Issues in multiuser diversity

1. Feedback and real-time traffic
2. Traffic prediction and uncertainty
3. Multiple antennas

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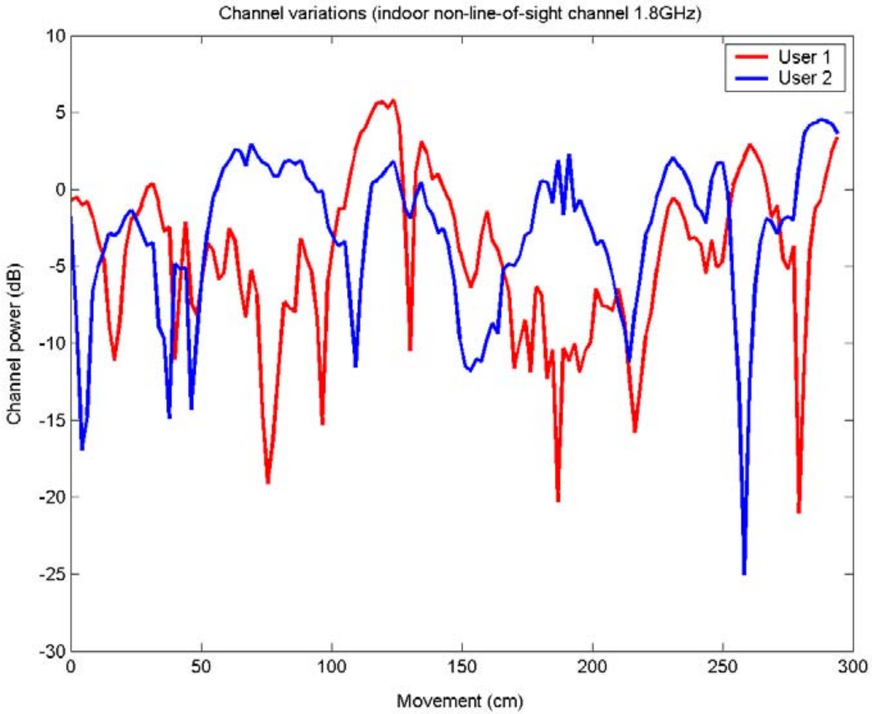
The benefit of channel variations

Consider a cellular downlink:

- Multiple users sharing the same resources
- Varying channel quality – ride the peaks!
  - More variations ⇒ more throughput



Multiuser diversity

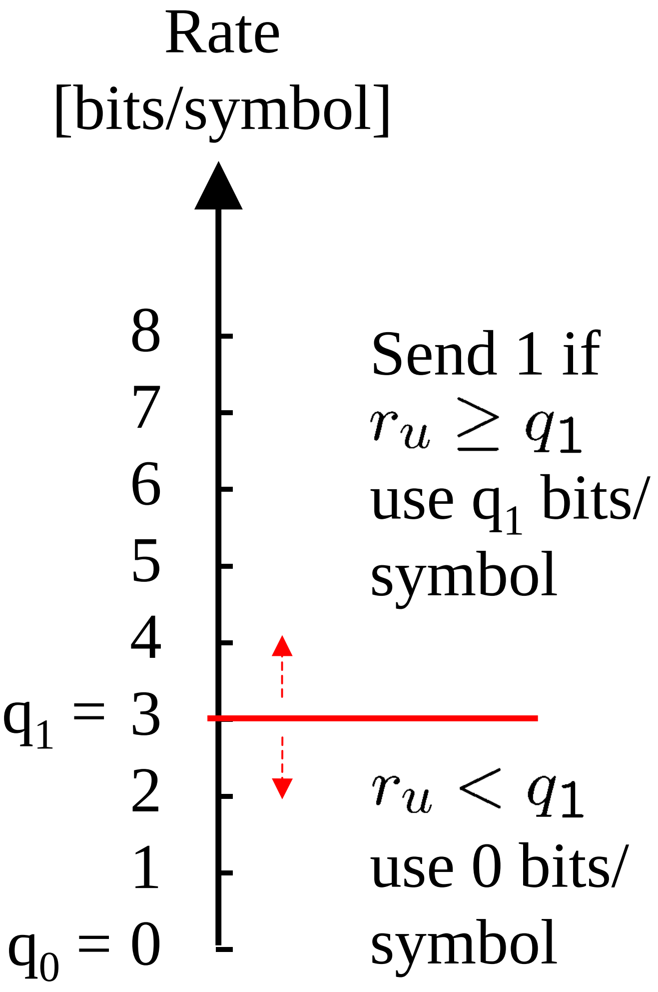
Limited channel feedback

Consider a downlink using adaptive modulation.

