Iterative Multiuser Decoding

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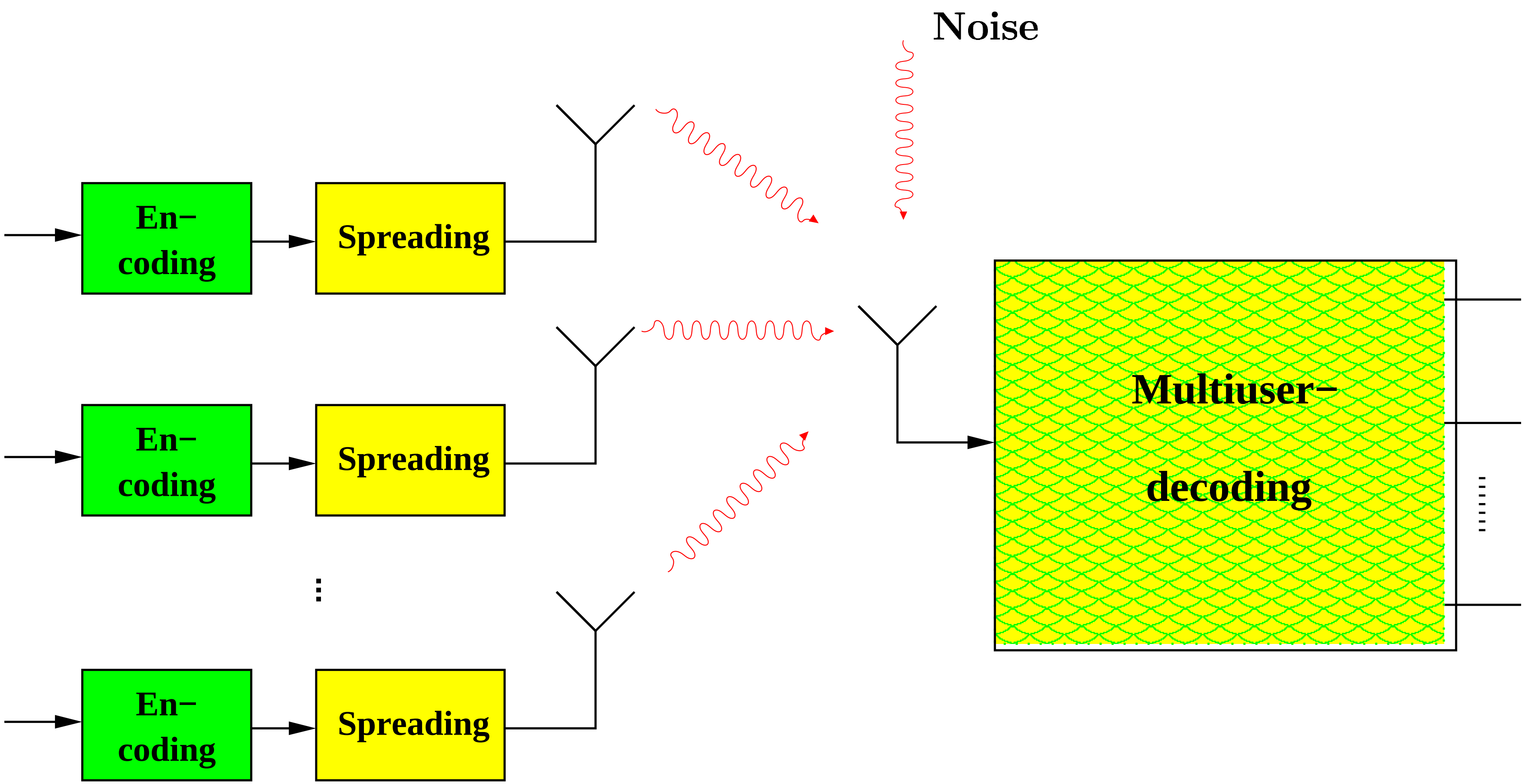
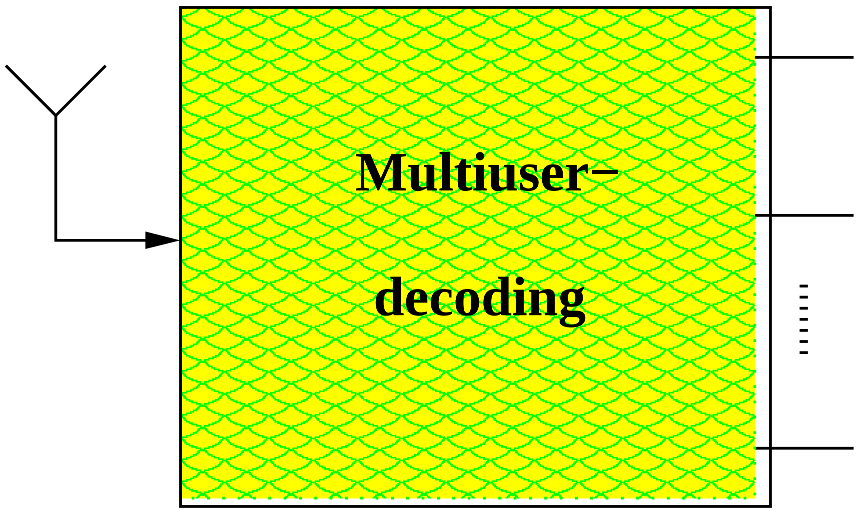
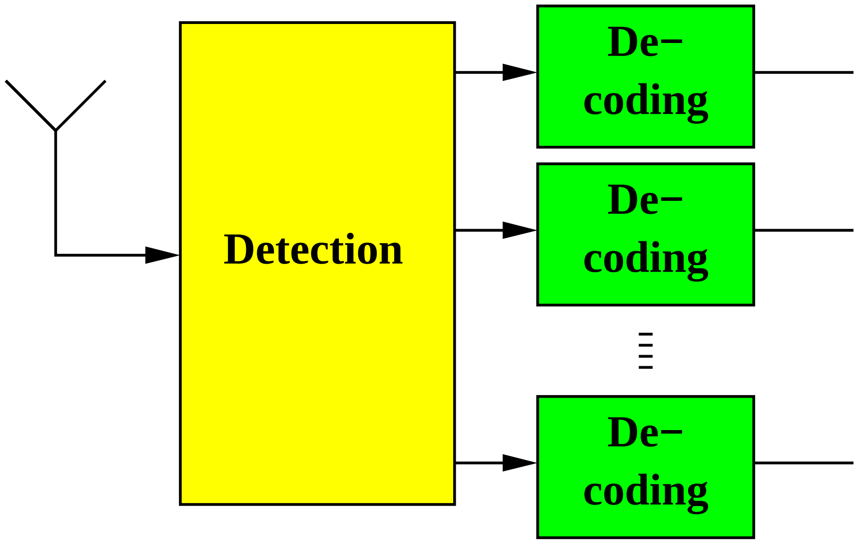
Coded CDMA


