Modelling and Simulation of MIMO Channels

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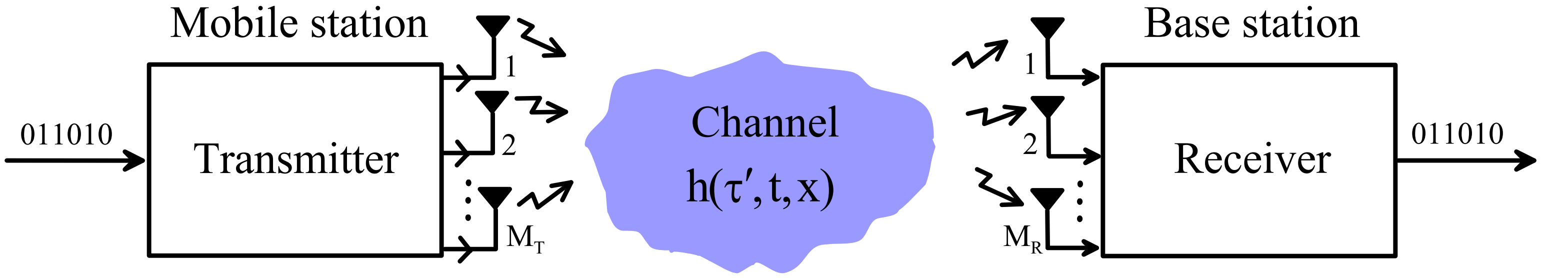
I. Introduction

Mobile Communications Using Smart Antennas



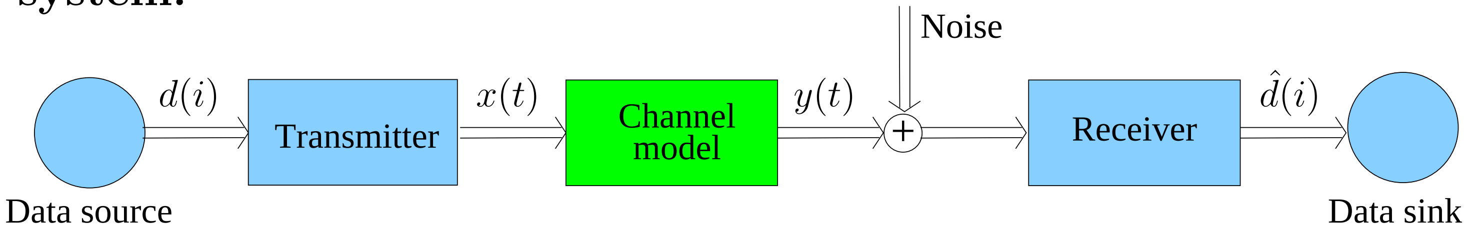
- Space-time coding
- Modulation

- Space-time signal processing
- Demodulation
- Decoding

Realistic spatio-temporal channel models are required for the design, test, and optimization of mobile communication systems employing smart antenna technologies.

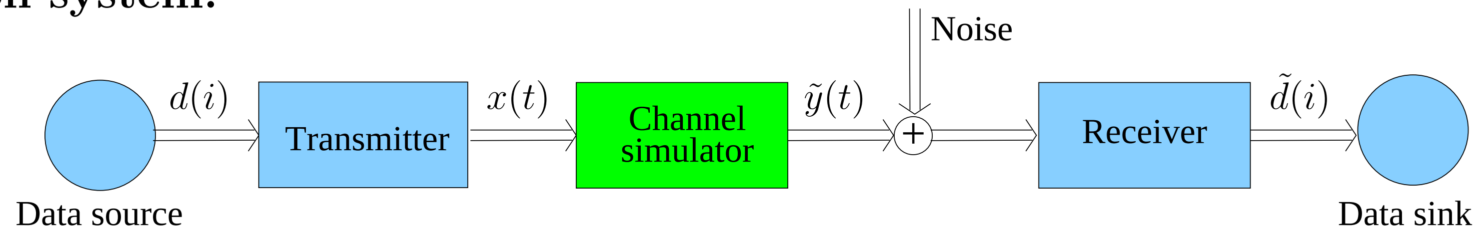
Meaning of Channel Modelling and Simulation for Mobile Communications

Reference system:



$\Rightarrow$ Performance measure: Bit error probability $P_b = f\left(\frac{E_b}{N_0}\right)$

Simulation system:



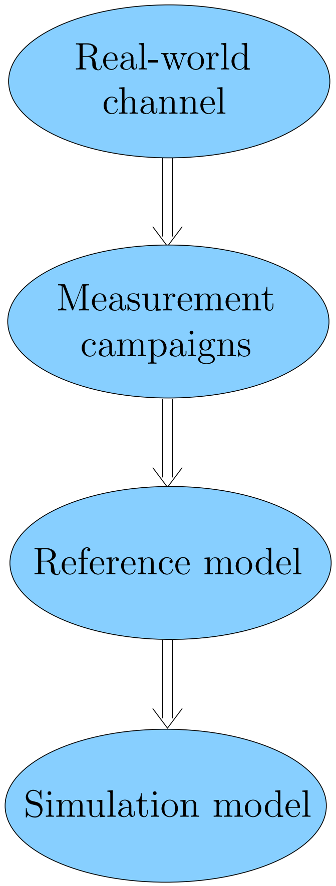
$\Rightarrow$ Performance measure: Bit error probability $\tilde{P}_b = f\left(\frac{E_b}{N_0}, \underset{\substack{\uparrow \\ \text{Model error}}}{\Delta\beta}\right) \approx P_b$

- The model error $\Delta\beta$ describes the essential error which is introduced by the channel simulator.

> Channel simulators are important for the test, the parameter optimization, and the performance analysis of mobile communication systems.

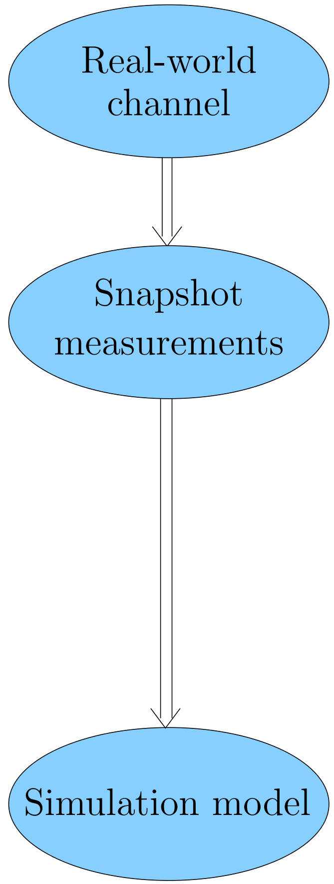
The Two Main Categories of Fading Channel Simulators

Simulators based on reference models:



Problem: Reference models are not close to real-world channels.

Simulators based on measurements:



Problem: Finding of typical scenarios.

$\Rightarrow$ The dilemma of mobile fading channel modelling.

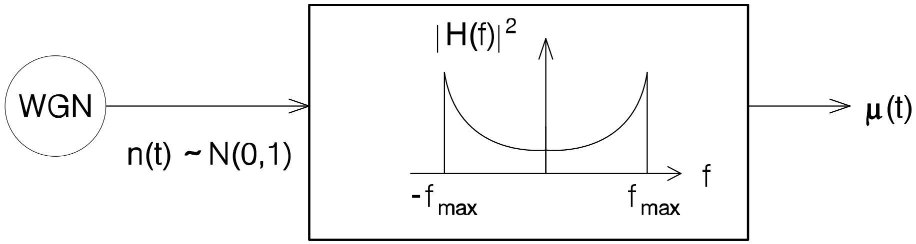
Some Requirements for Fading Channel Simulators

- High precision w.r.t. a given reference model or w.r.t. measured channels
- High efficiency
- The underlying stochastic channel simulator must be ergodic to reduce the number of trials
- Easy to determine the model parameters
- Easy to understand and easy to implement
- Reproducibility for enabling fair performance comparisons
- Easy to extend (frequency selectivity, spatial selectivity, multi-cluster propagation scenarios, number of antenna elements, etc.)
- Low complexity

II. Principles of Fading Channel Modelling

Two Fundamental Methods for Modelling of Coloured Gaussian Noise Processes

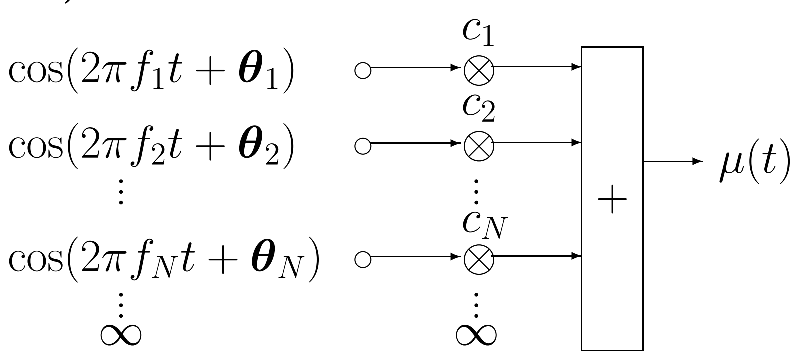
1. Filter method:



$\Rightarrow$ Stochastic process $\mu(t)$

$\Rightarrow$ Reference model

2. Sum-of-sinusoids (SOS) method:



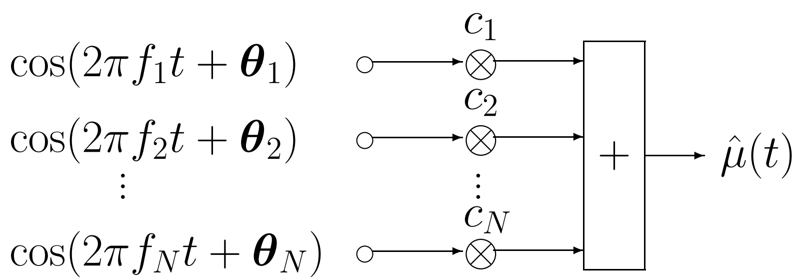
$\Rightarrow$ Stochastic process $\mu(t)$