Each timeslot, each mobile feeds back $\log_2(M+1)$ bits indicating which rate the channel supports given a desired BER

$$r_u \approx \log_2\left(1+\frac{\mathrm{SNR}_u}{f(\mathrm{BER}_u)}\right)$$

Example: M=1



Send 1 if $r_u \geq q_1$ use q_1 bits/symbol

$r_u < q_1$ use 0 bits/symbol

Quantization for maximum expected throughput

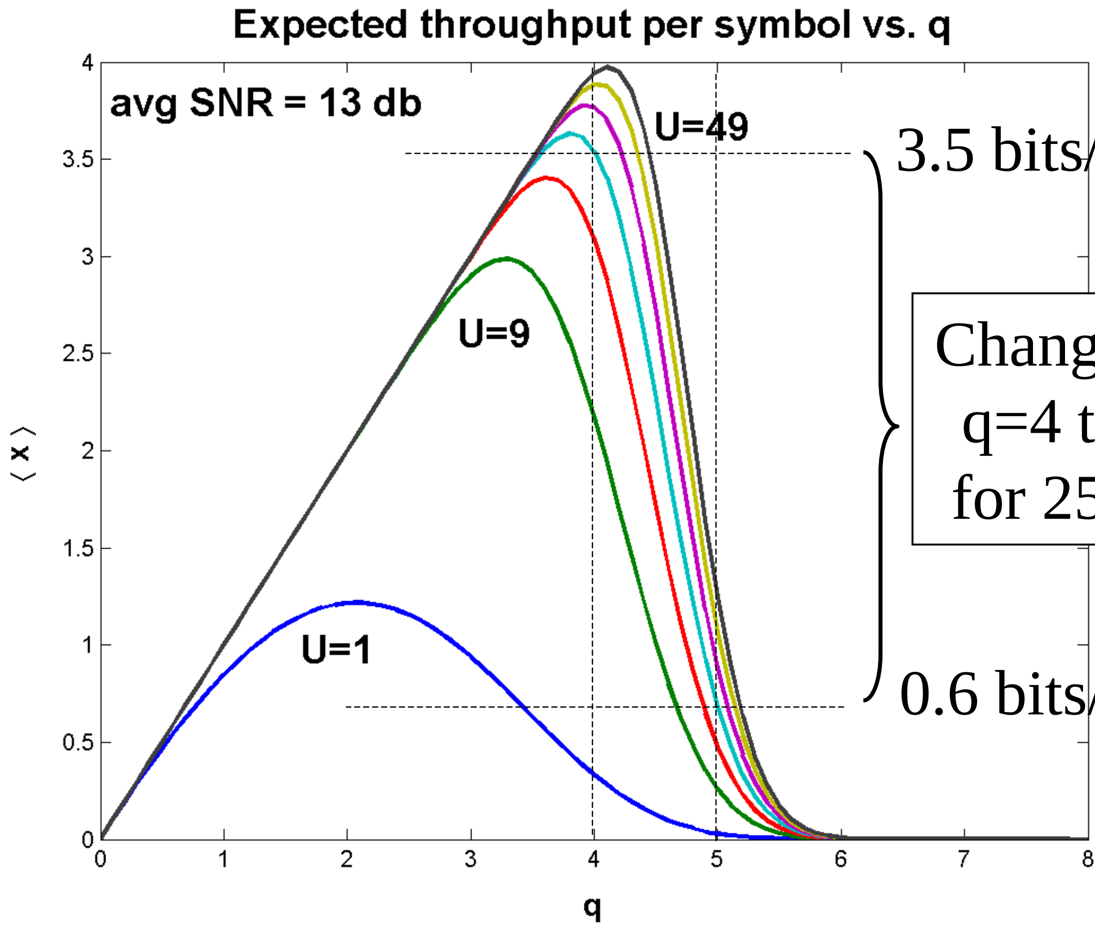
- The expected throughput with rate thresholds $q_0 \ldots q_M$ becomes

$$\langle x \rangle = \sum_{m=1}^{M} q_m \left(\prod_{u=1}^{U} \int_0^{q_m+1} p(r_u)dr_u - \prod_{u=1}^{U} \int_0^{q_m} p(r_u)dr_u \right)$$

  - *U* is the number of users
  - *p*(r_u) is the probability that user *u* can receive with rate r_u at a desired BER.
- 1-bit feedback:

$$\langle x \rangle = q \left(1 - \prod_{u=1}^{U} \int_0^{q} p(r_u)dr_u \right)$$

The downside



3.5 bits/symbol

Change from
q=4 to q=5
for 25 users

0.6 bits/symbol

As the number of users increases, the throughput becomes extremely sensitive to the choice of *q*.

