Transmitting over Ill-behaved MIMO Channels

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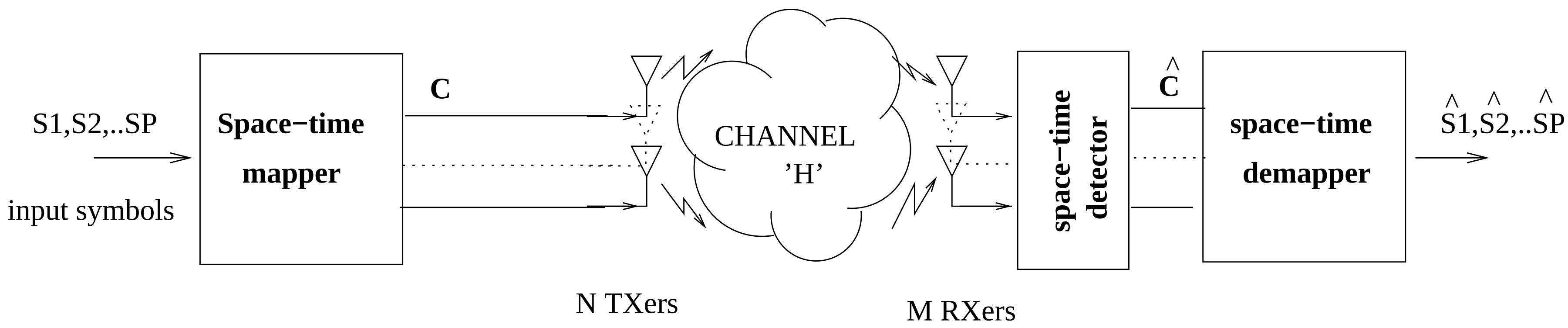
Outline

- Space-time codes
- Ill-behaved MIMO channels
- A Precoding approach
  - For low-rate codes
  - For high-rate codes

Space-time codes



Space-time codes: General model

Step 1: P input (QAM) modulation symbols are mapped to a *codeword matrix* of size $M_t \times K$.

$$\{s_1, s_2, .., s_P\} \Rightarrow \boldsymbol{C} = \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & & \vdots \\ c_{M_t}(1) & c_{M_t}(2) & \dots & c_{M_t}(K) \end{bmatrix}$$

Step 2: The codeword is launched into the channel. We receive:

$$\boldsymbol{Y} = \begin{bmatrix} h_{11} & \dots & h_{1M_t} \\ \vdots & & \vdots \\ h_{M_r1} & h_{M_r2} & h_{M_rM_t} \end{bmatrix} \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & & \vdots \\ c_{M_t}(1) & c_{M_t}(2) & \dots & c_{M_t}(K) \end{bmatrix} + Noise.$$

Low rate vs. high rate codes

Low rate codes

- Diversity (redundancy) oriented: E.g. $P = K$ (rate 1 symbol per channel use).
- Orthogonal ST codes are such that $\boldsymbol{C}$ is a unitary matrix (up to a scalar).
- For uncorrelated channels: Diversity order is $M_t M_r$.
- Sensitive to correlation. Moderately sensitive to rank.

High rate codes

- Rate (low redundancy) oriented: E.g. $P = KM_t$ (rate M_t symbols per channel use).
- Sensitive to correlation. Very sensitive to rank.
- Unable to extract capacity in strongly Ricean or correlated channels.