Noise
En−
coding
Spreading
En−
coding
Spreading
En−
coding
Spreading
Multiuser−
decoding

optimal **low complexity**

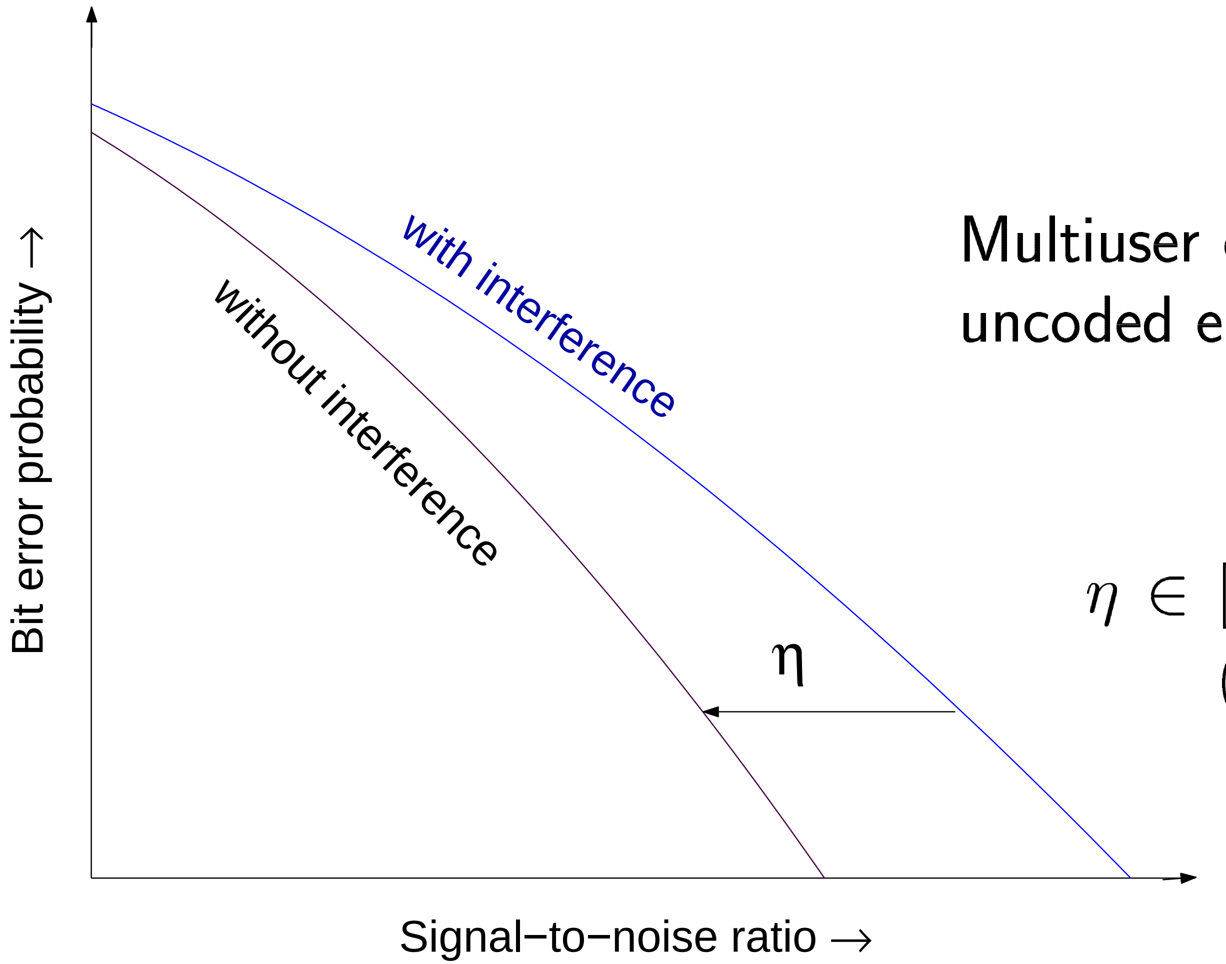




very high complexity

inefficient ???

Multiuser Efficiency



Multiuser efficiency measures uncoded error probability.

$\eta \in [0; 1]$

$(-\infty \text{ dB}; 0 \text{ dB}]$

Loss due to Separation of Detection & Decoding

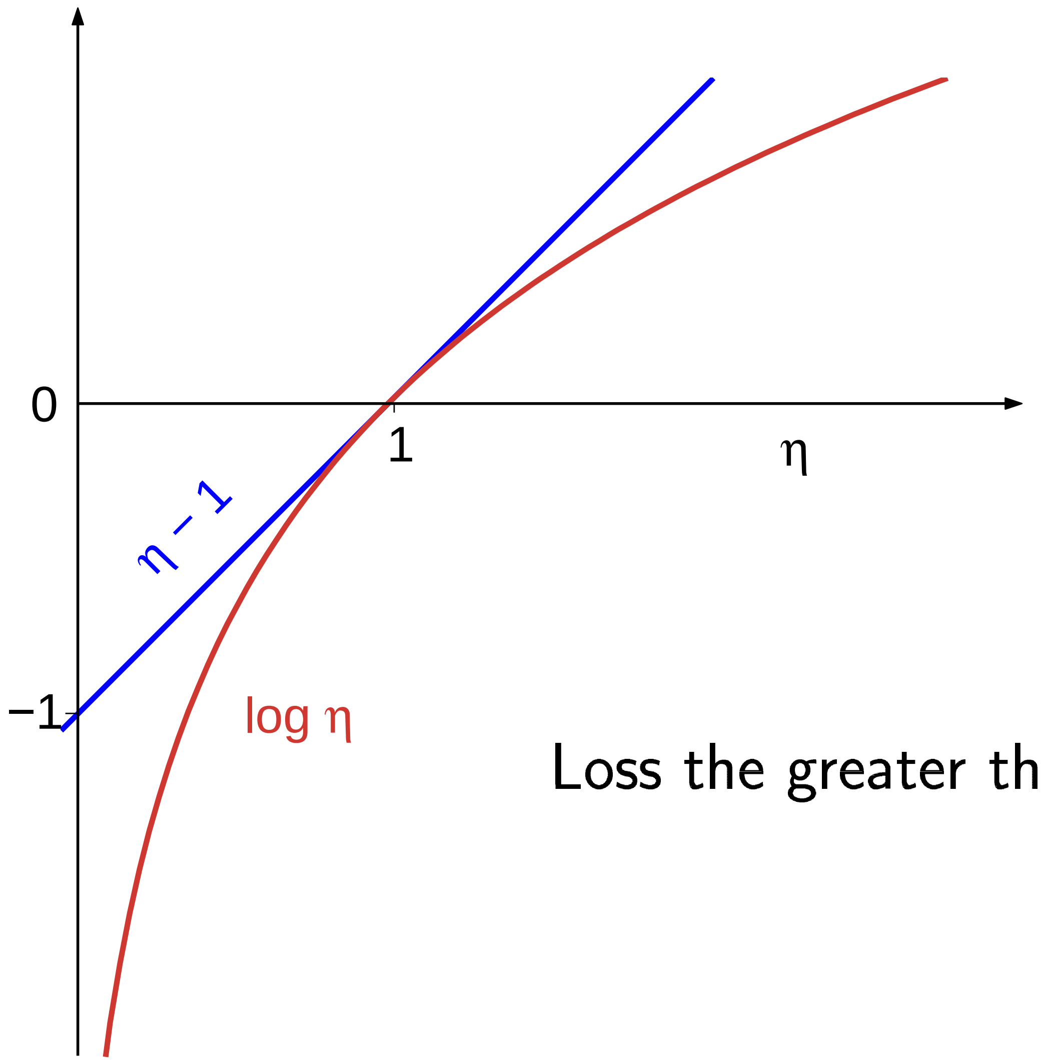
Complex AWGN channel, random spreading, many users

$$\Delta I = \eta - 1 - \log \eta \qquad \text{[nats]}$$

Gaussian input: S. Shamai and S. Verdú, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Trans. Inform. Theory*, vol. 47, no. 4, pp. 1302-1327, May 2001.