Implications of 1-bit channel feedback

- Theoretically, strict multiuser diversity is not badly affected by limited feedback
- but in practice, an extreme sensitivity to correct quantizations leads to drastic performance drops
- Note also that unfairness increases when feedback is reduced

Possible remedies

- Individual thresholds
  - decrease the sensitivity
  - but optimal individual thresholds depend on other users’ thresholds
- Avoid using strict multiuser diversity
- Increase the channel feedback

We will combine the two first suggestions and at the same time attain short-term fairness

Diversity-Enhanced Equal Access

- A modified fair multiuser-diversity strategy
  - Round-robin tournament:
    - In each time slot transmit to the user with best channel of the users that have not yet accessed the channel.
    - When all users have obtained access, repeat the tournament.
  - Determine individual 1-bit quantizations locally at the mobile terminal
- Ideal for real-time traffic
- Unsensitive to channel quantization errors

Performance

- Note that the scheduler uses a strict multiuser-diversity strategy with a decreasing number of users
- On average, *U/2* users compete
  - Performance will on average equal that of strict multiuser diversity with *U/2* users
- Ex: Rayleigh fading, 16 users spread out
  - Throughput becomes 268% of that of a round-robin scheme with an optimal common 1-bit quantization

Performance

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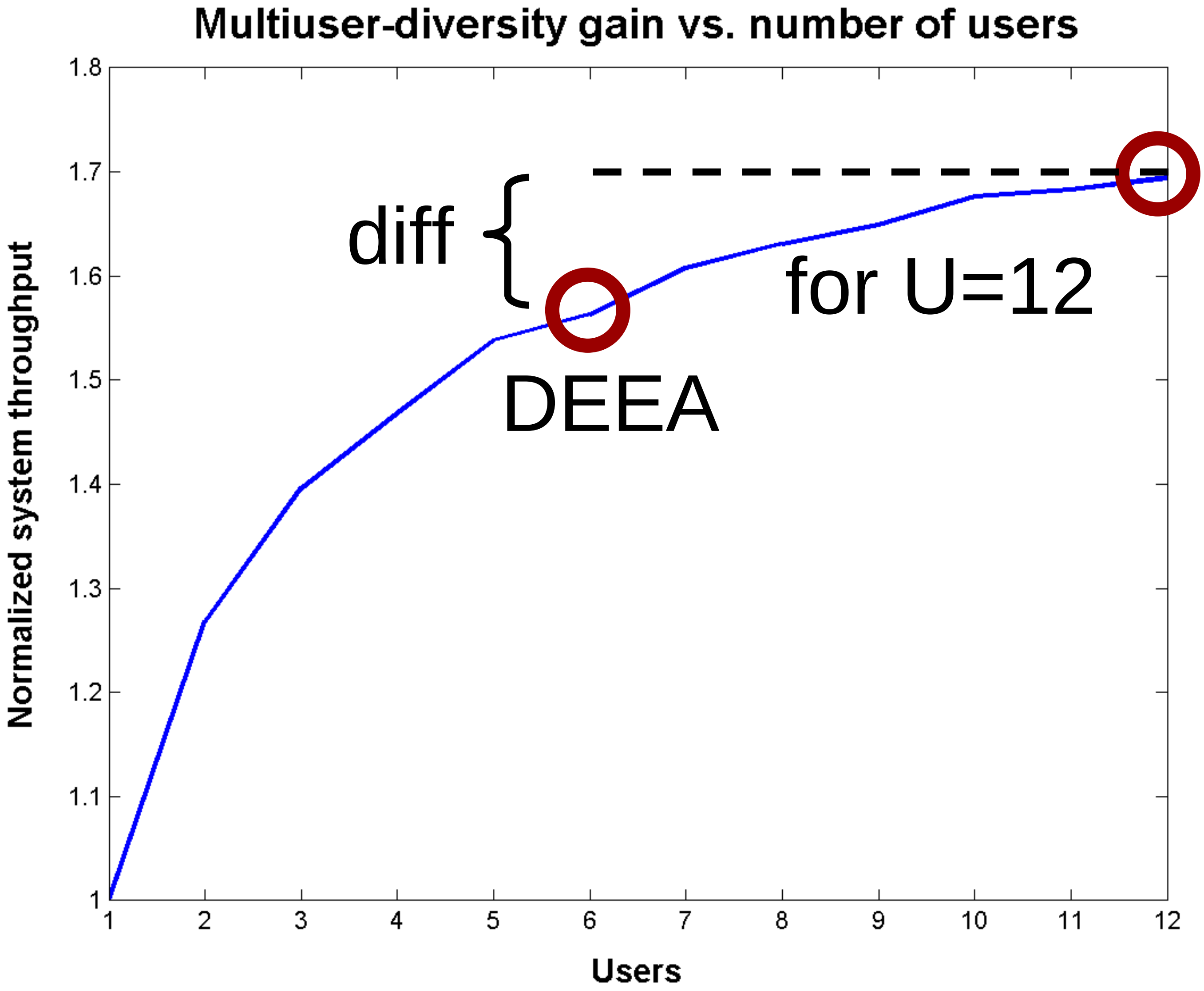