Ill-behaved MIMO channels

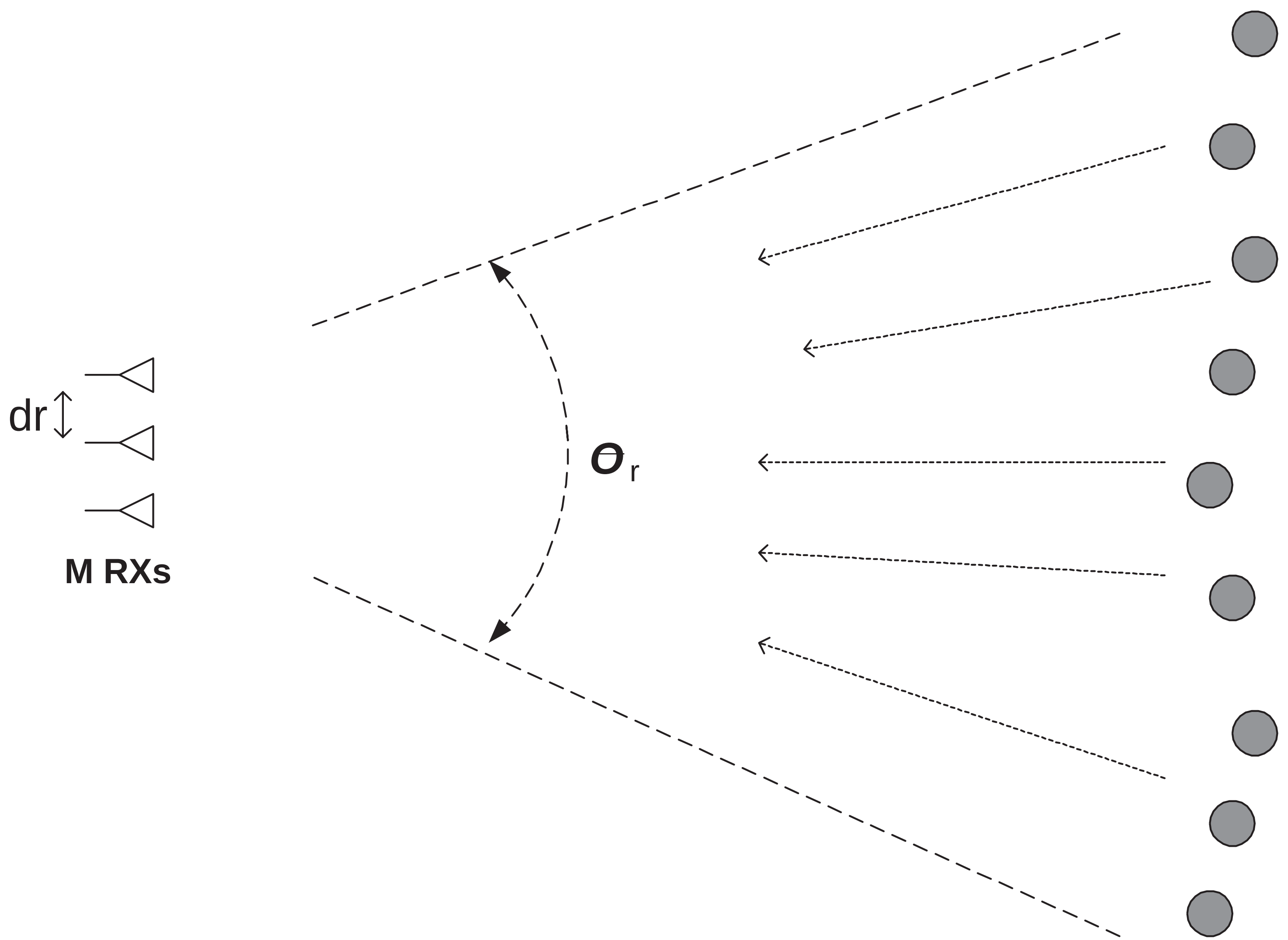
Ill-behaved channels occur due to:

- Fading correlation at TXer and/or RXer
- Correlation model is non separable (Kronecker model not realized)
- Low rank Rice component dominates

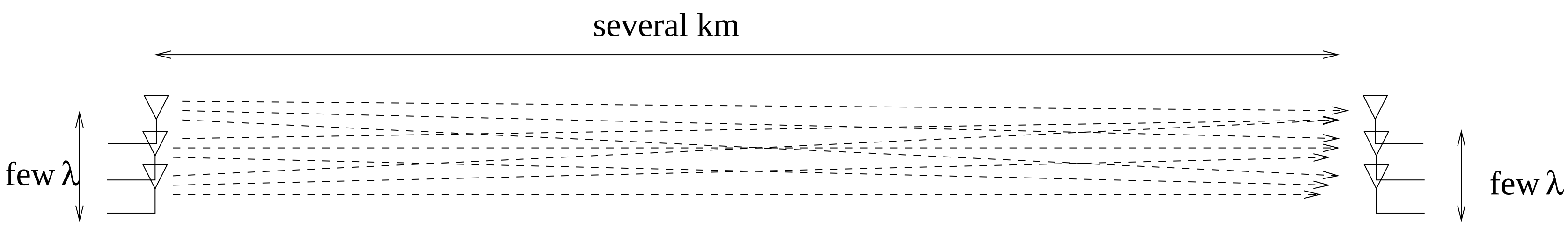
Leading to:

- *Effective diversity* order reduced
- *Effective rank* reduced

Antenna Array Correlation



Rank behavior of Rice component



The rank of LOS component is full iff [Gesbert et al. TCOM Dec. 2002]

$$A_t A_r \geq \frac{4\lambda R}{M_r}$$

where A_t, (resp. A_r) is TX (resp. RX) array aperture, R is range...never realized!

Dealing with Ill-behaved MIMO channels

- Propagation (Rice, correlation) scenario hard to predict
- Transmitter design must be *adaptive*.
- Low-rate statistical feedback is realistic.
- We want to keep rate constant.
- We want low-complexity closed-form solution where possible!