$\Rightarrow$ Reference model

For a finite number of harmonic functions N, we obtain:

Parameters: ensemble

$$\begin{aligned} \boldsymbol{\theta}_n &= \text{Random variable (RV)} \\ f_n &= \text{const.} \\ c_n &= \text{const.} \end{aligned}$$

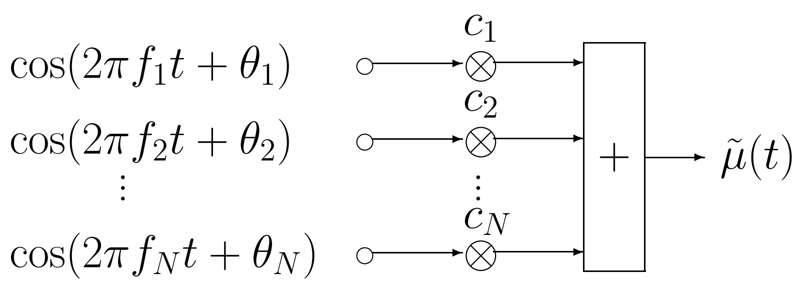


$\Rightarrow$ Stochastic process $\hat{\mu}(t)$

$\Rightarrow$ Stochastic simulation model

Parameters: single realization

$$\begin{aligned} \theta_n &= \text{const.} \\ f_n &= \text{const.} \\ c_n &= \text{const.} \end{aligned}$$



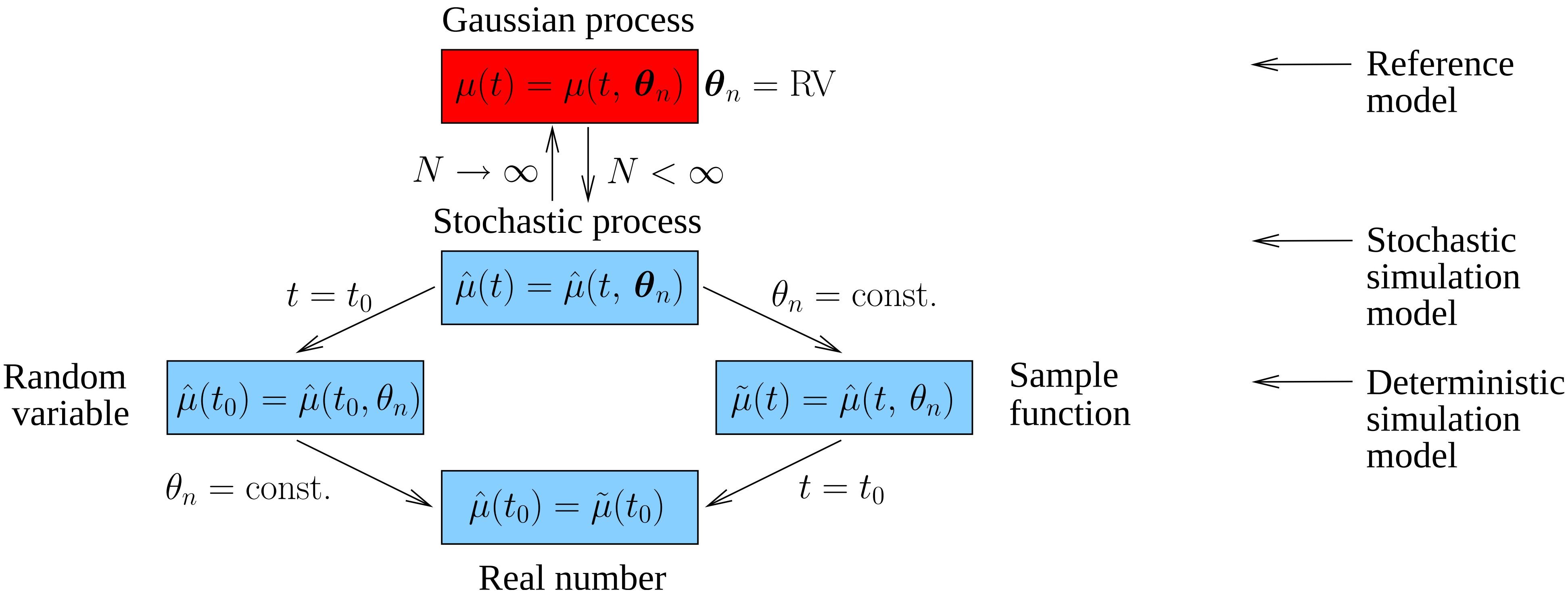
$\Rightarrow$ Deterministic process $\tilde{\mu}(t)$

$\Rightarrow$ Deterministic simulation model

Relationships Between Stochastic Processes, Random Variables, Sample Functions, and Real-Valued Numbers

Gaussian process: $\mu(t) = \lim_{N\to\infty} \sum_{n=1}^{N} c_n \cos(2\pi f_n t + \boldsymbol{\theta}_n)$ $\quad (c_n = \text{const.},\ f_n = \text{const.},\ \boldsymbol{\theta}_n = \text{RV})$

Stochastic process: $\hat{\mu}(t) = \sum_{n=1}^{N} c_n \cos(2\pi f_n t + \boldsymbol{\theta}_n)$ $\quad (c_n = \text{const.},\ f_n = \text{const.},\ \boldsymbol{\theta}_n = \text{RV})$



Principle of Deterministic Channel Modelling

Step 1: Starting point is a reference model derived from one, two or several Gaussian processes.

Step 2: Derive a stochastic simulation model from the reference model by replacing each Gaussian process by a sum-of-sinusoids with fixed gains, fixed frequencies, and random phases.

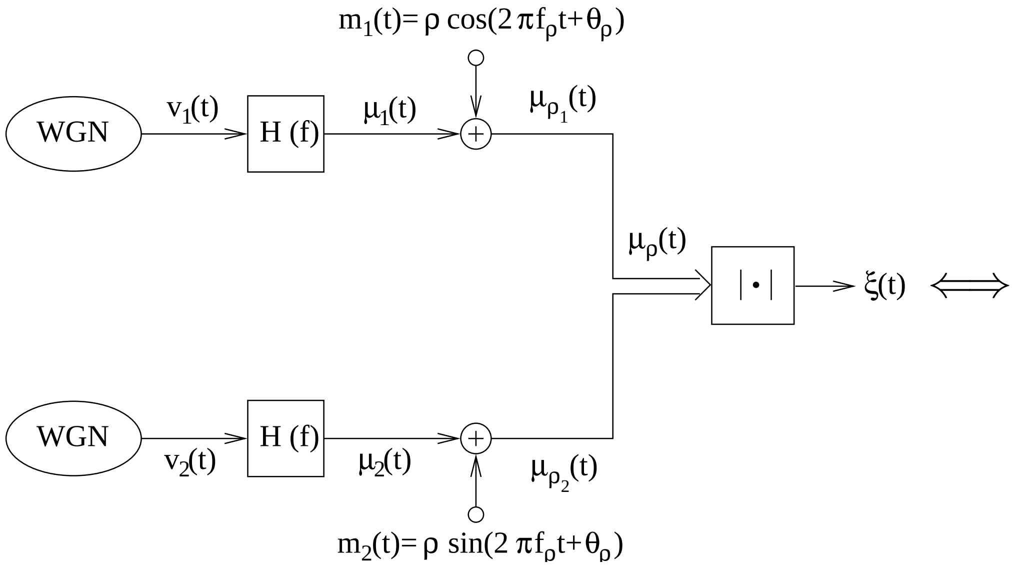
Step 3: Determine the deterministic simulation model by fixing all model parameters of the stochastic simulation model, including the phases.

Step 4: Compute the model parameters of the simulation model by using a proper parameter computation method.

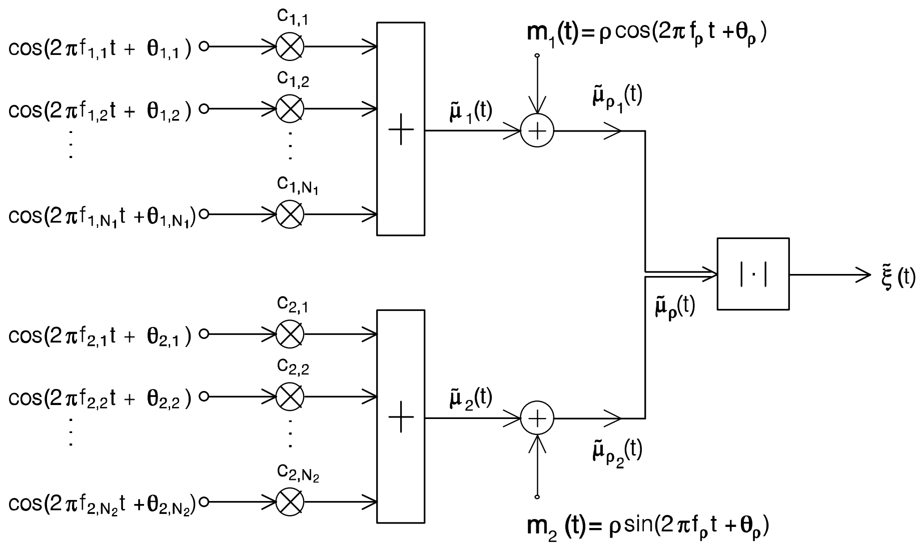
Step 5: Simulate one (or some few) sample functions (deterministic processes).

Example: Derivation of a channel simulator for Rice processes

Step 1: **Reference model**



Steps 2 & 3: **Simulation model**



$\Rightarrow$ Stochastic Rice process $\xi(t)$

$\Rightarrow$ Deterministic Rice process $\tilde{\xi}(t)$

Step 4: Compute the model parameters by fitting the statistics of the deterministic simulation model to those of the stochastic reference model.

Step 5: Simulate the deterministic Rice process $\tilde{\xi}(t)$.