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2. Traffic prediction and adaptation

- In theory, better performance can be obtained by scheduling over several time slots
  - Particularly with QoS constraints
- Requires channel prediction over longer periods
  - Should average criterion over pdf for channel to account for higher uncertainty
- Requires traffic prediction
  - ”Always data to send” unrealistic assumption

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Scheduling under uncertainty

- Minimize the expected total buffer contents after the scheduled horizon
  - Gives maximum expected throughput
- Constrain assignments as in DEEA to satisfy delay constraints

$$\langle L \rangle = \sum_{u=1}^{U} \sum_{n_u=0}^{\infty} \sum_{x_{ut}=0}^{\infty} p(n_u|I)p(x_{ut}|I)g\left(S_u + n_u - \sum_{t=1}^{T} x_{ut}\right)$$

$$g(x) = x \text{ if } x > 0, \text{ otherwise } g(x) = 0$$

Traffic prediction

- We wish to determine p(n|I) given
  I = past arrival statistics
- Imagine using histograms
  - Too few observations in comparison to possible inflow sizes
- Instead, partition the inflow-axis into a number of ’bins’
  - Count arrivals within each bin
  - Adapt the bin size to obtain high resolution at intervals of high intensity and lower elsewhere

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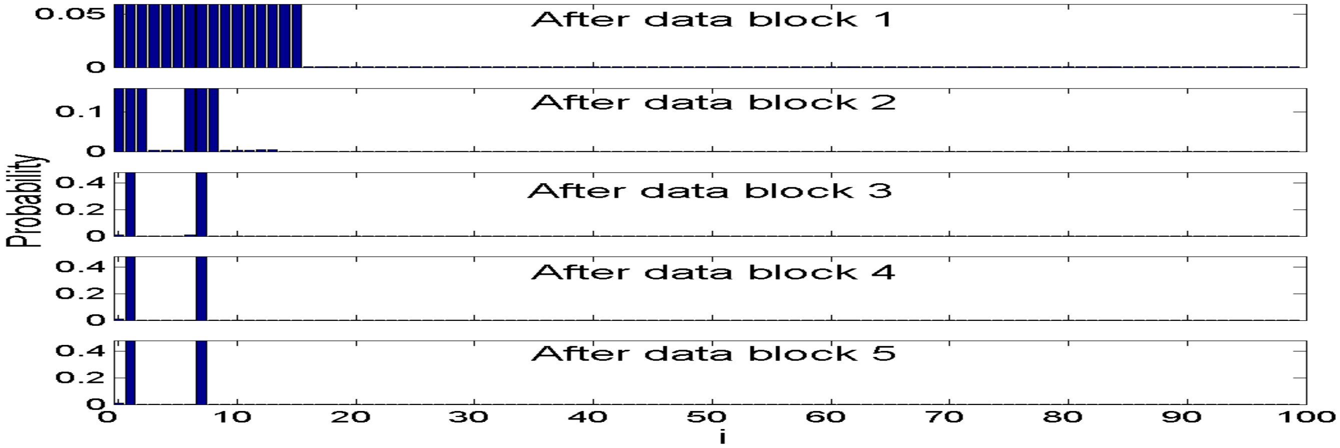
Traffic prediction – Bin probabilities



Using K bins and letting

- m_k = past number of arrivals of size within bin k
- M = total number of observations

we have (after some calculations...)

$$p(n \in \text{bin } k \mid m_k, M, I) = \frac{m_k + 1}{M + K}$$

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Traffic prediction – Adaptation

- Based on the bin probabilities, how do we adapt the bin positions and sizes?
  - Wish to have a quantized distribution which is as close to the exact distribution as possible.
- Formally, we wish to maximize the mutual information between the two distributions

Theorem:

Maximizing the mutual information is equivalent to maximizing the entropy of the bin probability distribution.

Traffic prediction – Adaptation

Proof:

$$\begin{aligned}
I(k,n) &= \sum_{k=1}^{K}\sum_{n=n_{min}}^{n_{max}} p(nk)\log\frac{p(nk)}{p(n)p(k)} \\
&= \sum_{k=1}^{K}\sum_{n=n_{min}}^{n_{max}} p(nk)\log\frac{p(k\mid n)}{p(k)} \\
&= -\sum_{k=1}^{K}\sum_{n\in\text{bin }k} p(n\mid k)p(k)\log p(k) \\
&= -\sum_{k=1}^{K} p(k)\log p(k)
\end{aligned}$$

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Traffic prediction

- The optimum bin partition is adapted according to the M most recent arrivals:
  - Assume a uniform probability distribution within each bin,
    p(n) = bin probability / bin width
  - Redistribute the bins so that each bin has equal probability mass (=max entropy)
  - Approximate low-complexity solution requires single sweep over the possible arrival sizes.