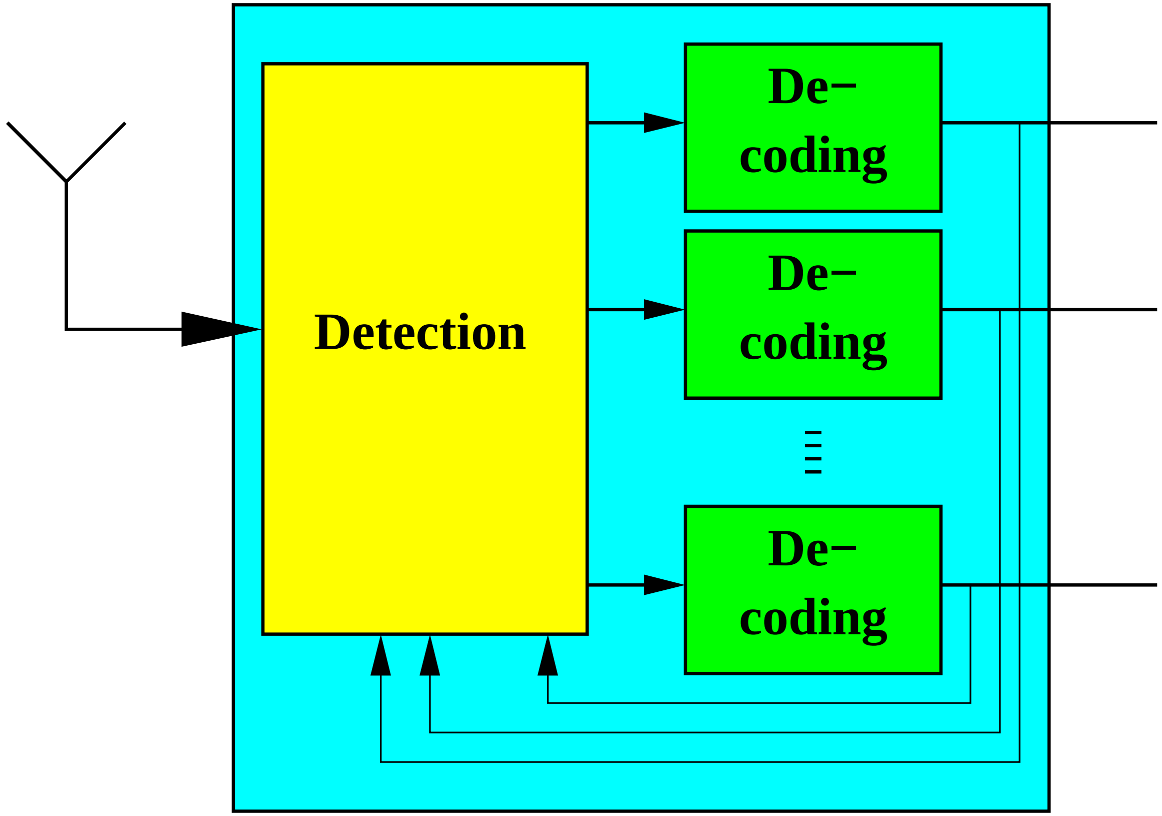
Binary input: R. Müller and W. Gerstacker, "On the capacity loss due to separation of detection and decoding," *IEEE Trans. Inform. Theory*, vol. 50, no. 8, pp. 1769-1778, August 2004.

Arbitrary input: D. Guo and S. Verdú, "CDMA spectral efficiency: Asymptotics via statistical physics," *IEEE Trans. Inform. Theory*, submitted.



Loss the greater the worse detector performs.

Iterative Receiver



- Knowledge gained by the code laws improves detection
- Feedback of conditional probabilities
- Convergence not ensured

Dynamics of the Iterative Receiver

Asking about the convergence of the iterative receiver is equivalent to asking for the dynamics of a multi-dimensional non-linear system with a random excitation.

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Approximations:

1. *The spreading sequences are random and statistically independent.*
2. *The number of users K and the spreading factor N are large, i.e. $K \gg 1 \ll N$.*

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System becomes one-dimensional.

Spin Glasses

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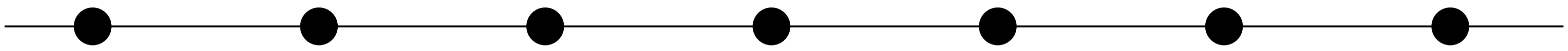

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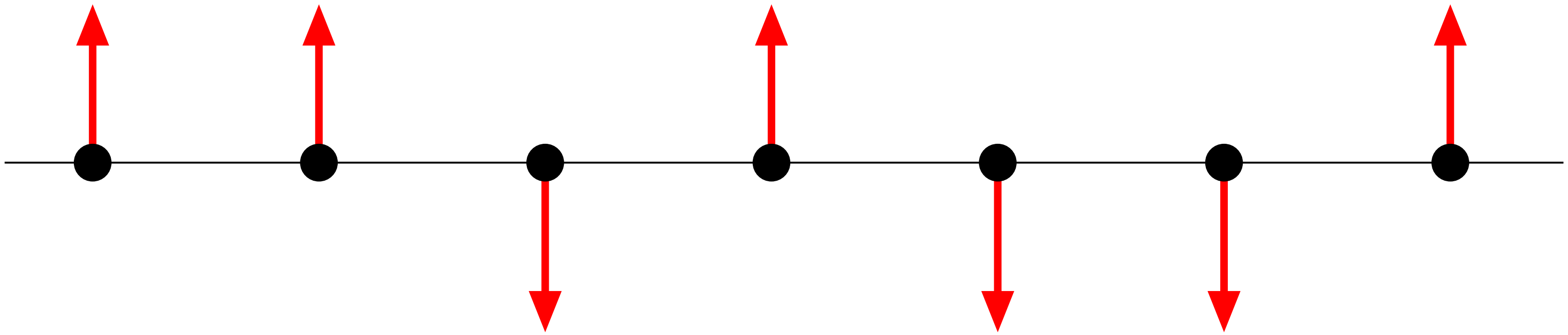

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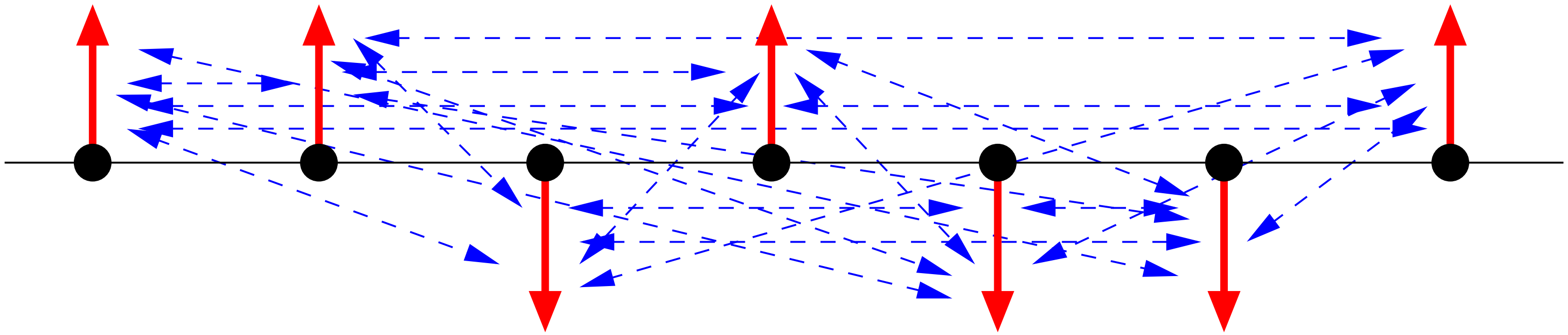