Methods for the Calculation of the Model Parameters

Deterministic process: $\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n})$

Model parameters: gains ($c_{i,n}$), frequencies ($f_{i,n}$), phases ($\theta_{i,n}$)

Historical overview

Methods	Disadvantages
Rice method (Rice, 1944)	Small period
Jakes method (Jakes, 1974)	Special case only
Monte Carlo method (Schulze, 1988)	Non-ergodic
Method of equal distances (Pätzold, 1994)	Small period
Method of equal areas (Pätzold, 1994)	Slow convergency
Harmonic decomposition technique (Crespo, 1995)	Small period
Mean-square-error method (Pätzold, 1996)	Small period
Method of exact Doppler spread (Pätzold, 1996)	Special case only
L_p-norm method (Pätzold, 1996)	–
Method porposed by Zheng and Xiao (2002)	Non-ergodic

Classes of SOS Channel Simulators and Their Statistical Properties

Deterministic process: $\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n})$

Class	Gains $c_{i,n}$	Frequencies $f_{i,n}$	Phases $\theta_{i,n}$	First-order stationary	Wide-sense stationary	Mean-ergodic	Autocor.-ergodic
I	const.	const.	const.	–	–	–	–
II	const.	const.	RV	yes	yes	yes	yes
III	const.	RV	const.	no/yes [a]	no/yes [a]	no/ yes [a]	no
IV	const.	RV	RV	yes	yes	yes	no
V	RV	const.	const.	no	no	no/yes [a]	no
VI	RV	const.	RV	yes	yes	yes	no
VII	RV	RV	const.	no/yes [a]	no/yes [a]	no/yes [a]	no
VIII	RV	RV	RV	yes	yes	yes	no

[a] If certain boundary conditions are fulfilled.

Application of the Scheme to Parameter Computation Methods

Stochastic process: $\hat{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \boldsymbol{\theta}_{i,n})$

Model parameters: gains, frequencies, phases (RVs)

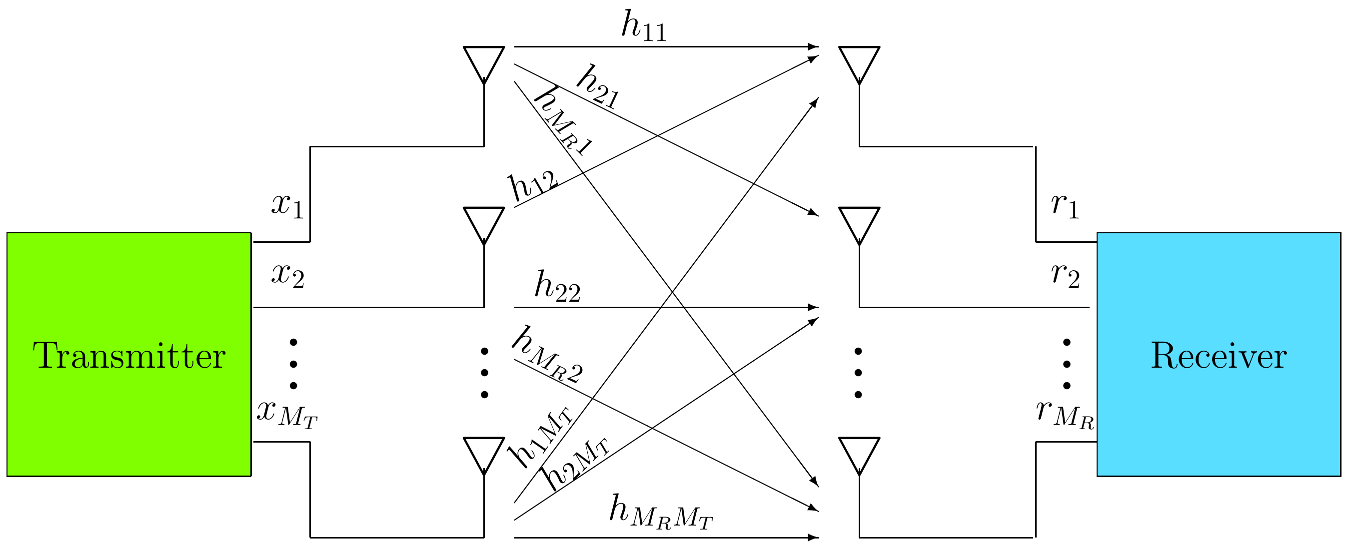
Parameter computation methods	Class	FOS	WSS	Mean-ergodic	Autocor.-ergodic
Rice method	II	yes	yes	yes	yes
Monte Carlo method	IV	yes	yes	yes	no
Jakes method	II	yes	yes	yes	yes
Harmonic decomposition technique	II	yes	yes	yes	yes
Method of equal distances	II	yes	yes	yes	yes
Method of equal areas	II	yes	yes	yes	yes
Mean-square-error method	II	yes	yes	yes	yes
Method of exact Doppler spread	II	yes	yes	yes	yes
L_p-norm method	II	yes	yes	yes	yes
Method proposed by Zheng and Xiao	IV	yes	yes	yes	no

Applications of the Principle of Deterministic Channel Modelling

- Frequency-nonselective channels (Rayleigh, Rice, ext. Suzuki, Nakagami, etc.),
- Frequency-selective channels (COST 207, CODIT),
- Space-time wideband channels (COST 259),
- Multiple cross-correlated Rayleigh fading channels,
- Multiple uncorrelated Rayleigh fading channels,
- Perfect modelling and simulation of measured 2-D and 3-D channels,
- Frequency-hopping fading channels,
- Design of ultra fast channel simulators (tables system),
- Design of burst error models,
- Development of MIMO channels.

III. Modelling of MIMO Channels

Block Diagram of a MIMO Mobile Communication System

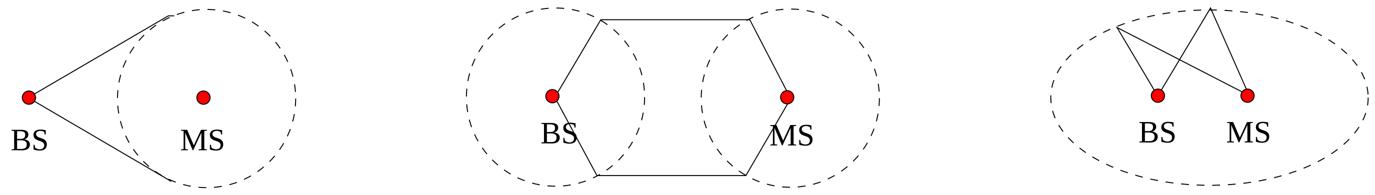


- **Channel coefficients:** The channel coefficient $h_{ij}(t)$ describes the transmission behavior of the channel from the jth transmit antenna to the ith receive antenna.
- **Channel matrix:** $\mathbf{H}(t) = [h_{ij}(t)] \in \mathbb{C}^{M_R \times M_T}$
- **Channel capacity:** $(M_R \geq M_T)$ $$C(t) = \log_2 \det \left(\mathbf{I}_{M_T} + \frac{S_{\text{BS, total}}}{M_T N_{\text{noise}}} \mathbf{H}(t)\mathbf{H}^H(t) \right) \quad \text{[bits/s/Hz]}$$

Generalized Principle of Deterministic Channel Modelling

Step 1: Starting point is a geometrical model with an infinite number of scatterers, e.g.,

(a) one-ring model (b) two-ring model (c) elliptical model



Step 2: Derive a stochastic reference model from the geometrical model.

Step 3: Derive an ergodic stochastic simulation model from the reference model by using only a finite number of N scatterers.

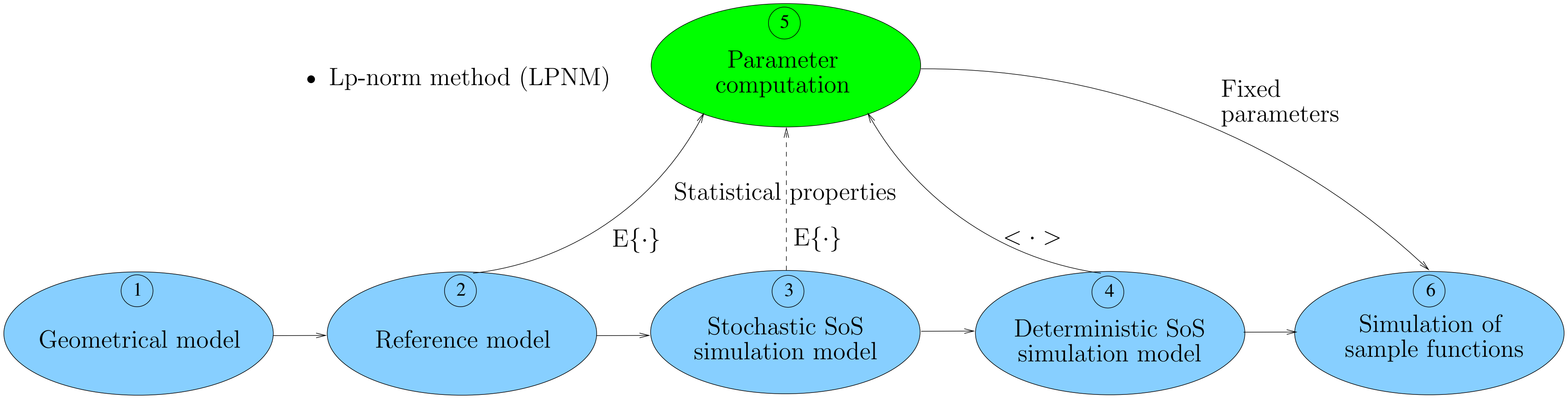
Step 4: Determine the deterministic simulation model by fixing all model parameters of the stochastic simulation model.

Step 5: Compute the parameters of the simulation model by using a proper parameter computation method, e.g., the L_p-norm method (LPNM).

Step 6: Generate one (or some few) sample functions by using the deterministic simulation model with fixed parameters.