Traffic prediction – Simulation set-up

- K = 6 bins,
- M = 100 observations between bin updates
- Min arrival rate=0, Max=100 [bit/time unit]
- Arrivals generated as
  - 50% 1-bit packets,
  - 50% 7-bit packets

  (switching between 2 fixed rates with equal frequency)

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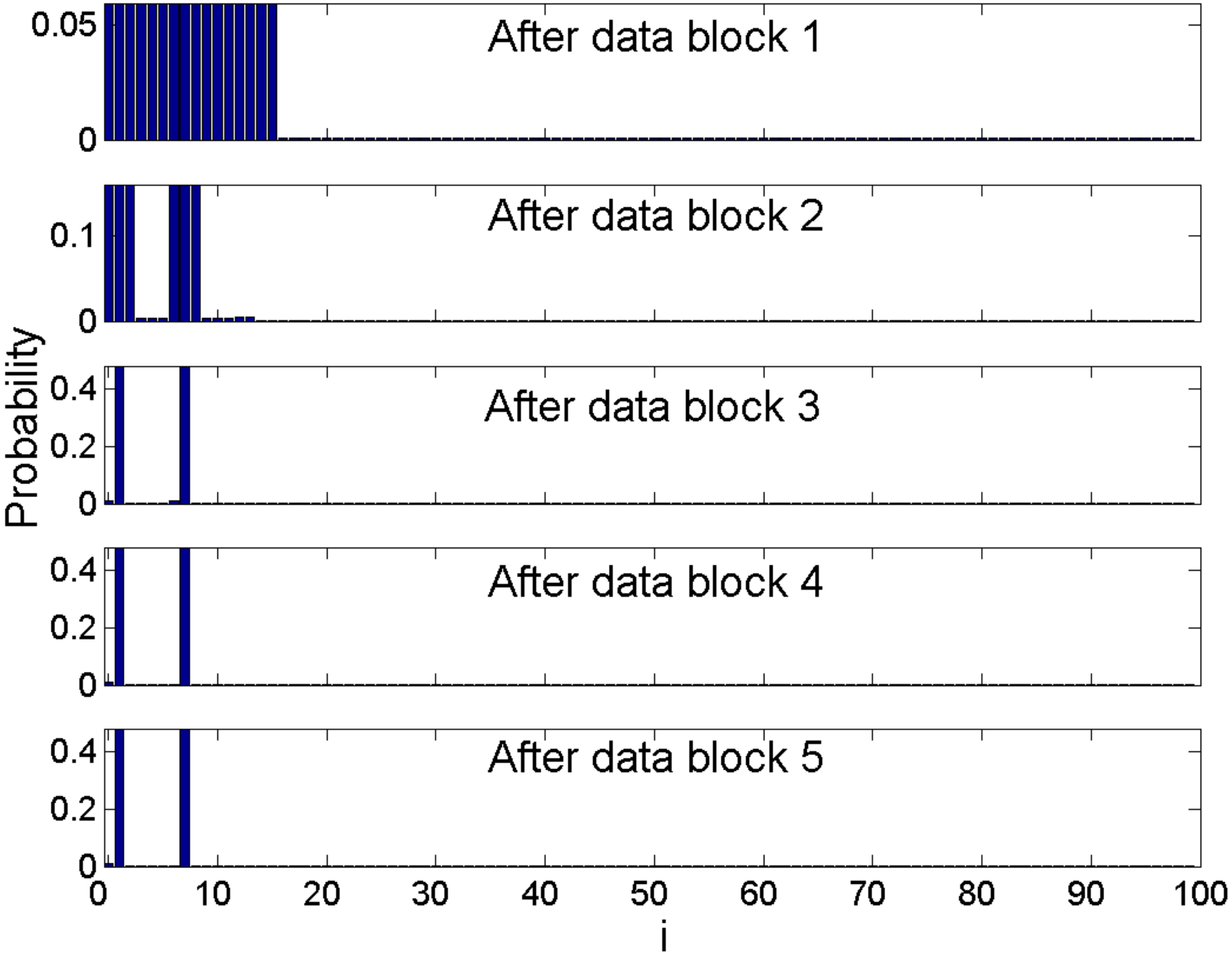
Traffic prediction – Results



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3. MIMO-Multiuser Diversity

- With a single channel, capacity-optimal schedule seds to one user at a time (Knopp&Humblet -95)

  $C \sim \log(\log U)$

- Recently, the Shannon capacity for the MIMO case has been found (Caire&Shamai 2001, Vishwanath&Jindal&Goldsmith 2002, Viswanath&Tse 2002)
- The capacity-optimal scheme requires full channel knowledge at the transmitter and is achieved by Costa-precoding (extremely complex)
- Single-user transmission no longer optimal!