Idea

- Design a linear precoder based on statistical feedback
  - Part 1: for low-rate space-time block codes
  - Part 2: for high-rate space-time (multiplexing) codes

Part one

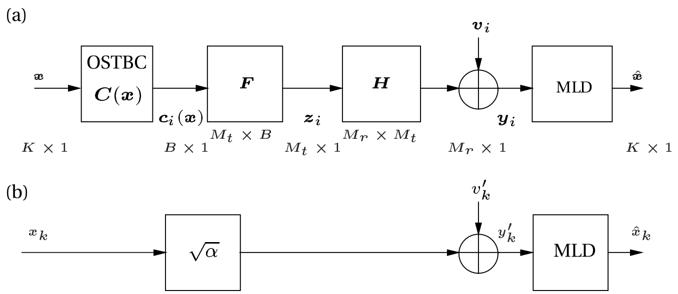
- Linear precoding for low-rate space-time orthogonal block codes

The interesting questions!

Let us assume the TX knows the full correlation information (TX,RX) but not the instantaneous channel...

1. Can we tackle transmitter correlation using a linear precoder?
   - Answer: Yes! [Giannakis et al][Sampath et al][others]
2. Can we find the optimal precoder in minimum SER sense?
   - Answer: Unknown! (Only bounds on pairwise error prob. used so far)
3. Can we find the precoder for correlated *receiver* case? Is there any gain?
   - Answer: Unknown! (Ergodic capacity says no gain!)
4. Can we find the precoder for non-separable TX-RX correlations?
   - Answer: Unknown!
5. Can we find a good precoder in closed-form for correlated receiver case?
   - Answer: Unknown!

Precoded Space-time Block Coding



$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{F}\boldsymbol{C}(\boldsymbol{s}) + \boldsymbol{V}. \tag{1}$$

Correlated MIMO channels

General case: Joint correlation at TX and RX

$$\mathrm{vec}\left(\boldsymbol{H}\right) = \boldsymbol{R}^{1/2}\,\mathrm{vec}\left(\boldsymbol{H}_w\right) \tag{2}$$

where $\boldsymbol{H}_w$ is i.i.d. Rayleigh MIMO matrix

The Kronecker case (Correlation at TX and RX are separable)

$$\boldsymbol{R} = E\left[\mathrm{vec}\left(\boldsymbol{H}\right)\mathrm{vec}^H\left(\boldsymbol{H}\right)\right] = \boldsymbol{R}_t^T \otimes \boldsymbol{R}_r, \tag{3}$$

...is not always true! [Bonek et al. 04]

Expressions for Symbol Error Rate (SER)

For M-QAM (also available for M-PSK,M-PAM [Simon & Alouini]):

γ is the instantaneous SNR, depending on channel, precoder

$$SER_\gamma = \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-\frac{g_{\text{QAM}}\gamma}{\sin^2(\theta)}} d\theta + \frac{1}{\sqrt{M}}\int_0^{\frac{\pi}{4}} e^{-\frac{g_{\text{QAM}}\gamma}{\sin^2(\theta)}} d\theta\right], \tag{4}$$

Average SER:

$$SER \triangleq \Pr\{\text{Error}\} = \int_0^\infty SER_\gamma p_\gamma(\gamma) d\gamma. \tag{5}$$

Looks like the defintion of a moment generating function $\phi_\gamma(s) = \int_0^\infty p_\gamma(\gamma)e^{s\gamma}d\gamma$.

Exact SER as function of precoder

For M-QAM (also available for M-PSK,M-PAM) [Hjorungnes & Gesbert 04]:

$$SER = \frac{4}{\pi}\frac{\sqrt{M}-1}{\sqrt{M}}\left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{d\theta}{\det\left(\boldsymbol{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2\theta}\boldsymbol{\Phi}\right)} + \frac{1}{\sqrt{M}}\int_{0}^{\frac{\pi}{4}} \frac{d\theta}{\det\left(\boldsymbol{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2\theta}\boldsymbol{\Phi}\right)}\right] \quad (6)$$

Where matrix $\boldsymbol{\Phi}$ defined by

$$\boldsymbol{\Phi} = \boldsymbol{R}^{1/2}\left[\left(\boldsymbol{F}^{*}\boldsymbol{F}^{T}\right) \otimes \boldsymbol{I}_{M_r}\right]\boldsymbol{R}^{1/2}. \quad (7)$$

The optimum precoder: problems and properties

Optimum precoder:

$$\min_{\{\boldsymbol{F}\in C^{M_t\times B}\}} SER \tag{8}$$

$$\text{subject to} \quad Ka\sigma_x^2 \,\mathrm{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = P \tag{9}$$

Some properties:

- If $\boldsymbol{F}$ is an optimal precoder, then $\boldsymbol{F}\boldsymbol{U}$, where $\boldsymbol{U}$ is unitary, is also optimal.
- if SNR$\rightarrow\ \infty$ then the optimal precoder is given by the trivial precoder $\boldsymbol{F} = \beta\boldsymbol{I}_{M_t}$ for the M-PSK, M-PAM, and M-QAM constellations.
- If $\boldsymbol{R} = \boldsymbol{I}_{M_tM_r}$, then the optimal precoder is given by the trivial precoder $\boldsymbol{F} = \beta\boldsymbol{I}_{M_t}$ for the M-PSK, M-PAM, and M-QAM constellations.
- If Kronecker model holds and $\boldsymbol{R}_t = \boldsymbol{I}_{M_t}$, then $\boldsymbol{F} = \beta\boldsymbol{I}_{M_t}$.