Illustration of the Generalized Principle of Deterministic Channel Modelling

- Lp-norm method (LPNM)



- Infinite complexity ($N \to \infty$)
- Infinite number of sample functions

$\Rightarrow$ Non-realizable

- Finite complexity ($N \approx 25$)
- Infinite number of sample functions

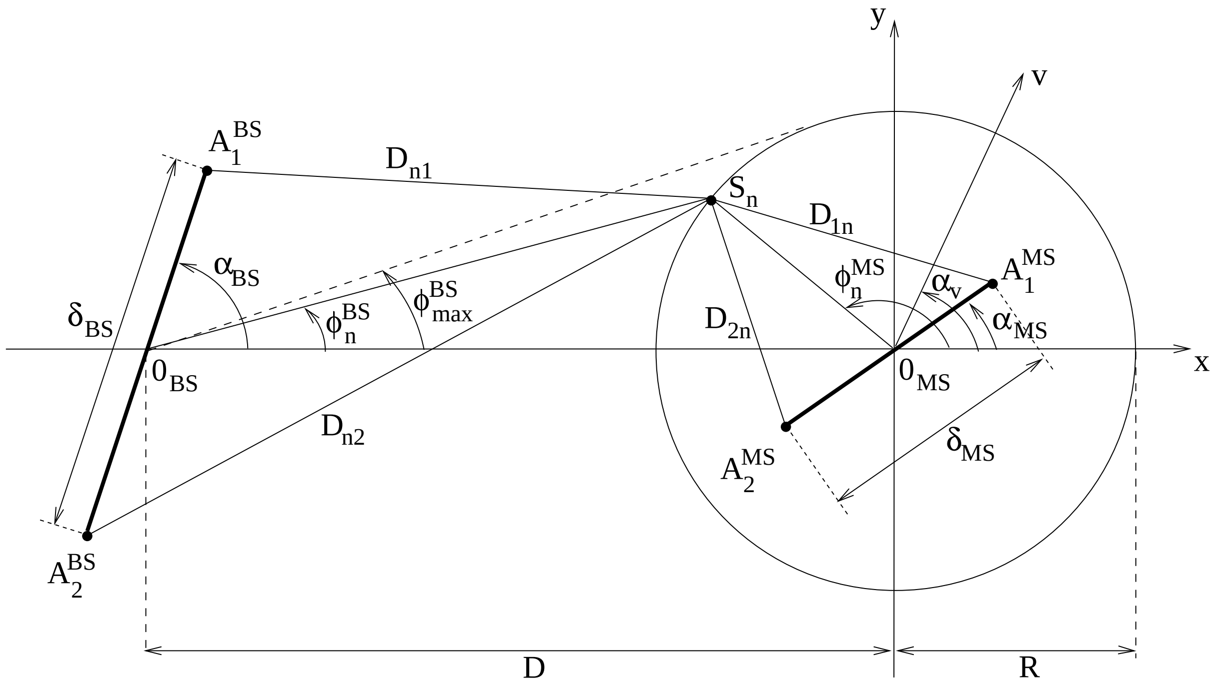
$\Rightarrow$ Non-realizable

- Finite complexity ($N \approx 25$)
- One (or some few) sample functions

$\Rightarrow$ Realizable

Application of the Generalized Principle of Deterministic Channel Modelling

A Geometrical Model for a MIMO Channel



- The geometrical model is known as the *one-ring model* for a 2×2 channel.
- The local scatterers are laying on a ring around the MS.
- If the number of scatterers $N \to \infty$, then the discrete AOA ϕ_n^{MS} tends to a continuous RV ϕ^{MS} with a given distribution $p(\phi^{\text{MS}})$, e.g., the uniform distribution, the von Mises distribution, the Laplacian distribution, etc.

The Reference Model

- **Channel coefficients:**

$$h_{11}(t) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n b_n e^{\,j(2\pi f_n t + \theta_n)}$$

$$h_{12}(t) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n^* b_n e^{\,j(2\pi f_n t + \theta_n)}$$

$$h_{21}(t) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n b_n^* e^{\,j(2\pi f_n t + \theta_n)}$$

$$h_{22}(t) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n^* b_n^* e^{\,j(2\pi f_n t + \theta_n)}$$

where the phases θ_n are i.i.d. RVs and

$$a_n = e^{\,j\pi(\delta_{\mathrm{BS}}/\lambda)\left[\cos(\alpha_{BS}) + \phi_{\max}^{BS}\sin(\alpha_{BS})\sin(\phi_n^{MS})\right]}$$

$$b_n = e^{\,j\pi(\delta_{\mathrm{MS}}/\lambda)\cos(\phi_n^{MS} - \alpha_{MS})}$$

$$f_n = f_{\max}\cos(\phi_n^{\mathrm{MS}} - \alpha_{\mathrm{v}})$$

- **Channel matrix:**

$$\mathbf{H}(t) = [h_{ij}(t)] = \begin{pmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{pmatrix}$$

- **Channel capacity:**

$$C(t) = \log_2 \det\left(\mathbf{I}_2 + \frac{S_{\text{BS, total}}}{2N_{\text{noise}}}\mathbf{H}(t)\mathbf{H}^H(t)\right) \quad \text{[bits/s/Hz]}$$

Statistical Properties of the Reference Model

- **Space-time CCF:**

$$\begin{aligned}\rho_{11,22}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}, \tau) &:= E\{h_{11}(t) h_{22}^*(t+\tau)\} \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} a_n^2\, b_n^2 e^{\,-j2\pi f_n \tau} \\ &= \int_{-\pi}^{\pi} a_n^2(\delta_{\mathrm{BS}}) b_n^2(\delta_{\mathrm{MS}}) e^{\,-j2\pi f_n \tau} p(\phi^{\mathrm{MS}}) d\phi^{\mathrm{MS}}\end{aligned}$$

- **Time ACF:**

$$\begin{aligned}r_{h_{11}}(\tau) &:= E\{h_{11}(t) h_{11}^*(t+\tau)\} \\ &= \int_{-\pi}^{\pi} e^{\,-j2\pi f_{\max} \cos(\phi^{\mathrm{MS}} - \alpha_{\mathrm{v}})\tau} p(\phi^{\mathrm{MS}})\, d\phi^{\mathrm{MS}}\end{aligned}$$

Relationships: $r_{h_{11}}(\tau) = r_{h_{12}}(\tau) = r_{h_{21}}(\tau) = r_{h_{22}}(\tau) = \rho_{11,22}(0,\ 0,\ \tau)$

- **2D space CCF:**

$$\begin{aligned}\rho(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}) &:= \rho_{11,22}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}, 0) \\ &= \int_{-\pi}^{\pi} a_n^2(\delta_{\mathrm{BS}}) b_n^2(\delta_{\mathrm{MS}}) p(\phi^{\mathrm{MS}}) d\phi^{\mathrm{MS}}\end{aligned}$$

The Stochastic Simulation Model

- **Channel coefficients:** $\hat{h}_{11}(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n b_n e^{\,j(2\pi f_n t + \theta_n)}$

  $\Rightarrow$ The phases θ_n are i.i.d. RVs

- **Channel matrix:** $\hat{\mathbf{H}}(t) = \begin{pmatrix} \hat{h}_{11}(t) & \hat{h}_{12}(t) \\ \hat{h}_{21}(t) & \hat{h}_{22}(t) \end{pmatrix}$

  $\Rightarrow$ Stochastic channel matrix

- **Channel capacity:** $\hat{C}(t) = \log_2 \det \left(\mathbf{I}_2 + \frac{S_{\text{BS, total}}}{2N_{\text{noise}}} \hat{\mathbf{H}}(t) \hat{\mathbf{H}}^H(t) \right)$ [bits/s/Hz]

  $\Rightarrow$ Stochastic process

  $\Rightarrow$ The analysis of $\hat{C}(t)$ has to be performed by using statistical averages, e.g., $C_s = E\{\hat{C}(t)\}$.

Statistical Properties of the Stochastic Simulation Model

- **Space-time CCF:**
$$\begin{aligned}\hat{\rho}_{11,22}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}, \tau) &:= \mathrm{E}\{\hat{h}_{11}(t)\hat{h}^*_{22}(t+\tau)\} \\ &= \frac{1}{N}\sum_{n=1}^{N} a_n^2(\delta_{\mathrm{BS}}) b_n^2(\delta_{\mathrm{MS}}) e^{-j2\pi f_n \tau}\end{aligned}$$

- **Time ACF:**
$$\begin{aligned}\hat{r}_{h_{11}}(\tau) &:= \mathrm{E}\{\hat{h}_{11}(t)\hat{h}^*_{11}(t+\tau)\} \\ &= \frac{1}{N}\sum_{n=1}^{N} e^{-j2\pi f_{\max}\cos(\phi_n^{\mathrm{MS}}-\alpha_{\mathrm{v}})\tau}\end{aligned}$$

Relationships: $\hat{r}_{h_{11}}(\tau) = \hat{r}_{h_{12}}(\tau) = \hat{r}_{h_{21}}(\tau) = \hat{r}_{h_{22}}(\tau) = \hat{\rho}_{11,22}(0,\, 0,\, \tau)$

- **2D space CCF:**
$$\begin{aligned}\hat{\rho}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}) &:= \hat{\rho}_{11,22}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}, 0) \\ &= \frac{1}{N}\sum_{n=1}^{N} a_n^2(\delta_{\mathrm{BS}}) b_n^2(\delta_{\mathrm{MS}}) \\ &= \frac{1}{N}\sum_{n=1}^{N} e^{j2\pi(\delta_{\mathrm{BS}}/\lambda)[\cos(\alpha_{\mathrm{BS}})+\phi_{\max}^{\mathrm{BS}}\sin(\alpha_{\mathrm{BS}})\sin(\phi_n^{\mathrm{MS}})]} \cdot e^{j2\pi(\delta_{\mathrm{MS}}/\lambda)\cos(\phi_n^{\mathrm{MS}}-\alpha_{\mathrm{MS}})}\end{aligned}$$

The Deterministic Simulation Model

- **Channel coefficients:** $\tilde{h}_{11}(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n b_n e^{\,j(2\pi f_n t + \theta_n)}$

  $\Rightarrow$ The phases θ_n are constant quantities

- **Channel matrix:** $\tilde{\mathbf{H}}(t) = \begin{pmatrix} \tilde{h}_{11}(t) & \tilde{h}_{12}(t) \\ \tilde{h}_{21}(t) & \tilde{h}_{22}(t) \end{pmatrix}$

  $\Rightarrow$ Deterministic channel matrix

- **Channel capacity:** $\tilde{C}(t) = \log_2 \det \left(\mathbf{I}_2 + \frac{S_{\text{BS, total}}}{2N_{\text{noise}}} \tilde{\mathbf{H}}(t) \tilde{\mathbf{H}}^H(t) \right) \quad$ [bits/s/Hz]

  $\Rightarrow$ Deterministic process

  $\Rightarrow$ The analysis of $\tilde{C}(t)$ has to be performed by using time averages, e.g., $C_t = < \tilde{C}(t) >$.