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MIMO-Multiuser Diversity

- Consider M Tx antennas, and 1 Rx antenna and full channel knowledge at transmitter and receiver
- Then, capacity for Gaussian MIMO broadcast channel for large U is

  $$C \sim M \log(\log U)$$

  (Sharif&Hassibi -03, submitted)
- Sharif & Hassibi proposes using M random beams where each user feeds back best beam and SINR
  - approaches capacity when U → ∞

A simple MIMO-multiuser approach

- Transmit to different users on each antenna
- Let each user feed back best SINR and best antenna
- On each antenna, transmit to the user with highest SINR

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A simple MIMO-multiuser approach

- Assuming M = 2 Tx antennas, and that noise is small compared to the interference from the other antenna, the rate is

$$\boxed{\begin{aligned} R_u &\leq \log\left(1 + \frac{|h_{1u}|^2}{\sigma^2 + |h_{2u}|^2}\right) \\ &\approx \log\left(1 + \frac{|h_{1u}|^2}{|h_{2u}|^2}\right) \end{aligned}}$$

A simple MIMO-multiuser approach

- Assume independent, identically-distributed flat Rayleigh fading on both antennas. (Exponentially distributed $|h|^2$).
- The distribution of a ratio of two exponentially distributed numbers with the same mean is independent of that mean.

Thus, the performance will not depend on the average channel gain!

- Unfairness no longer a big problem

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Performance

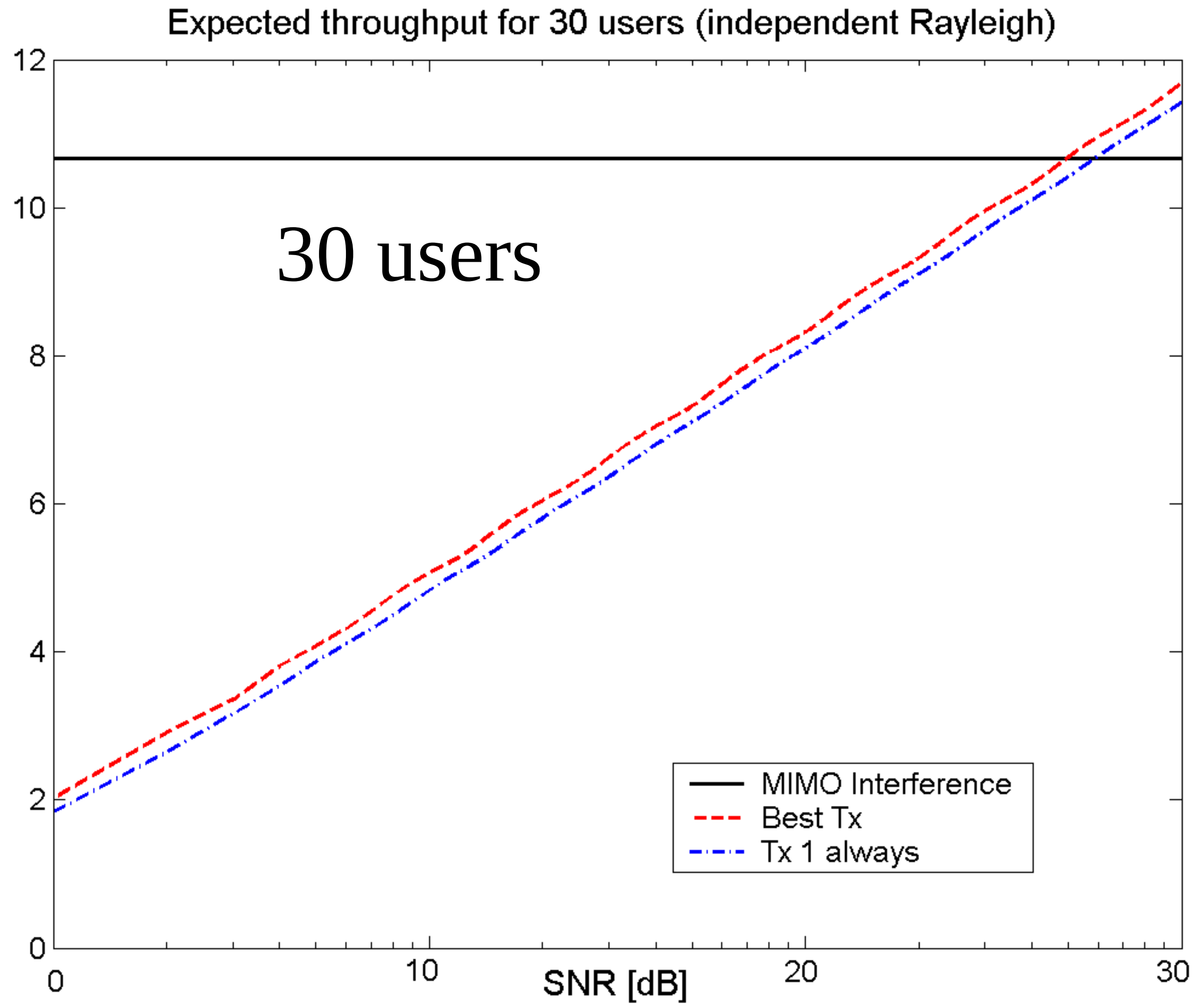
- The expected throughput depends on the probability for having high gain on one antenna, low gain on the other.

