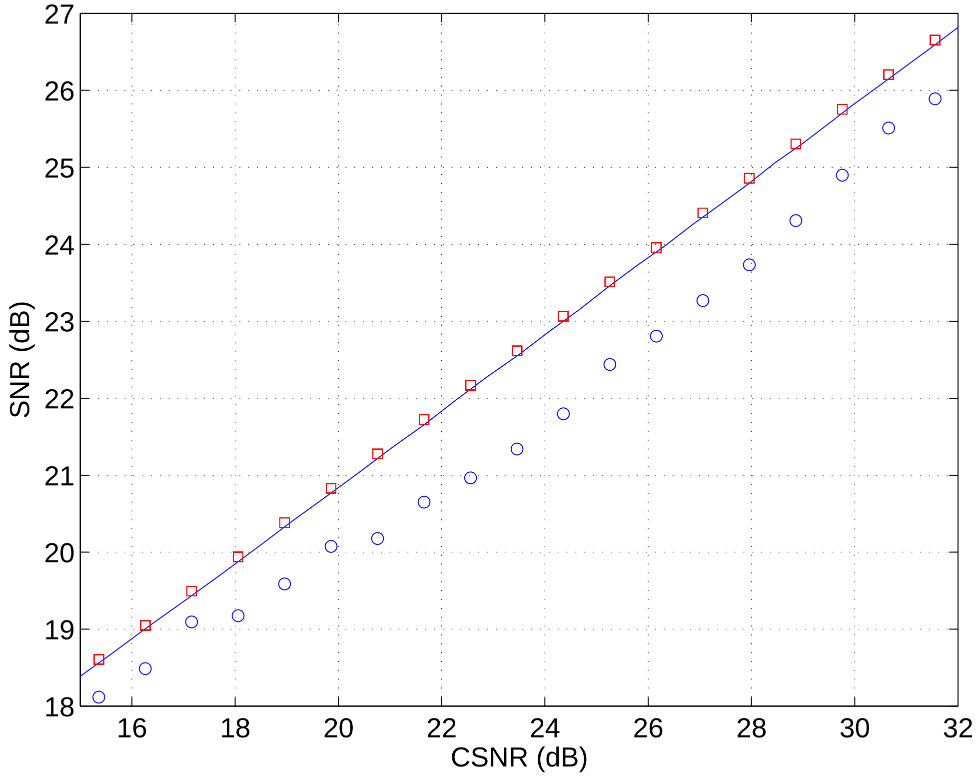
blue curve: positive amplitudes

red curve: negative amplitudes

Loss: approx. 1.5 dB compared to OPTA

Finite number of mappings (preliminary result)

Use rates 3, 2, 1, 3/2, 1/2, 1/3, 1/4, 0



Even greater loss using practical mappings

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Further work

Develop bandwidth allocation procedure when a small set of mappings is available

Look at alternative ways of optimization (Calculus of variations)

Design system: Subband decomposition-mappings-OFDM including adaptive allocation of maps

Channel estimation: Theory requires exact channel knowledge. Use pilots to find approximate channel states and transmit states in feedback channel. Find optimal optimal number of pilots and their power levels