Statistical Properties of the Deterministic Simulation Model

- **Space-time CCF:**

$$\tilde{\rho}_{11,22}(\delta_{\text{BS}}, \delta_{\text{MS}}, \tau) := < \tilde{h}_{11}(t)\tilde{h}^*_{22}(t+\tau) > = \frac{1}{N}\sum_{n=1}^{N} a_n^2(\delta_{\text{BS}})b_n^2(\delta_{\text{MS}})e^{-j2\pi f_n \tau}$$

- **Time ACF:**

$$\tilde{r}_{h_{11}}(\tau) := < \tilde{h}_{11}(t)\tilde{h}^*_{11}(t+\tau) > = \frac{1}{N}\sum_{n=1}^{N} e^{-j2\pi f_{\max}\cos(\phi_n^{\text{MS}}-\alpha_{\text{v}})\tau}$$

Relationships: $\tilde{r}_{h_{11}}(\tau) = \tilde{r}_{h_{12}}(\tau) = \tilde{r}_{h_{21}}(\tau) = \tilde{r}_{h_{22}}(\tau) = \tilde{\rho}_{11,22}(0,\, 0,\, \tau)$

- **2D space CCF:**

$$\tilde{\rho}(\delta_{\text{BS}}, \delta_{\text{MS}}) := \tilde{\rho}_{11,22}(\delta_{\text{BS}}, \delta_{\text{MS}}, 0) = \frac{1}{N}\sum_{n=1}^{N} a_n^2(\delta_{\text{BS}})b_n^2(\delta_{\text{MS}})$$

$$= \frac{1}{N}\sum_{n=1}^{N} e^{j2\pi(\delta_{\text{BS}}/\lambda)[\cos(\alpha_{\text{BS}})+\phi_{\max}^{\text{BS}}\sin(\alpha_{\text{BS}})\sin(\phi_n^{\text{MS}})]} \cdot e^{j2\pi(\delta_{\text{MS}}/\lambda)\cos(\phi_n^{\text{MS}}-\alpha_{\text{MS}})}$$

Parameter Computation Method

Parameters: The model parameters to be determined are the discrete AOAs ϕ_n^{MS} $(n = 1, 2, \ldots, N)$.

Aim: Determine the model parameters ϕ_n^{MS} such that

$$\rho_{11,22}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}, \tau) \approx \tilde{\rho}_{11,22}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}, \tau)\,.$$

Solution: L_p-norm method

$$E_1^{(p)} := \left\{ \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} |r_{h_{11}}(\tau) - \tilde{r}_{h_{11}}(\tau)|^p d\tau \right\}^{1/p}$$

$$E_2^{(p)} := \left\{ \frac{1}{\delta_{\max}^{\mathrm{BS}}\,\delta_{\max}^{\mathrm{MS}}} \int_0^{\delta_{\max}^{\mathrm{BS}}} \int_0^{\delta_{\max}^{\mathrm{MS}}} |\rho(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}}) - \tilde{\rho}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}})|^p d\delta_{\mathrm{MS}}\, d\delta_{\mathrm{BS}} \right\}^{1/p}$$

Two alternatives:

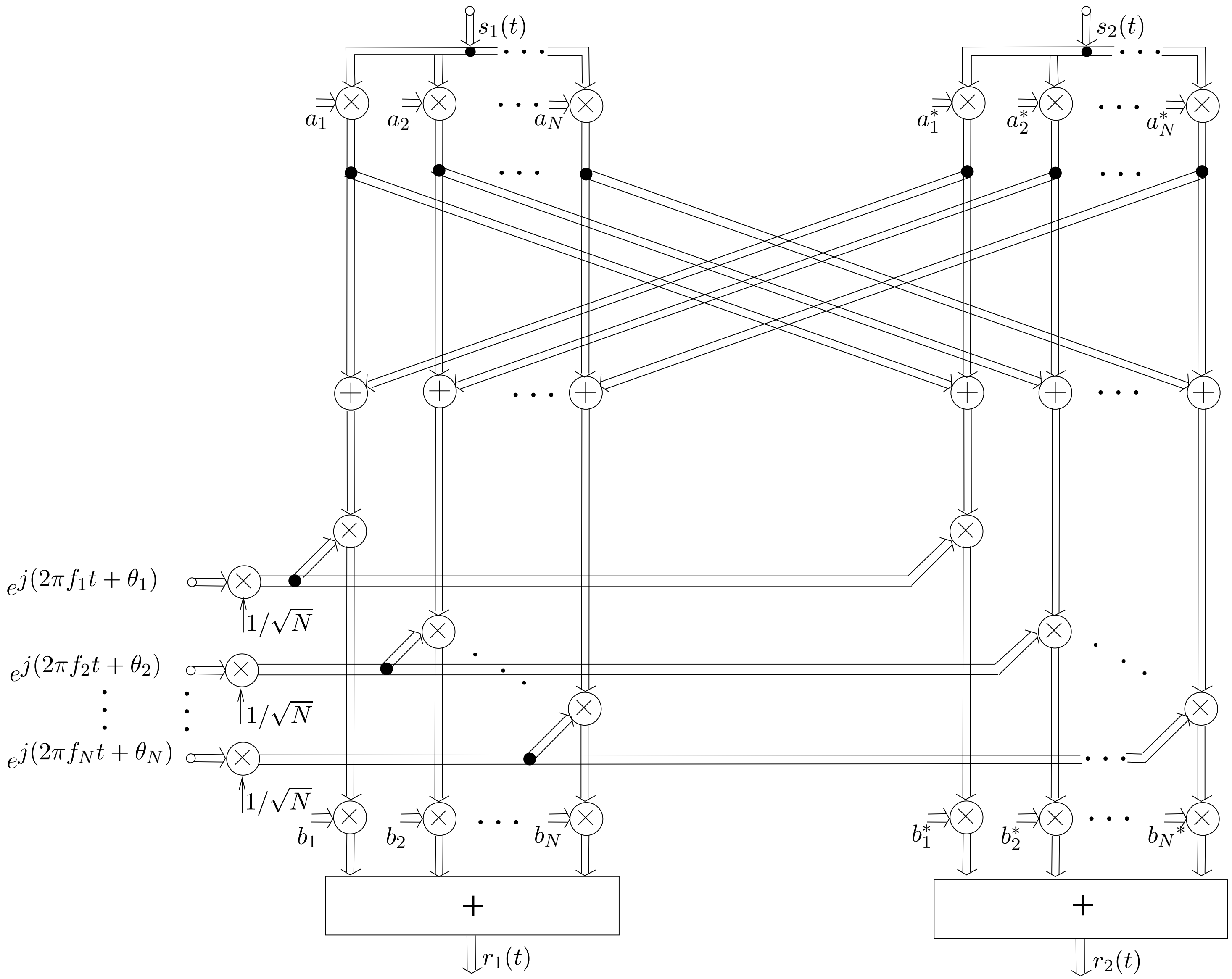
(i) Joint optimization: $E^{(p)} = w_1 E_1^{(p)} + w_2 E_2^{(p)}$

w_1, w_2: weighting factors

(ii) Using two independent sets $\{\phi_n^{\prime\mathrm{MS}}\}$ and $\{\phi_n^{\mathrm{MS}}\}$ in $\tilde{r}_{h_{11}}(\tau)$ and $\tilde{\rho}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}})$, respectively.

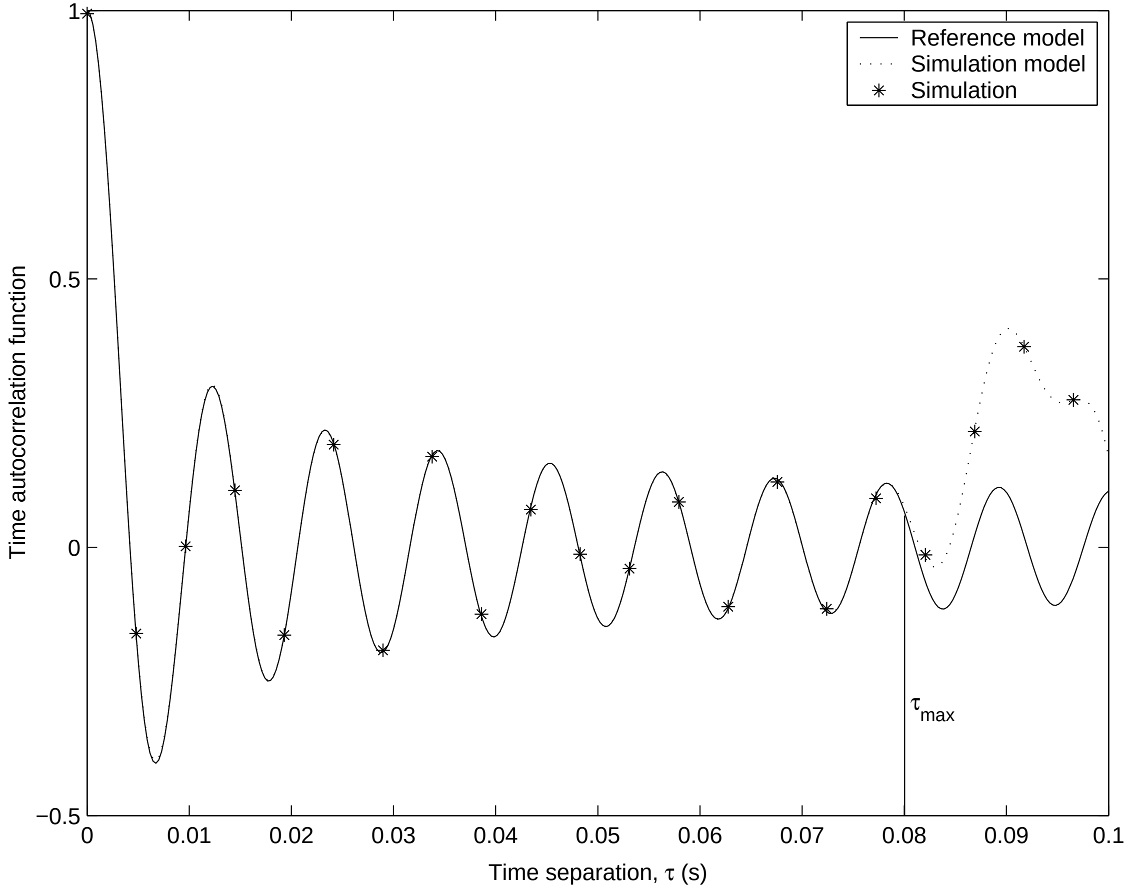
$\Rightarrow$ Orthogonalization of the optimization problem.