$$R_u \approx \log\left(1 + \frac{|h_{1u}|^2}{|h_{2u}|^2}\right)$$

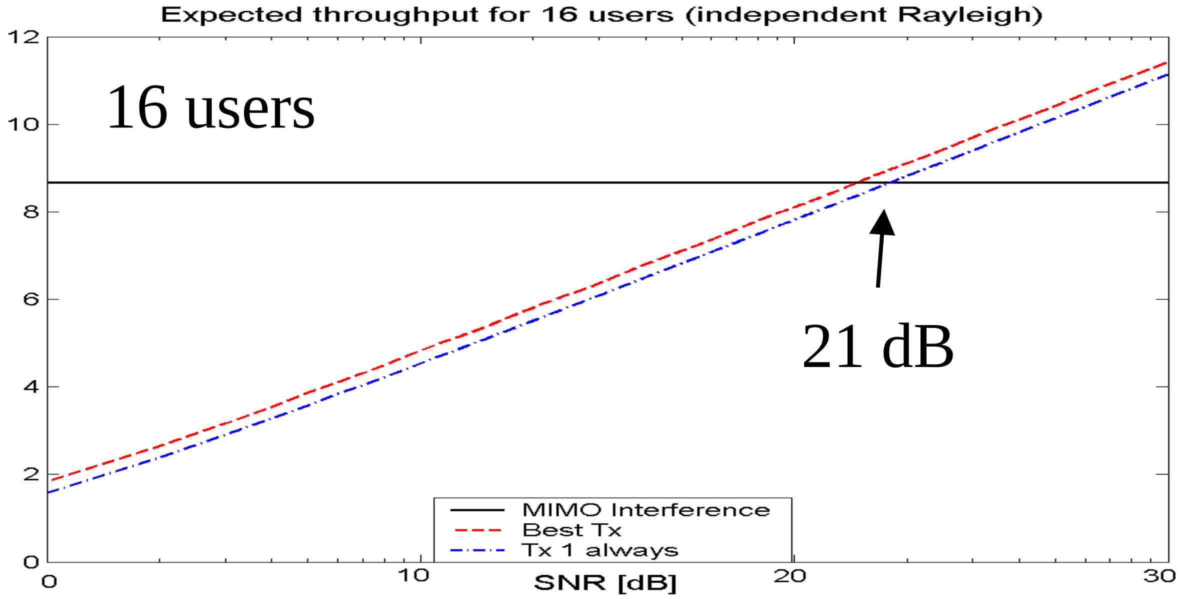
- More users ⇔ Higher probability!

- Compare the expected throughput of this scheme to that of
  - always sending only on antenna 1,
  - sending only to the best user on the best antenna (equal to one-antenna case with twice as many users)

Performance



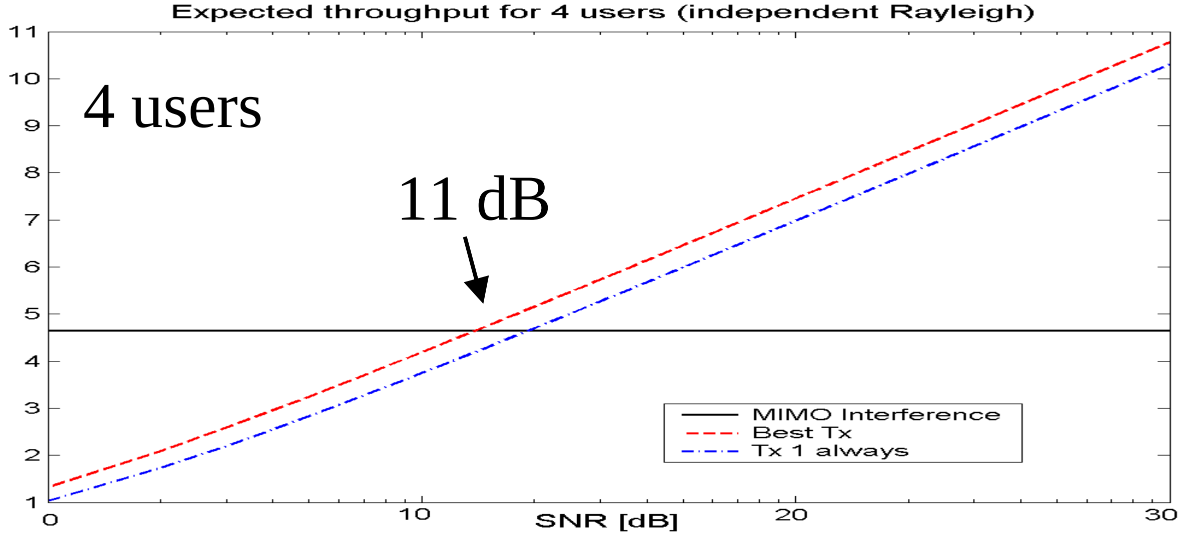
Performance



Expected throughput for 16 users (independent Rayleigh)