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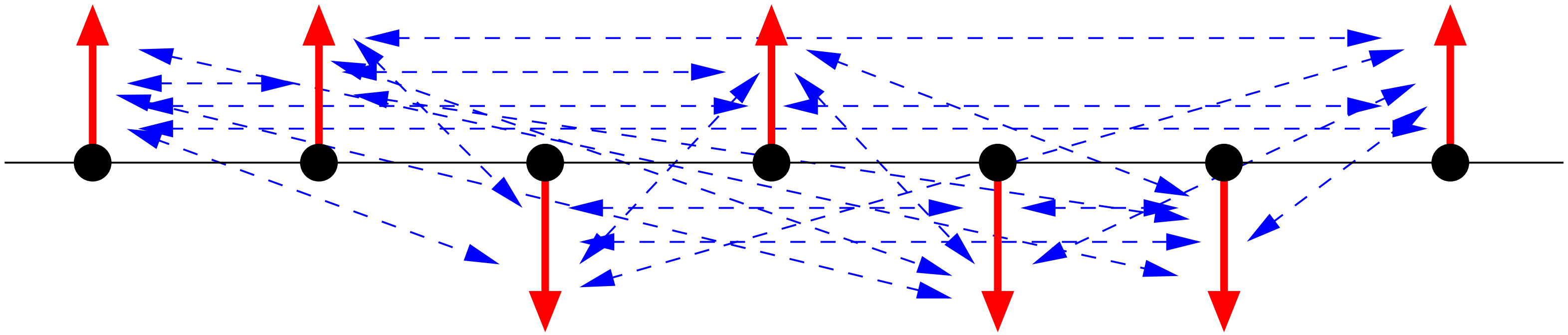

Energy function (Hamiltonian):

$$-\sum_i \sum_{j<i} r_{ij} x_i x_j$$

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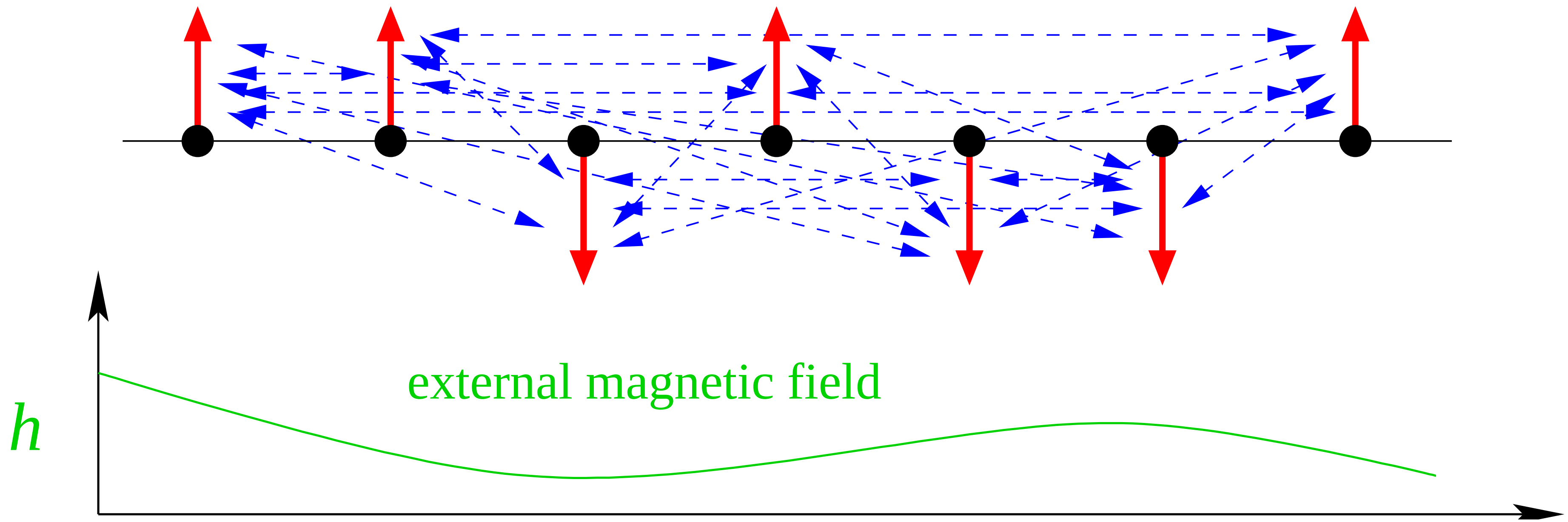

Energy function (Hamiltonian):

$$-\sum_i \sum_{j<i} r_{ij} x_i x_j - \sum_i h_i x_i$$

Detection of CDMA

$$\boldsymbol{y} = \boldsymbol{S}\boldsymbol{x} + \boldsymbol{n}$$

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$$\hat{\boldsymbol{x}} = \operatorname*{argmin}_{\boldsymbol{x} \in \{\pm 1\}^K} ||\boldsymbol{y} - \boldsymbol{S}\boldsymbol{x}||$$

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Minimisation of the energy function of a spin glass!

Optimal Large System Multiuser Efficiency

$$\frac{1}{\eta_\ell} = 1 + \frac{K}{N} \int \gamma \left(1 - p^2\right) \int \frac{1 - \tanh\left(z\sqrt{\gamma\eta_\ell} + \gamma\eta_\ell\right)}{1 - p^2 \tanh^2\left(z\sqrt{\gamma\eta_\ell} + \gamma\eta_\ell\right)} Dz \, dF(\gamma, p)$$

with

$$p_{k,n} = 2 \Pr\left(x_{k,n} = 1 \middle| \text{iteration } \ell - 1\right) - 1,$$

the signal-to-noise ratio γ_k and the Gaussian measure $Dz \triangleq \frac{\exp(-z^2/2)}{\sqrt{2\pi}} dz$.

G. Caire, R. Müller and T. Tanaka, "Iterative multiuser joint decoding: Optimal power allocation and low-complexity implementation," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, September 2004.

(cond.) LMMSE Large System Multiuser Efficiency

Performance of best linear MMSE detector

$$\frac{1}{\eta_\ell} = 1 + \frac{K}{N} \int \frac{\gamma \left(1 - p^2\right)}{1 + \eta_\ell \gamma \left(1 - p^2\right)} \, dF(\gamma, p)$$

The interference powers in the filter reflect the dynamics of the interference power level from iteration to iteration.

G. Caire, R. Müller and T. Tanaka, "Iterative multiuser joint decoding: Optimal power allocation and low-complexity implementation," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, September 2004.

(uncond.) LMMSE Large System Multiuser Efficiency

Performance of suboptimal linear MMSE detector

$$\frac{1}{\eta_\ell} = 1 + \frac{K}{N} \int \frac{\gamma \int 1 - p^2 dF(p|\gamma)}{1 + \eta_\ell \gamma \int 1 - p^2 dF(p|\gamma)} dF(\gamma)$$

The interference powers in the filter reflect the dynamics of the **average** interference power level from iteration to iteration.

G. Caire, R. Müller and T. Tanaka, "Iterative multiuser joint decoding: Optimal power allocation and low-complexity implementation," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, September 2004.

Gaussian Approximation for the Decoder Output

The distribution $F(p|\gamma)$ of the decoder output signal can be well approximated by a single parameter $\mu(\gamma\eta)$ which characterizes the average reliablility of the decoder output. [Gaussian approximation for the conditional log-likelihood ratios].

This average reliability depends on the structure of the codes, the signal-to-noise ratio γ and the multiuser efficiency η.

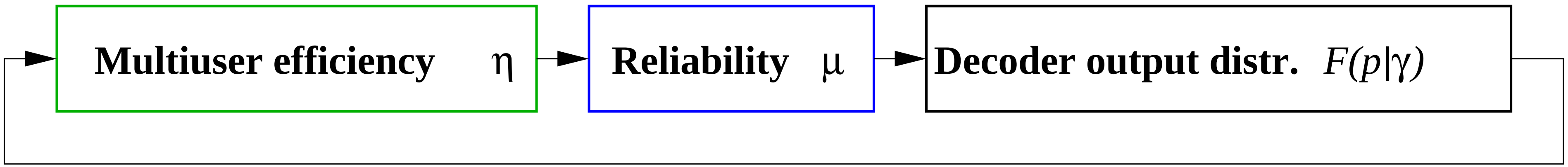
The scalar function

$$\mu(\gamma\eta)$$

can be determined by simulation and interpolation and fully characterizes the code properties (within the limits of the Gaussian approximation).