Structure of the Simulation Model



Performance Evaluation

- Comparison of the time ACFs $r_{h_{11}}(\tau)$ (reference model) and $\tilde{r}_{h_{11}}(\tau)$ (simulation model)

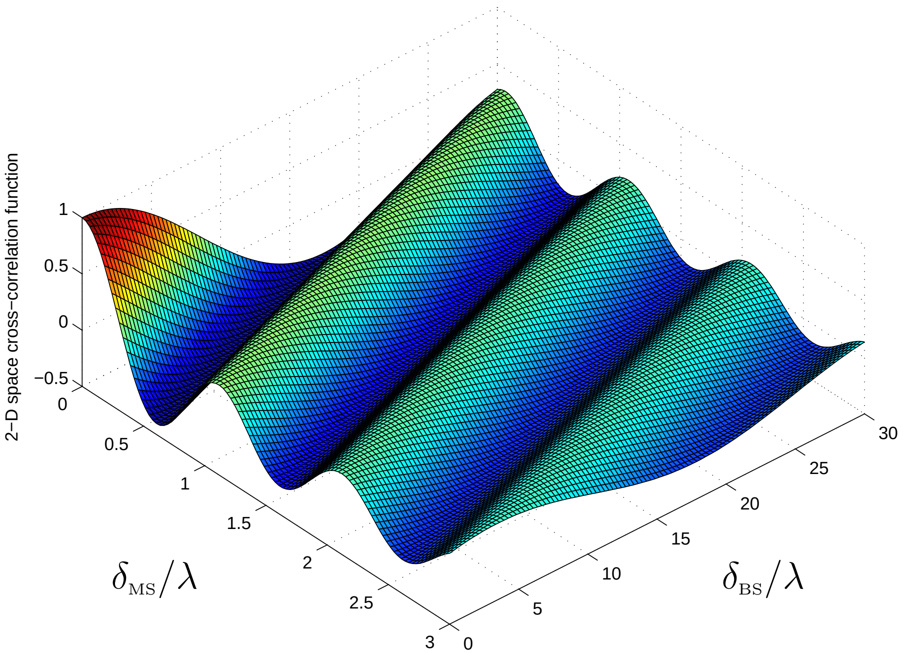


Reference model: Based on the one-ring model with uniformly distributed AOAs ϕ^{MS}.

Simulation model: Designed by using the L$_p$-norm method ($N = 25$, $p = 2$, $\tau_{\max} = 0.08$ s, $f_{\max} = 91$ Hz).

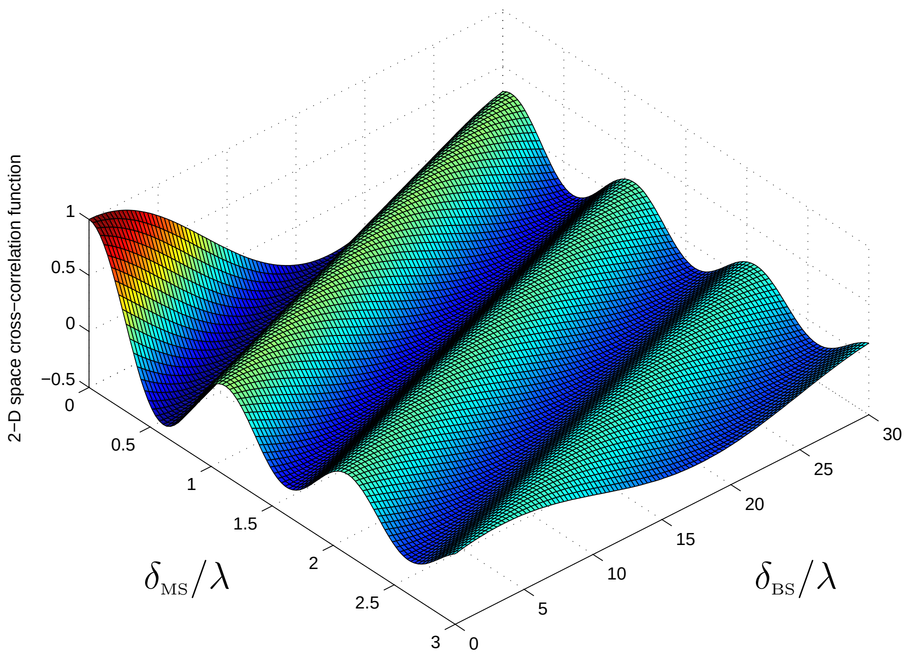
Reference model

2-D space CCF $\rho(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}})$



Simulation model

2-D space CCF $\tilde{\rho}(\delta_{\mathrm{BS}}, \delta_{\mathrm{MS}})$

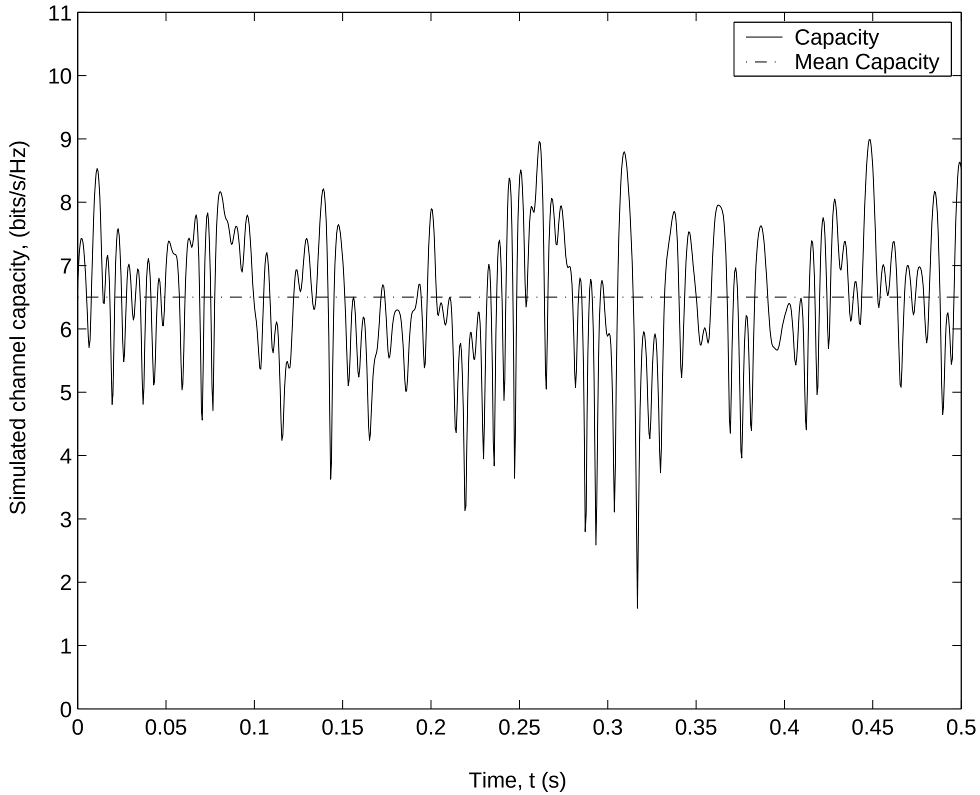


Reference model: Based on the one-ring model with uniformly distributed AOAs ϕ^{MS}.

Simulation model: Designed by using the Lp-norm method with $N = 25$ ($p = 2$, $\delta_{\max}^{\mathrm{BS}} = 30\lambda$, $\delta_{\max}^{\mathrm{MS}} = 3\lambda$, $\alpha_{\mathrm{BS}} = \alpha_{\mathrm{MS}} = 90°$, $\phi_{\max}^{\mathrm{BS}} = 2°$).