One more property

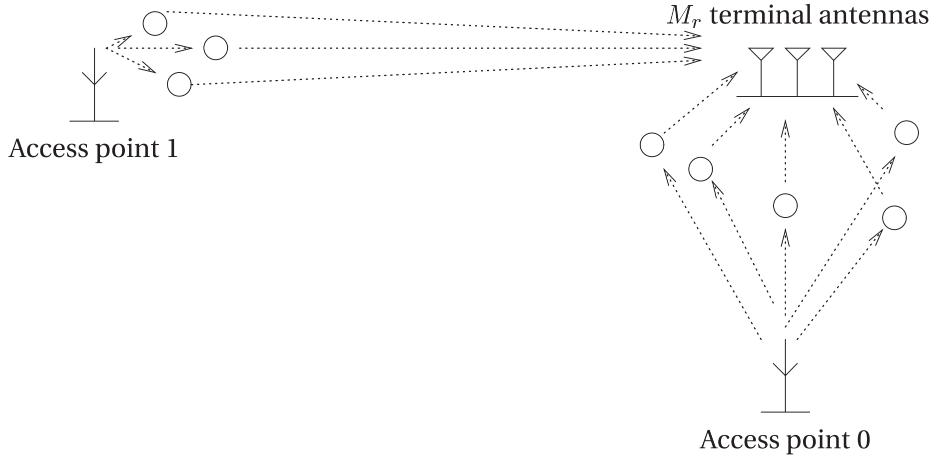
If no transmitter correlation, then the general-case correlation writes:

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{r_0} & \mathbf{0}_{M_r \times M_r} & \cdots & \mathbf{0}_{M_r \times M_r} \\ \mathbf{0}_{M_r \times M_r} & \boldsymbol{R}_{r_1} & \cdots & \mathbf{0}_{M_r \times M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{M_r \times M_r} & \mathbf{0}_{M_r \times M_r} & \cdots & \boldsymbol{R}_{r_{M_t - 1}} \end{bmatrix}, \tag{10}$$

Theorem:

- In this case, optimal $\boldsymbol{F}$ can be chosen diagonal, real, positive, up to a unitary matrix. (power scaling strategy).

Non separable-No TX correlation



Non separable-No TX correlation: A closed form solution

Equivalent SISO channel formulation for $M_t = 2$ case:

$$\alpha = f_0^2 \|\boldsymbol{h}_0\|^2 + f_1^2 \|\boldsymbol{h}_1\|^2 = f_0^2 \sum_{j=0}^{M_r-1} \lambda_{r_{0_j}} |h'_{w_{0_j}}|^2 + f_1^2 \sum_{j=0}^{M_r-1} \lambda_{r_{1_j}} |h'_{w_{1_j}}|^2. \tag{11}$$

where $\{\lambda_{r_{0_j}}\}$ and $\{\lambda_{r_{1_j}}\}$ are the eigenvalues of $\boldsymbol{R}_{r_0}$ and $\boldsymbol{R}_{r_1}$. $h_{w_{i_j}}$ are i.i.d. Gaussian.

Maximum diversity spread principle: *The symbol energy should be spread as much as possible over all independent components of the equivalent SISO channel.*