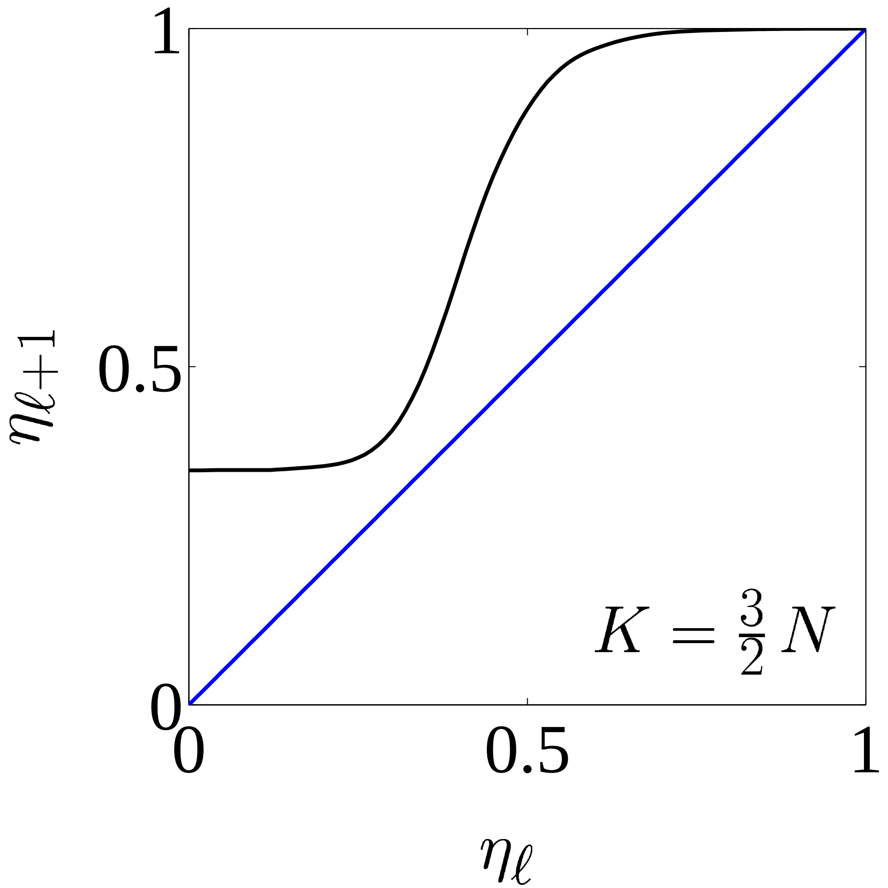
Dynamics of the Iterative Receiver

$$\eta_\ell \mapsto \mu_\ell \mapsto F_\ell(p|\gamma) \mapsto \eta_{\ell+1}$$



The multiuser efficiency characterises the dynamics of the iterative receiver.

Convergence Properties



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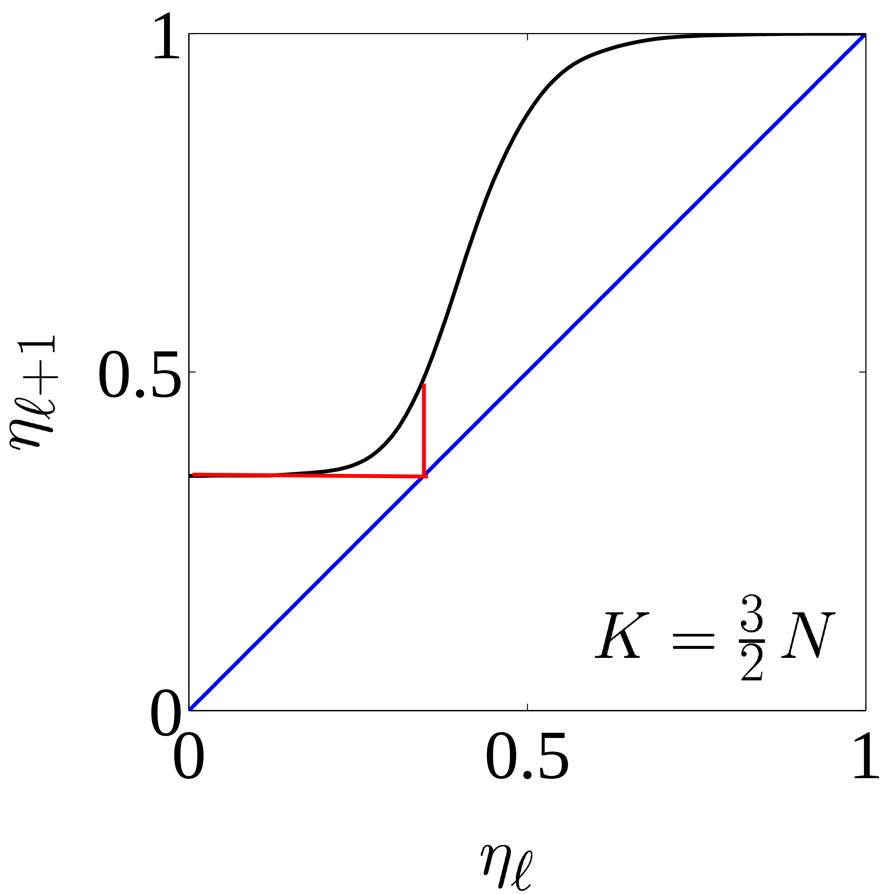

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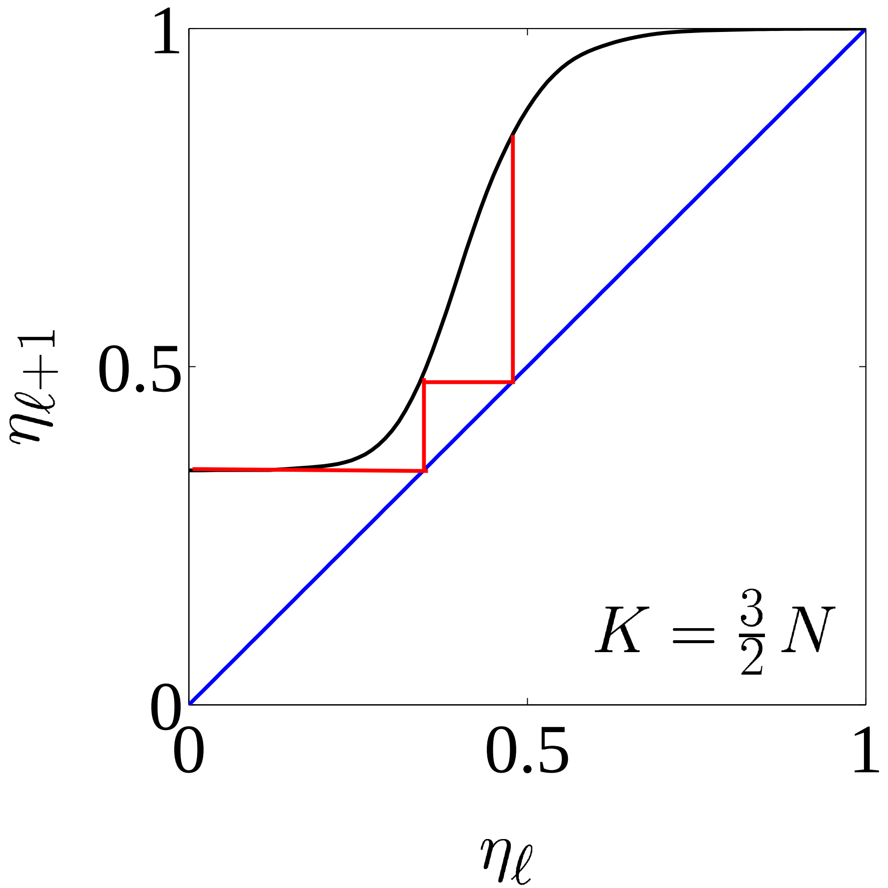