16 users

21 dB



Expected throughput for 4 users (independent Rayleigh)

4 users

11 dB

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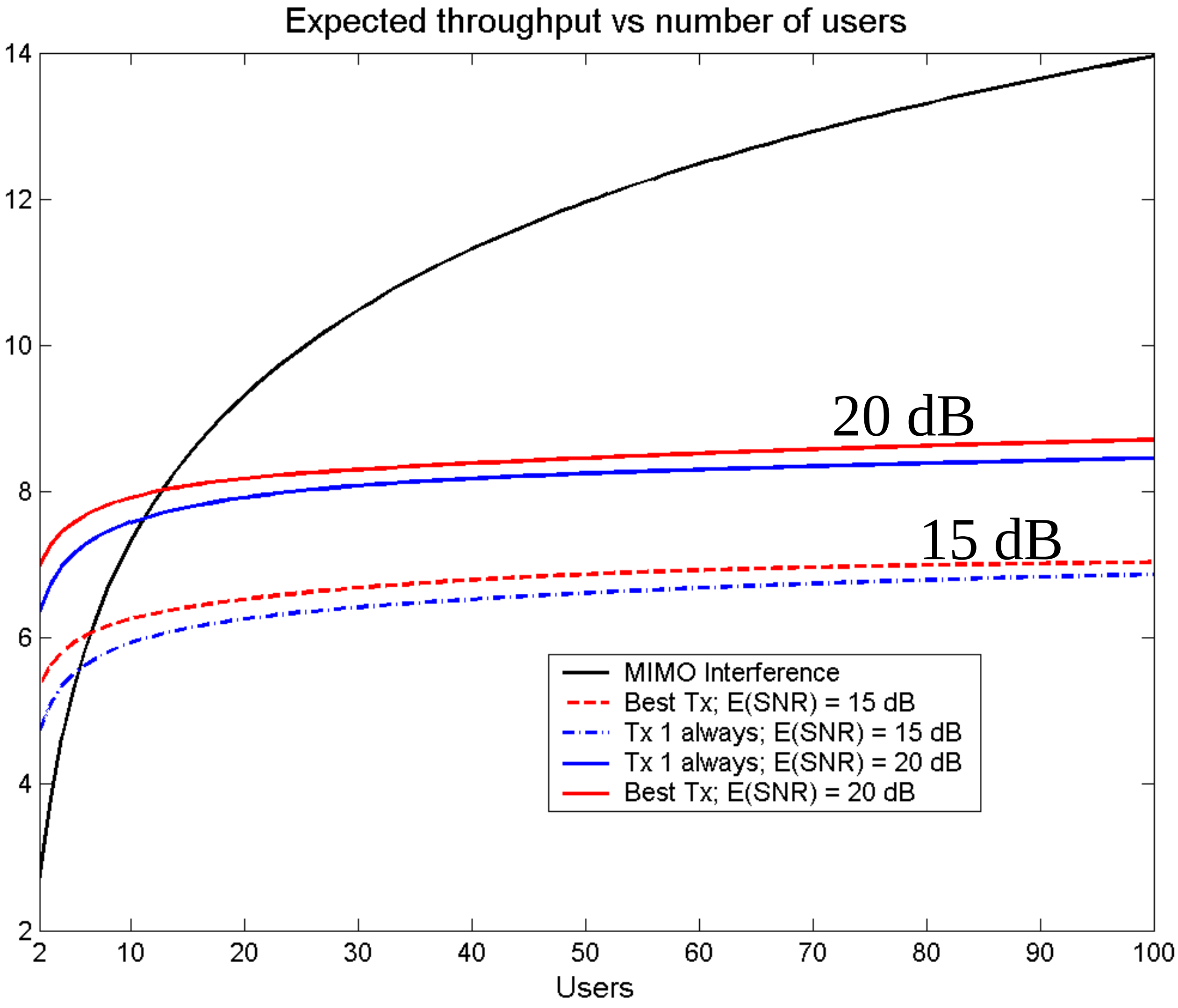
Multiuser-diversity gain



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Concluding remarks

- Real-time traffic and multiuser diversity is compatible for large U
  - Reason is log(logU) behavior of rate
- Traffic prediction will be required if scheduling over longer time horizons
  - presented new adaptive method
- Multiuser-MIMO requires transmitting mote than one data stream at a time to approach capacity
  - simple interference-channel approach seems promising