The Maximum Diversity Spread Solution

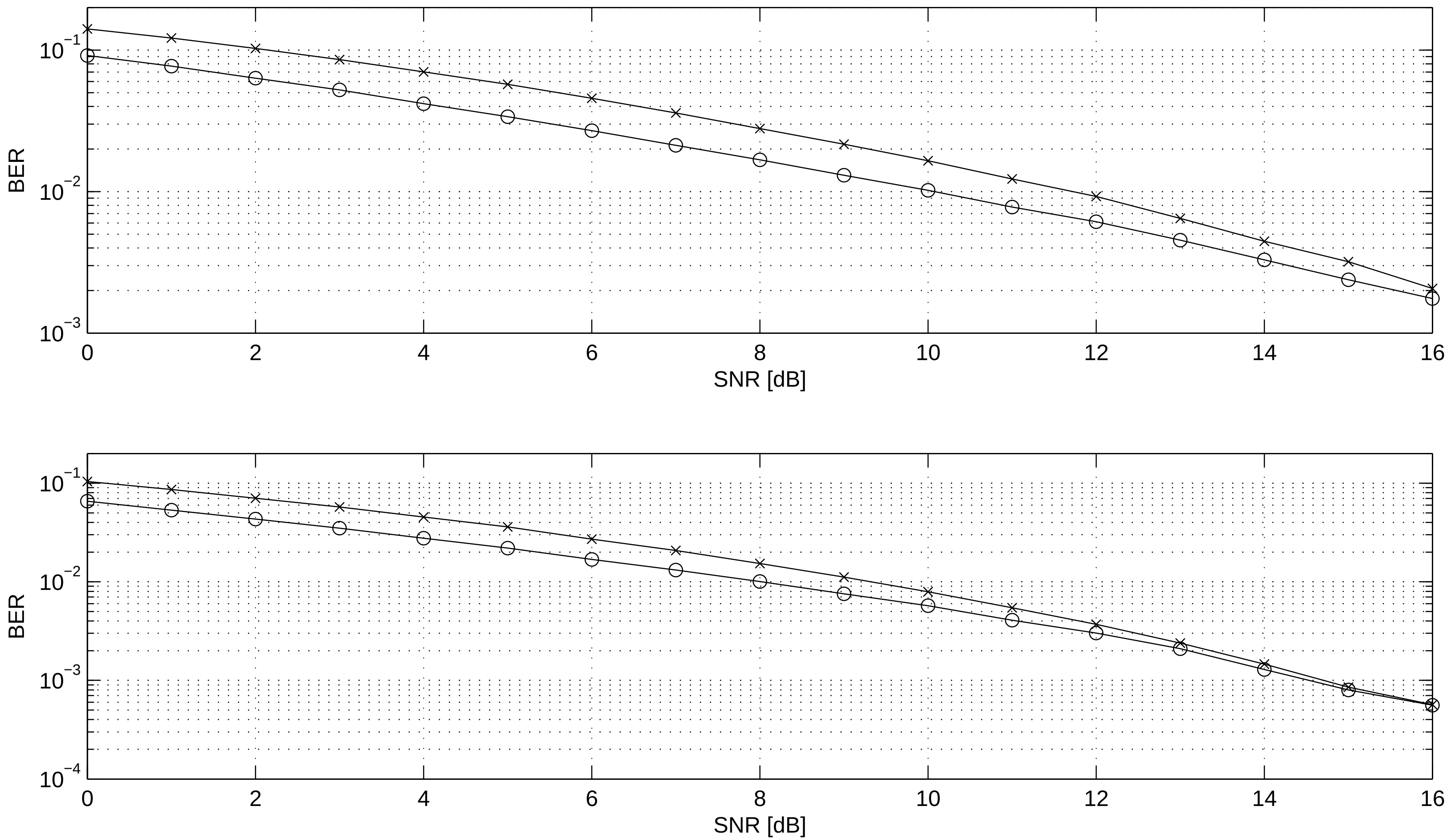
$$\min_{f_0,f_1\geq 0}\sum_{i=0}^{1}\sum_{j=0}^{M_r-1}\left(f_i^2\lambda_{r_{i_j}}-\frac{1}{2M_r}\sum_{i=0}^{1}\sum_{j=0}^{M_r-1}f_i^2\lambda_{r_{i_j}}\right)^2 \quad \text{subject to} \quad f_0^2+f_1^2=1, \quad (12)$$

With parameterization $f_0=\cos(\theta)$, $f_1=\sin(\theta)$:

$$\tan\theta=\sqrt{\frac{\sum_{j=0}^{M_r-1}\lambda_{r_{0_j}}^2}{\sum_{j=0}^{M_r-1}\lambda_{r_{1_j}}^2}}.$$