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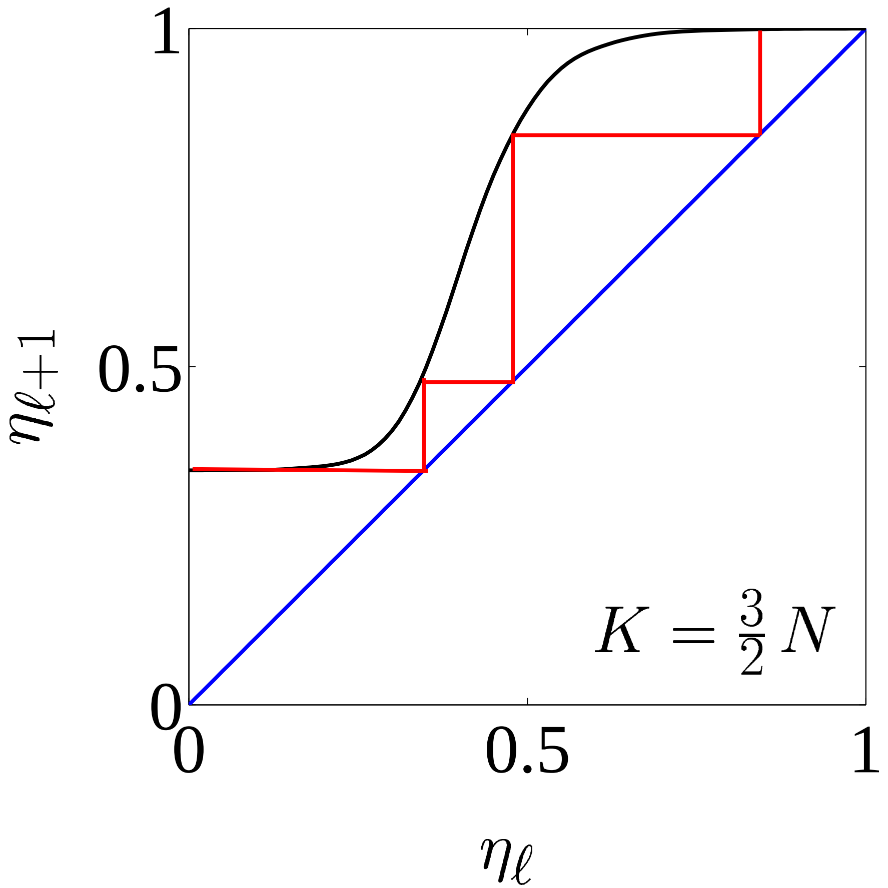

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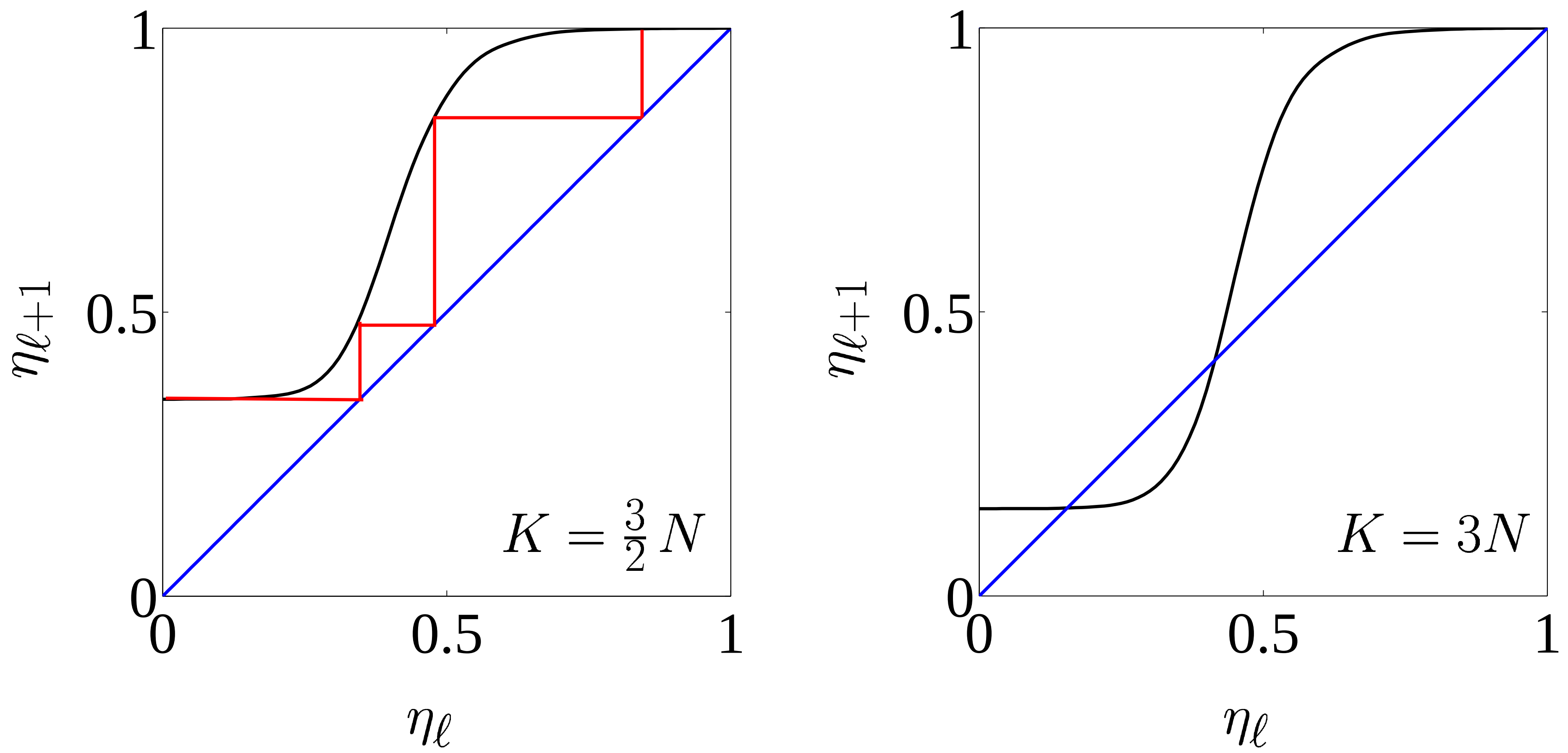

Power Optimisation

Convergence can be improved by disuniformising the users' power levels.