IV. Simulation of MIMO Channels

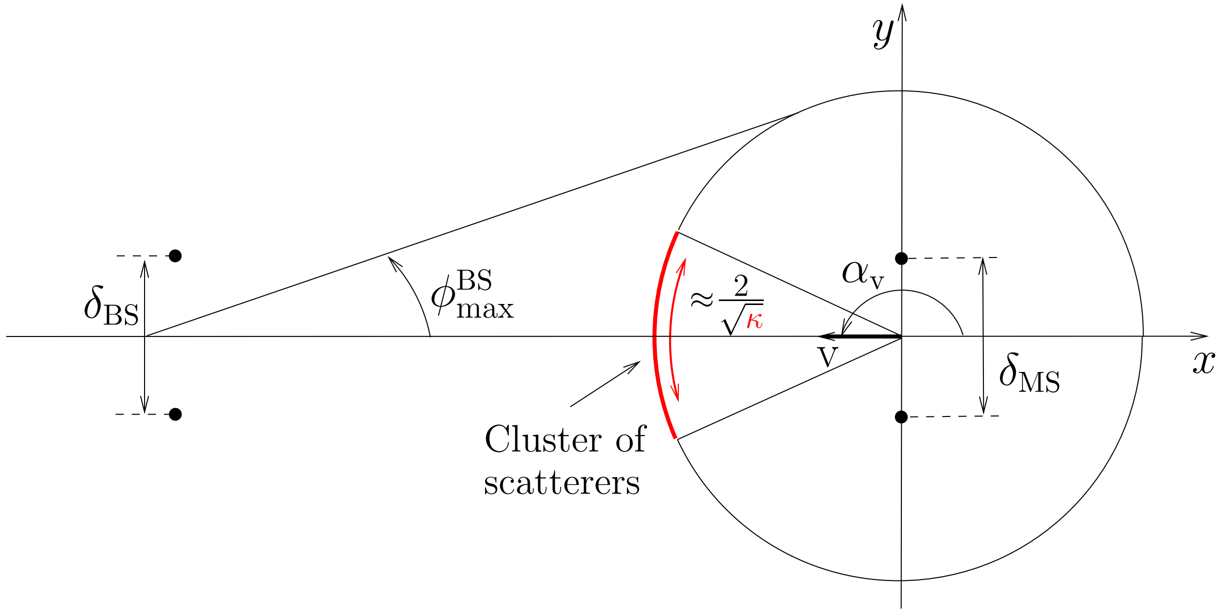
- **Channel capacity:** $\tilde{C}(t) := \log_2 \det \left(\mathbf{I}_2 + \frac{S_{\text{BS, total}}}{2N_{\text{noise}}} \tilde{\mathbf{H}}(t) \tilde{\mathbf{H}}^H(t) \right)$ [bits/s/Hz]



Parameters: $N = 25$, $M_{\text{BS}} = M_{\text{MS}} = 2$, $\delta_{\text{BS}} = \delta_{\text{MS}} = \lambda/2$, SNR $= 17$ dB

Scattering Scenario

Geometrical one-ring model:



Parameters:

$M_{BS} = M_{MS} = 2$

$\delta_{BS} = \delta_{MS} = \lambda/2$

$\phi_{max}^{BS} = 2°$

$\alpha_V = \phi_0^{MS} = 180°$

$f_{max} = 91$ Hz

Reference model: assuming the von Mises distribution for the AOA ϕ^{MS} :

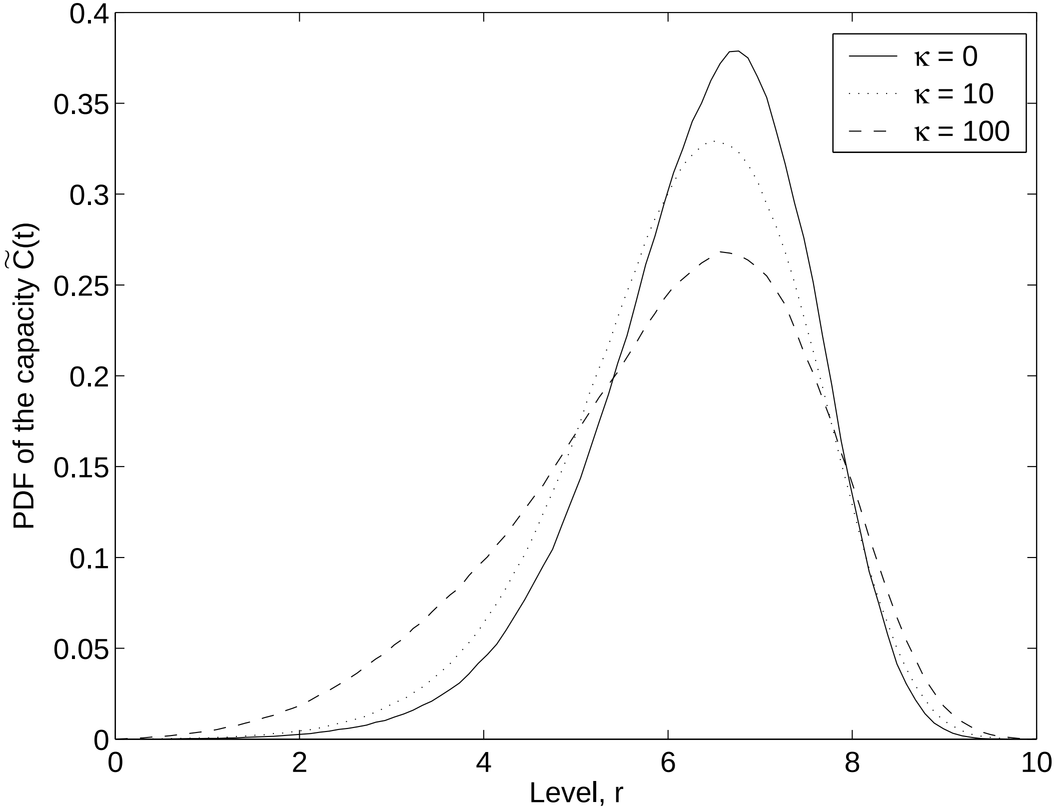
$$p(\phi^{MS}) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\phi^{MS} - \phi_0^{MS})}$$

$\kappa = 0 : \quad p(\phi^{MS}) = 1/(2\pi) \quad$ (isotropic scattering)

$\kappa \to \infty : p(\phi^{MS}) \to \delta(\phi^{MS} - \phi_0^{MS}) \quad$ (extremely nonisotropic scattering)

Simulation model: designed by using the LPNM with $N = 25$.

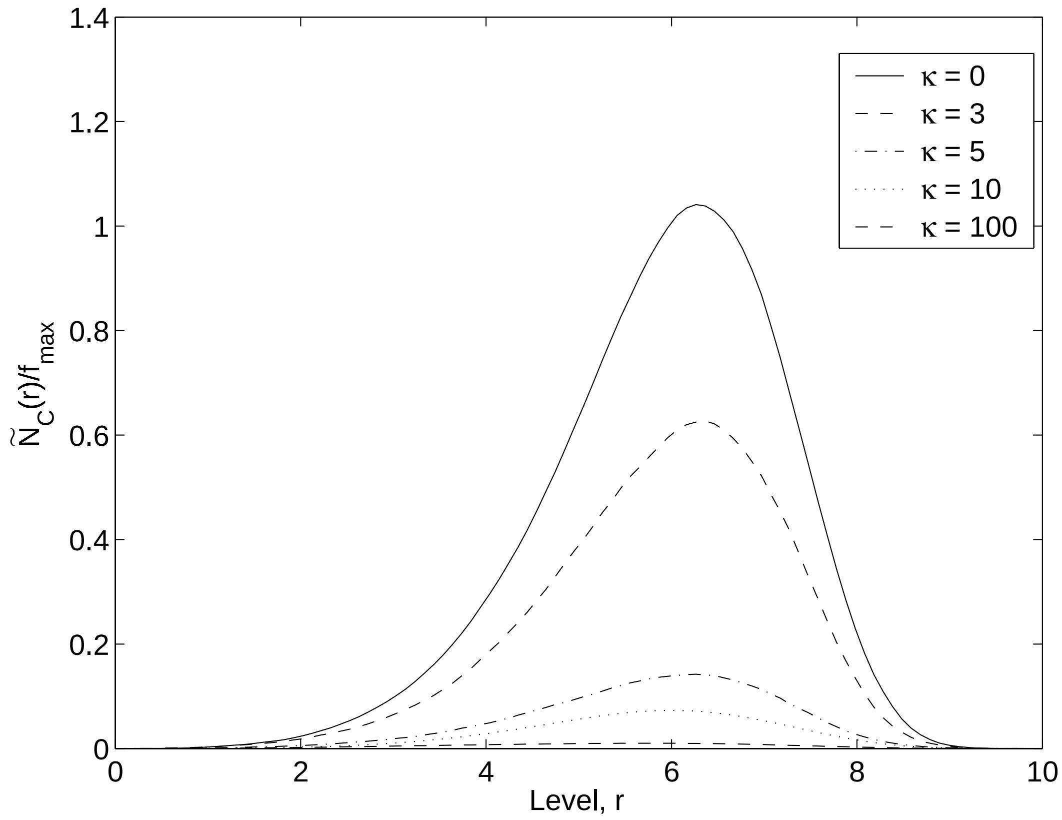
PDF of the Channel Capacity $\tilde{C}(t)$



Observations:

- κ influences the PDF $\tilde{p}_C(r)$ of the capacity $\tilde{C}(t)$
- If $\kappa \uparrow \rightsquigarrow$ angular spread of the AOA $\phi^{\mathrm{MS}} \downarrow \rightsquigarrow \mathrm{Var}\{\tilde{C}(t)\} \uparrow$