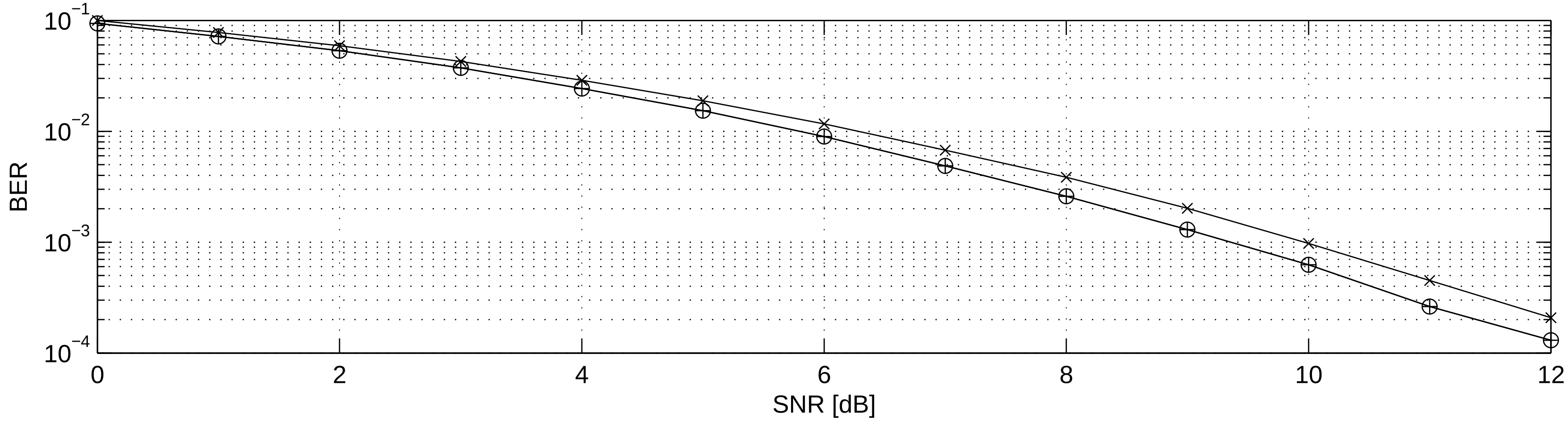
Simulation: Scenario 1

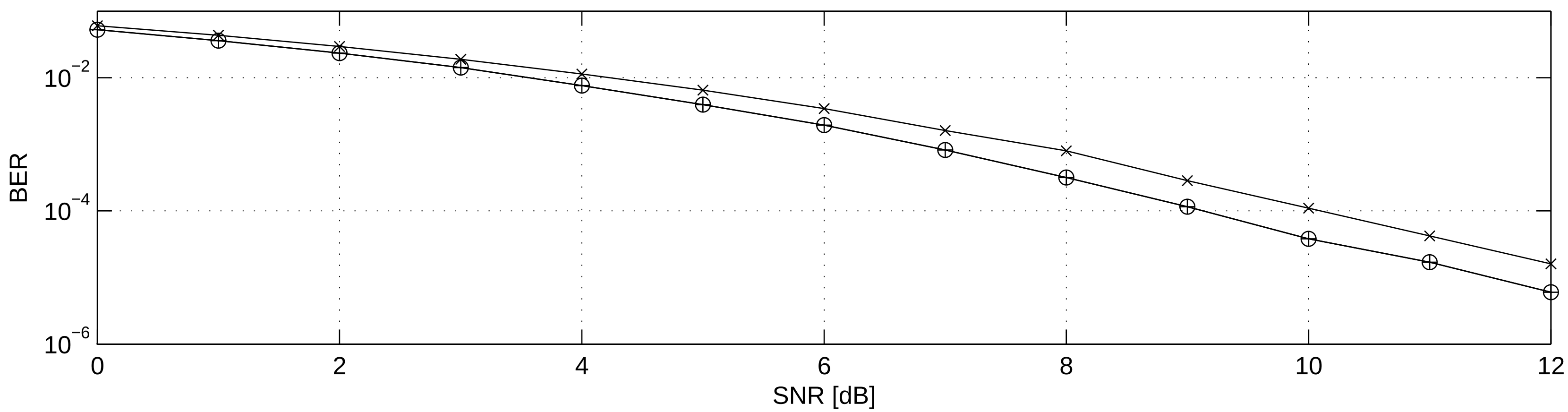
$M_t = 2$, Alamouti, $M_r = 4, 6$. $(\boldsymbol{R})_{k,l} = 0.99^{|k-l|}$ (Non separable)



Simulation: Scenario 2

$M_t = 2$, Alamouti, $M_r = 4, 6$. No transmit correlation. $\boldsymbol{R}_{r_0} = I$ and $\boldsymbol{R}_{r_1}$ ="all-ones".

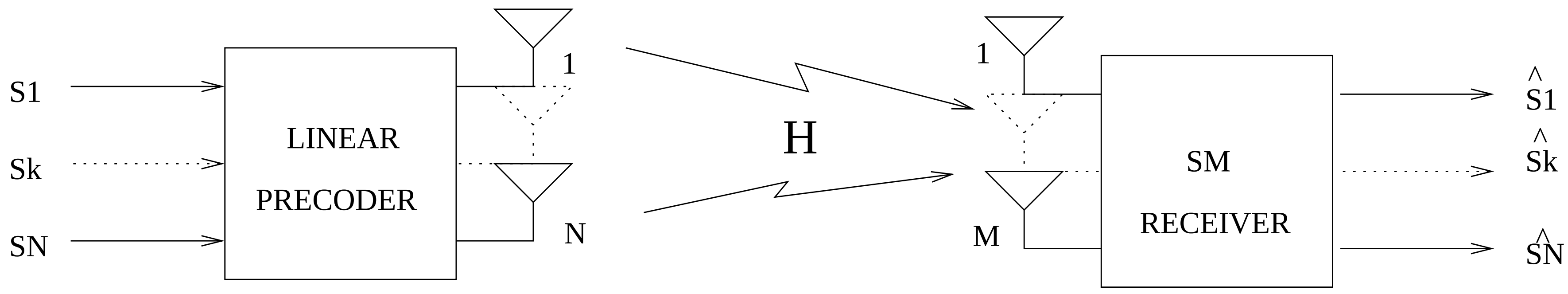




Part Two

- Linear precoding for high-rate codes

Spatial Multiplexing with Precoding



Proposed approaches

- Focus is on *low-complexity, closed form* solutions for transmitter optimization.(IEEE Globecom 2003, IEEE Trans. Wireless, to appear).
- *Previous work:* Nabar, Bolcskei, Paulraj (BER optimized). Ivrlac et al (capacity optimized).