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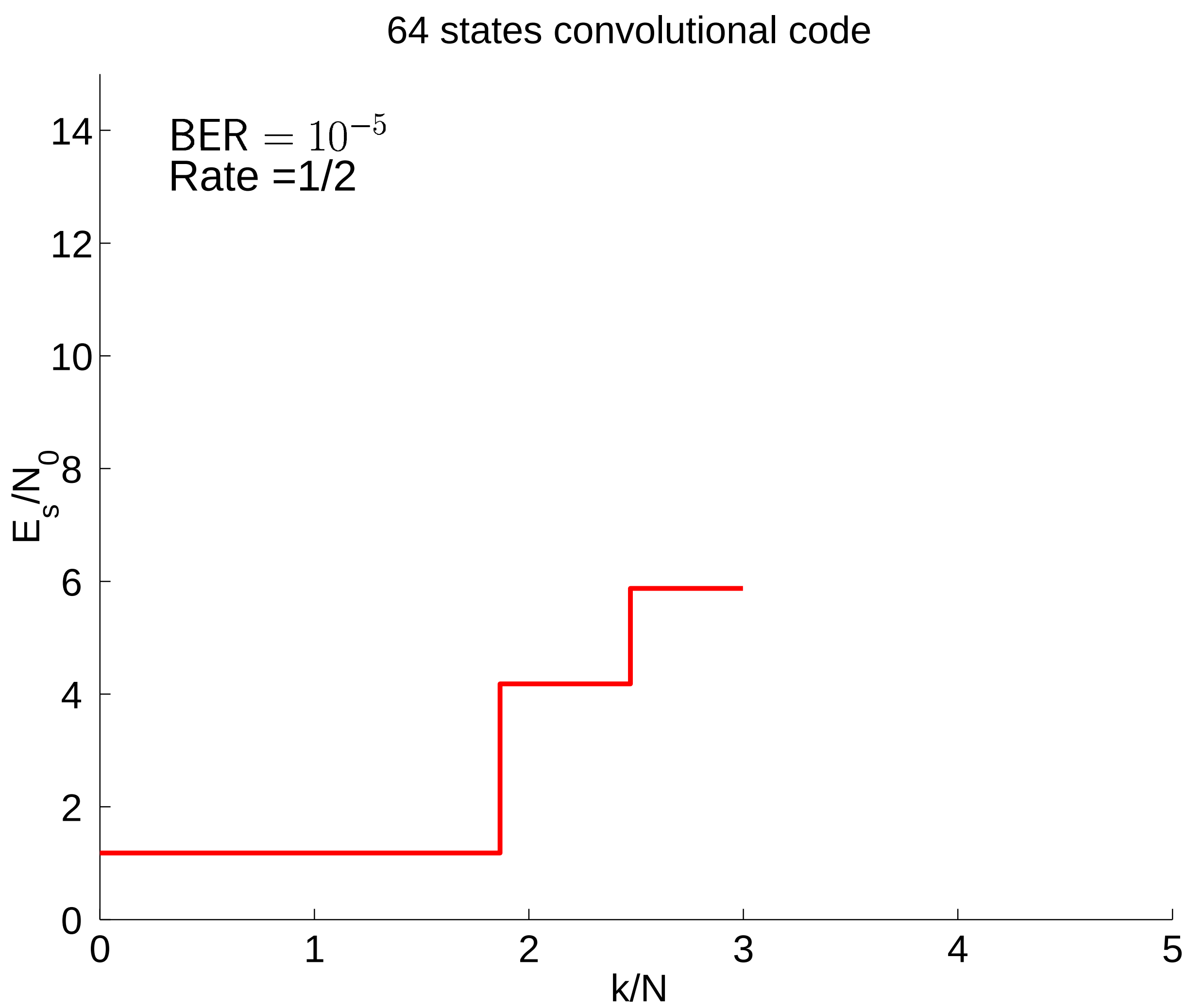
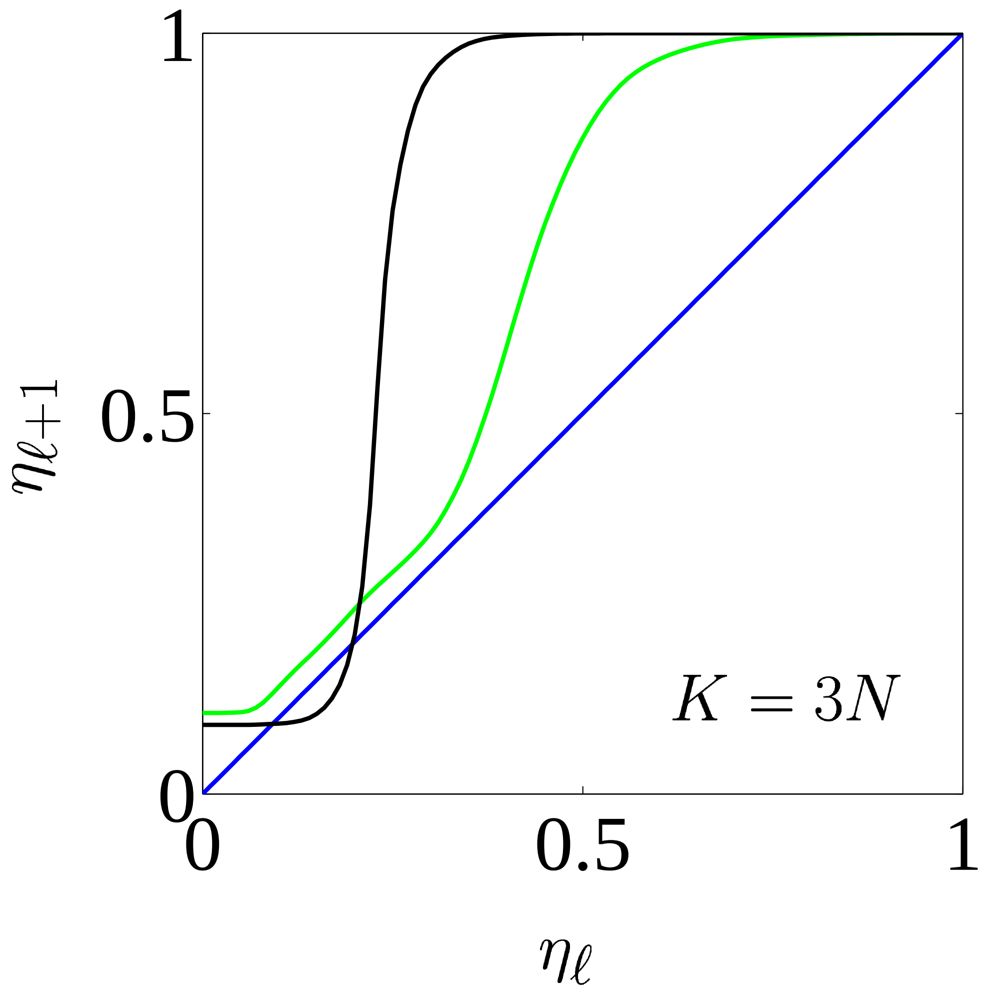
In the large system limit the power distribution with the minimum average power that allows for convergence of the multiuser efficiency towards at target multiuser efficiency (normally ≈ 1) can be found by solving a linear program.