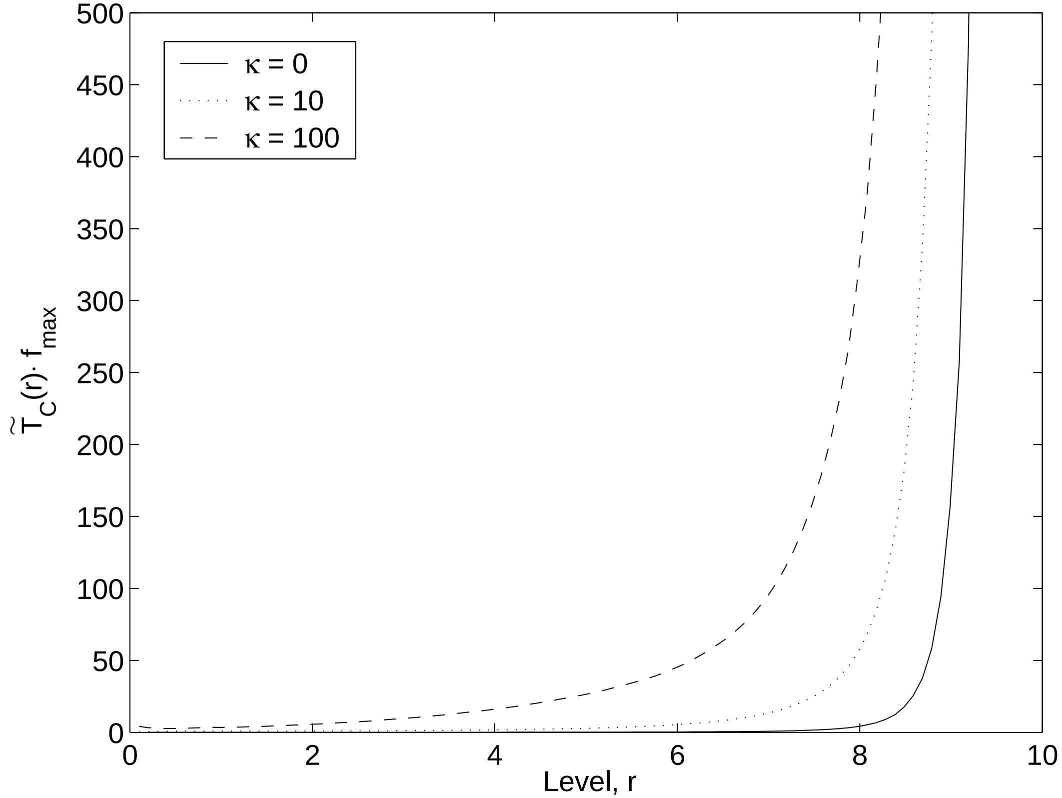
Normalized LCR of the Channel Capacity $\tilde{C}(t)$



Observations:

- κ has a strong influence on the LCR $\tilde{N}_C(r)$ of the capacity $\tilde{C}(t)$
- If $\kappa \uparrow \rightsquigarrow$ angular spread of the AOA $\phi^{\mathrm{MS}} \downarrow \rightsquigarrow \tilde{N}_C(r) \downarrow$

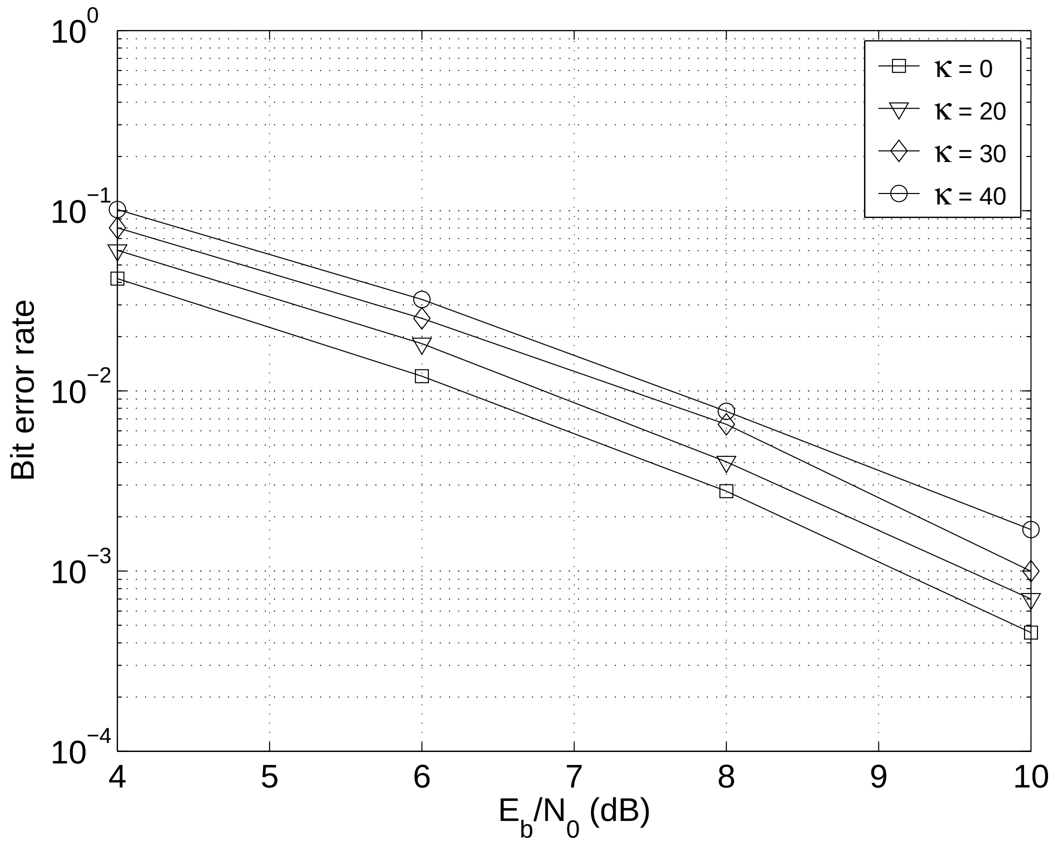
Normalized ADF of the Channel Capacity $\tilde{\mathbf{C}}(\mathbf{t})$



Observations:

- κ has a strong influence on the ADF $\tilde{T}_C(r)$ of the capacity $\tilde{C}(t)$
- If $\kappa \uparrow \rightsquigarrow$ angular spread of the AOA $\phi^{\mathrm{MS}} \downarrow \rightsquigarrow \tilde{T}_C(r) \uparrow$.

The Effect of κ on the BER Performance

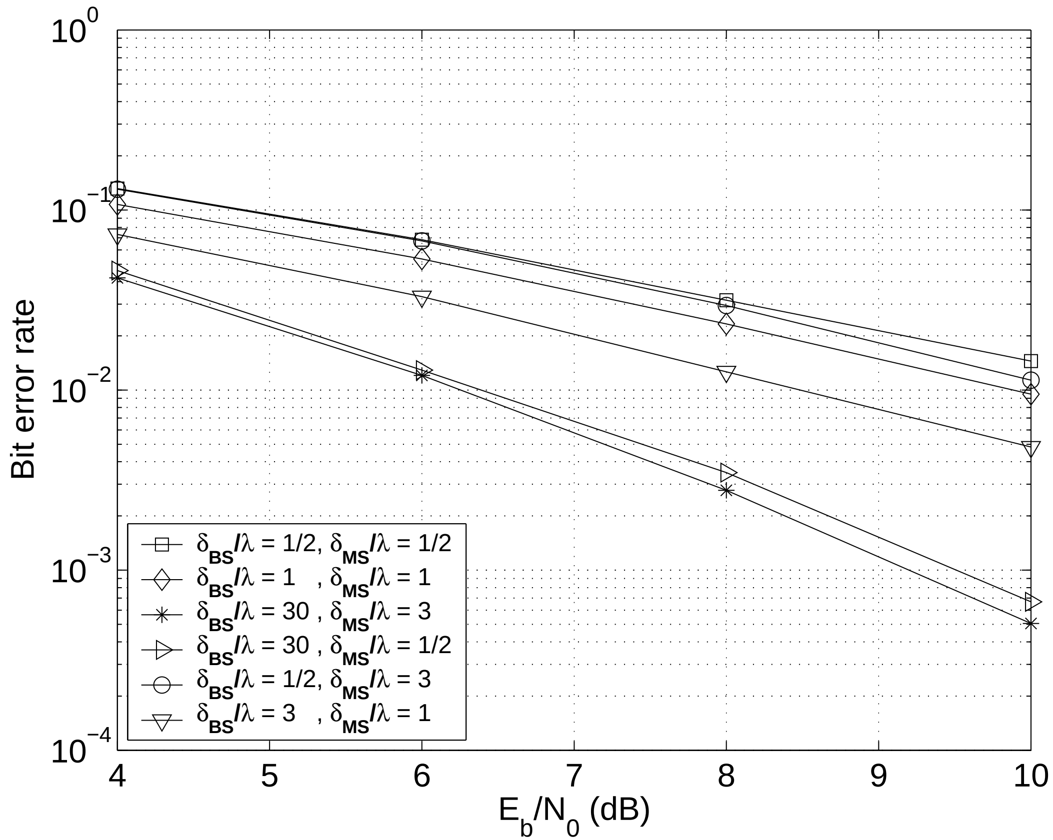


Coding system: 4-D 16QAM linear 32-state space-time trellis code with 2 transmit and 2 receive antennas ($\delta_{\mathrm{BS}} = 30\lambda$ and $\delta_{\mathrm{MS}} = 3\lambda$).

Observations:

- $\kappa \uparrow \leadsto$ angular spread $\downarrow \leadsto$ BER $\uparrow$
- If κ decreases from 40 to 0, then a gain of ca. 1.5 dB can be achieved at a BER of $2 \cdot 10^{-3}$.

The Effect of the Antenna Spacing on the BER Performance



Coding system: 4-D 16QAM linear 32-state space-time trellis code with 2 transmit and 2 receive antennas.

Observations:

- δ_{BS}, $\delta_{\mathrm{MS}} \uparrow \rightsquigarrow$ spatial correlation $\downarrow \rightsquigarrow$ BER $\downarrow$
- A large spacing among the antennas at the BS provides a better performance than a large antenna spacing at the MS.

VI. Conclusion

- The principle of deterministic channel modelling has been generalized.
- The generalized concept has been applied on the geometrical one-ring scattering model.
- It has been shown how the parameters of the simulation model can be determined by using the L_p-norm method (LPNM).
- The designed deterministic MIMO channel simulator with $N = 25$ scatterers has nearly the same statistics as the underlying stochastic reference model with $N = \infty$ scatterers.
- The new MIMO space-time channel simulator is very useful in the investigation of the PDF, LCR, and ADF of the channel capacity $C(t)$ from time-domain simulations.
- Finally, it has been demonstrated that the system performance decreases if the spatial correlation increases.