Correlated Ricean MIMO model

Channel model:

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}} + \sqrt{\frac{K}{K+1}}\mathbf{H}_{los}. \tag{13}$$

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{14}$$

Precoding as a phase/amplitude adjustment:

$$\mathbf{s} = [\sqrt{P}_1 s_1 \ \sqrt{P}_2 e^{j\phi_2} s_2 \ \ldots \ \sqrt{P}_{M_t} e^{j\phi_{M_t}} s_{M_t}]^T. \tag{15}$$

where $\sum_{i=1}^{M_t} P_i = 1$.

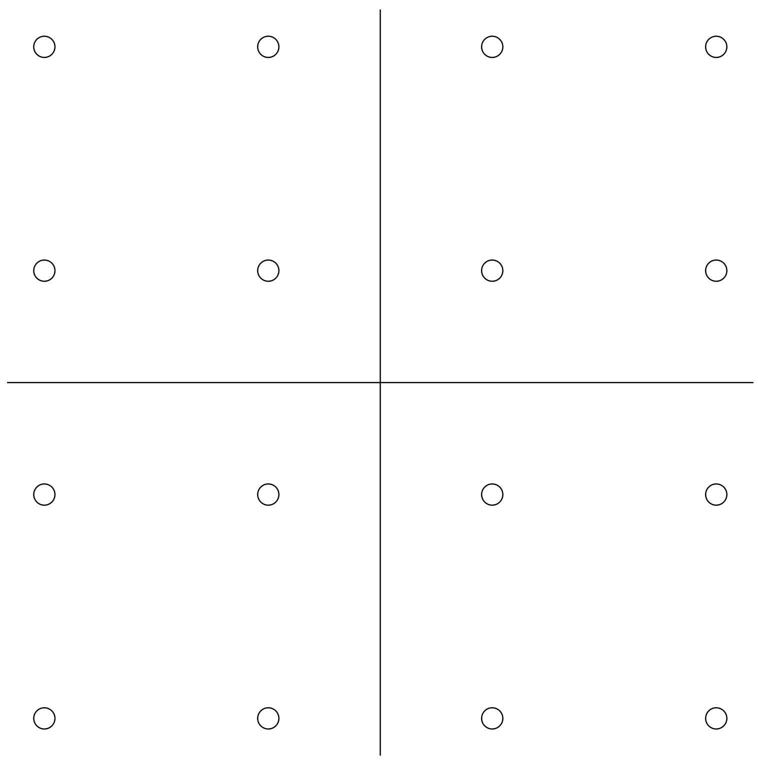
Constellation multiplexing

let s belong to a regular 2^{mM_t}-QAM constellation. Then there exist $r_m \in [0, 1]$ and $s_1, s_2, ..s_{M_t}$ symbols, each belonging to a 2^m-QAM constellation, such that s can be written in the form of:

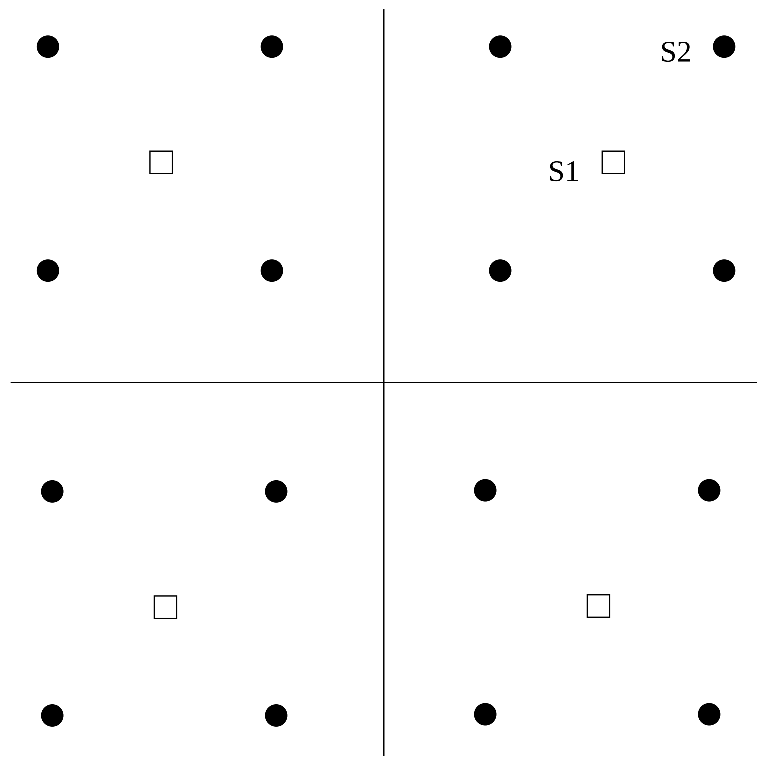
$$s = \sum_{k=1}^{M_t} r_m^k s_k \tag{16}$$

where r_m is such that $\sum_{k=1}^{M_t} r_m^{2k} = 1$

Constellation multiplexing



(a)