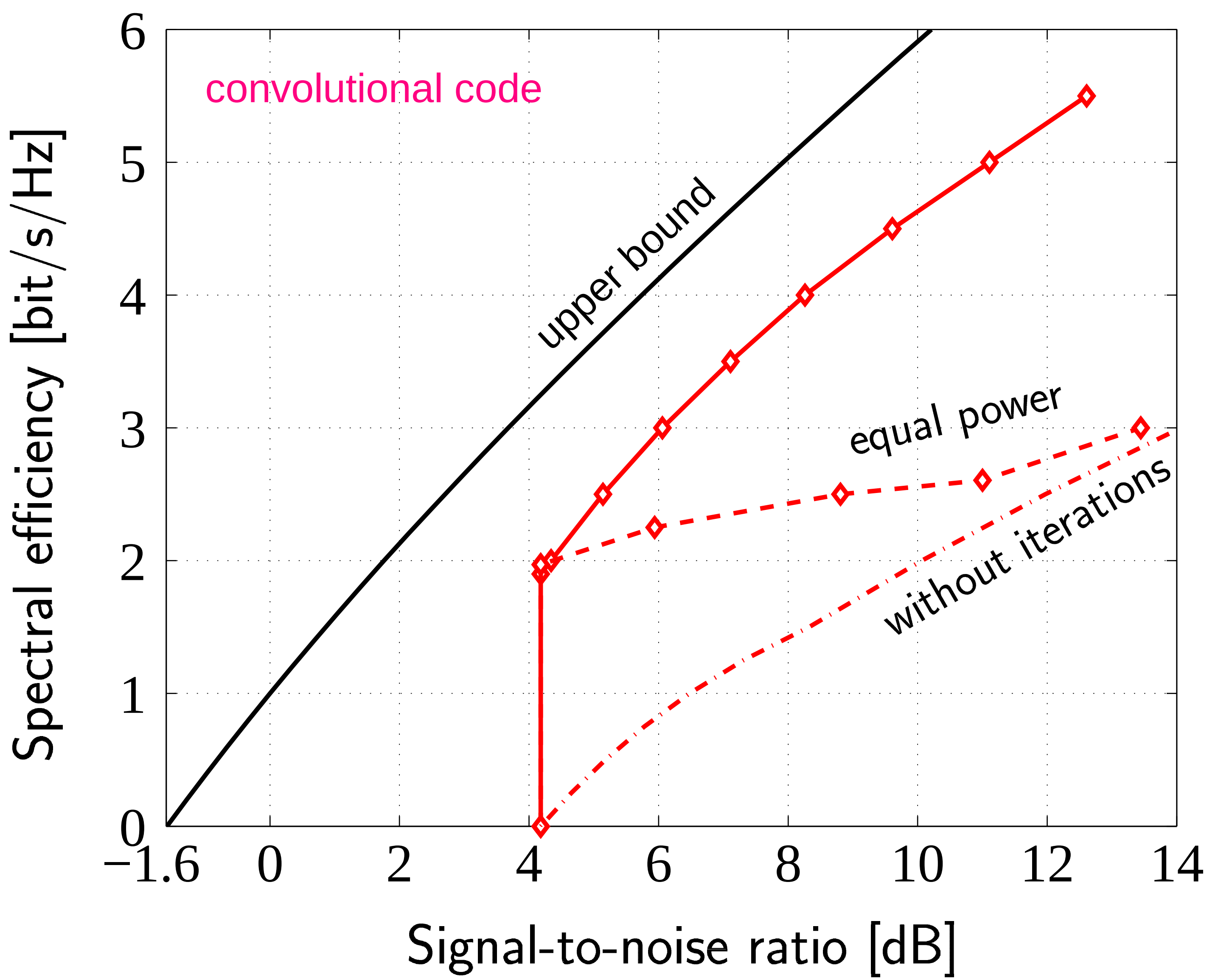
G. Caire, R. Müller and T. Tanaka, "Iterative multiuser joint decoding: Optimal power allocation and low-complexity implementation," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, September 2004.

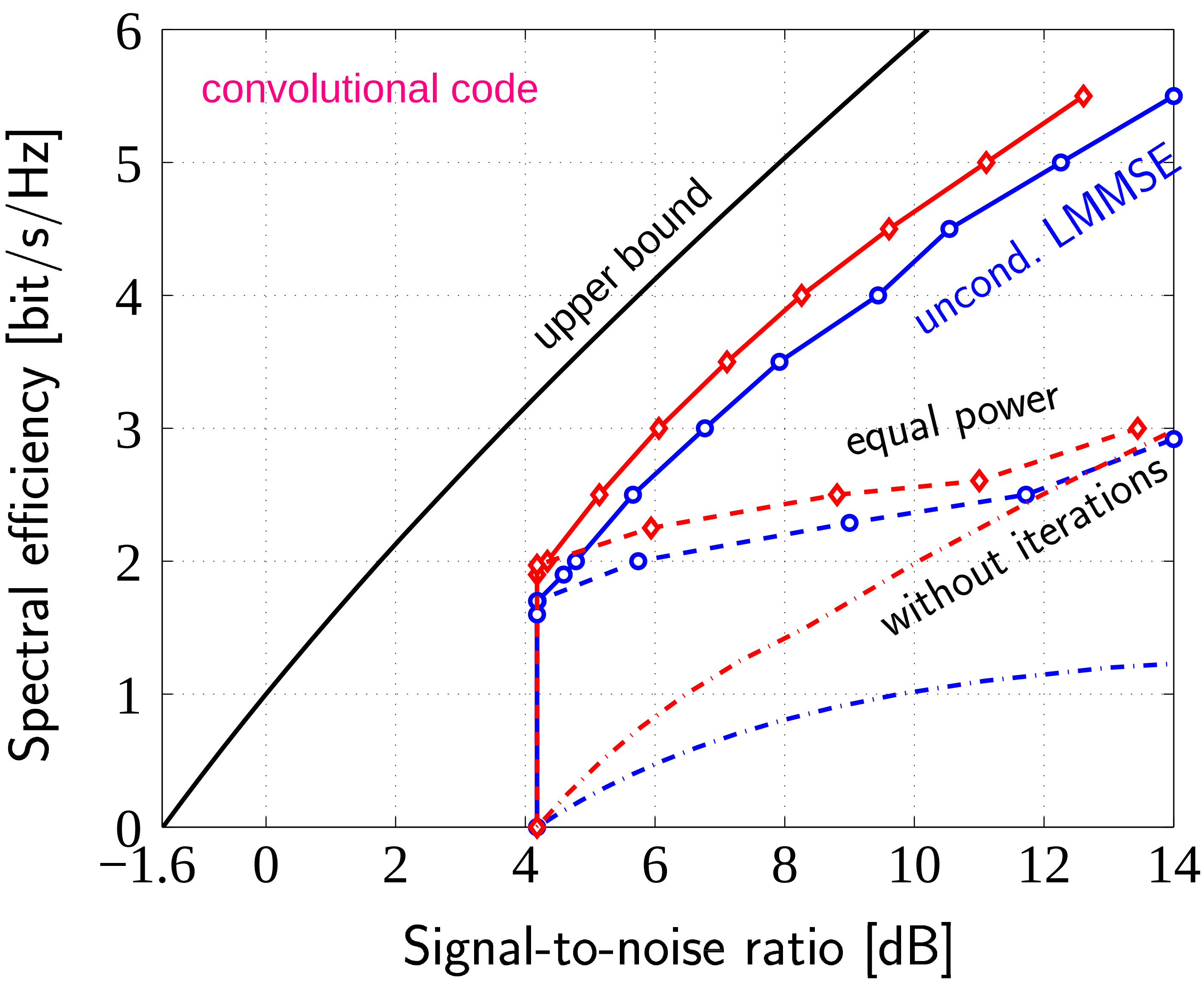
Example of Optimal Power Profile



Evolution Characteristic




convolutional code
upper bound
equal power
without iterations
Spectral efficiency [bit/s/Hz]
Signal-to-noise ratio [dB]
0
1
2
3
4
5
6
−1.6
0
2
4
6
8
10
12
14


convolutional code
upper bound
uncond. LMMSE
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"~~Apply sphere decoding to improve on the linear MMSE detector.~~"

There is little gain over the linear MMSE detector.
The stronger the code the less gain is possible.

Where Have the EXIT Charts Got Lost?

Instead of tracing mutual informations, multiuser efficiency was tracked.

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For any AWGN channel, any channel input distribution, the non-linear MMSE satisfies

$$\mathsf{MMSE}(\mathsf{SNR}) = 2 \frac{\partial}{\partial\, \mathsf{SNR}} \mathrm{I}(\mathsf{SNR}).$$

D. Guo, S. Shamai, and S. Verdú, "Mutual information and minimum mean-square error in Gaussian channels," *IEEE Trans. Inform. Theory*, submitted.