(b)

Precoding for low rank MIMO channels

Consider rank one channel : $\mathbf{H} = [\mathbf{h}, \mathbf{h}, .., \mathbf{h}]$

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n} = \mathbf{h} \sum_{k=1}^{M_t} \sqrt{P_k} e^{j\phi_k} s_k + \mathbf{n} = \mathbf{h}s + \mathbf{n} \tag{17}$$

where the precoding coefficients are selected as:

$$P_k = r_m^{2k} \quad k = 1..M_t \tag{18}$$

$$\phi_k = 0 \quad k = 1..M_t \tag{19}$$

$\Rightarrow$ transmitting over a rank one MIMO channel with an appropriate diagonal precoder is equivalent to transmitting over a higher-order constellation signal over a SIMO channel

MRC-based SIC detection

Goals of detection scheme

- Sole purpose of detection scheme is to derive closed-form precoder.
- Precoder should be function only of the long-term statistics $\mathbf{R}_t$, K, H_{los}.
- Ill conditioned channel components dealt with in a MRC-SIC manner rather than matrix inversion.

E.g: Closed-form Precoding for 2x2 case

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0 \begin{bmatrix} \alpha & \beta e^{j\psi} \\ \beta e^{-j\psi} & \alpha \end{bmatrix} + \sqrt{\frac{K}{K+1}} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \quad (20)$$

where $\alpha^2 + \beta^2 = 1$, and $\rho = 2\alpha\beta$ is the modulus of the antenna correlation coefficient

MRC-SIC detection statistics

$$z_1 = (\mathbf{H}^*)_{1,:}\mathbf{y} = \tau_1\sqrt{P}_1 s_1 + \tau_2\sqrt{P}_2 e^{j\phi_2} s_2 + (\mathbf{H}^*)_{1,:}\mathbf{n} \tag{21}$$

$$\hat{\mathbf{y}} = \mathbf{y} - \mathbf{H}_{:,1}\sqrt{P}_1 s_1. \tag{22}$$

$$z_2 = (\mathbf{H}^*)_{2,:}\hat{\mathbf{y}} = \tau_3\sqrt{P}_2 e^{j\phi_2} s_2 + (\mathbf{H}^*)_{2,:}\mathbf{n}, \tag{23}$$

with $\tau_1 = (\mathbf{H}^*)_{1,:}\mathbf{H}_{:,1}$, $\tau_2 = (\mathbf{H}^*)_{1,:}\mathbf{H}_{:,2}$ and $\tau_3 = (\mathbf{H}^*)_{2,:}\mathbf{H}_{:,2}$.

min distances under average channel behavior

$$\delta_1 = E\{\tau_1\}\sqrt{P}_1 d_{min} - \overline{E\{\tau_2\}}\sqrt{P}_2 d_{max}, \tag{24}$$

$$\delta_2 = E\{\tau_3\}\sqrt{P}_2 d_{min}. \tag{25}$$

where

$$E\{\tau_1\} = \frac{1}{K+1}(2 + K(h_{1,1}^* h_{1,1} + h_{2,1}^* h_{2,1})) \tag{26}$$

$$\overline{E\{\tau_2\}} = \frac{1}{K+1}(2\rho + K|h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}|) \tag{27}$$

$$E\{\tau_3\} = \frac{1}{K+1}(2 + K(h_{1,2}^* h_{1,2} + h_{2,2}^* h_{2,2})). \tag{28}$$

Precoder optimization via the BBC

"Bit Error Rate Balancing Criterion" (BBC) involves equating minimum distances $\delta_1 = \delta_2$, under constraint $P_1 + P_2 = 1$

Result:

$$P_1 = \frac{[d_{max}(2\rho + K|\alpha|) + d_{min}(2 + K(|h_{1,2}|^2 + |h_{2,2}|^2)]^2}{d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2)^2 + [d_{max}(2\rho + K|\alpha|) + d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2)]^2} \quad (29)$$

and

$$P_2 = \frac{d_{min}^2(2 + K(|h_{1,1}|^2 * + |h_{2,1}|^2)}{d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2)^2 + [d_{max}(2\rho + K|\alpha|) + d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2)]^2} \quad (30)$$

where $\alpha = h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}$.

Interpretations

No LOS case:

$$P_1 = \frac{(1+\frac{d_{max}}{d_{min}}\rho)^2}{1+(1+\frac{d_{max}}{d_{min}}\rho)^2}, \; P_2 = \frac{1}{1+(1+\frac{d_{max}}{d_{min}}\rho)^2}. \tag{31}$$

- **Uncorrelated:** yields equal power transmission
- **Fully correlated :** eg: 4QAM. Yields $P_1 = 0.8$ and $P_2 = 0.2$ (equivalent to 16QAM!)

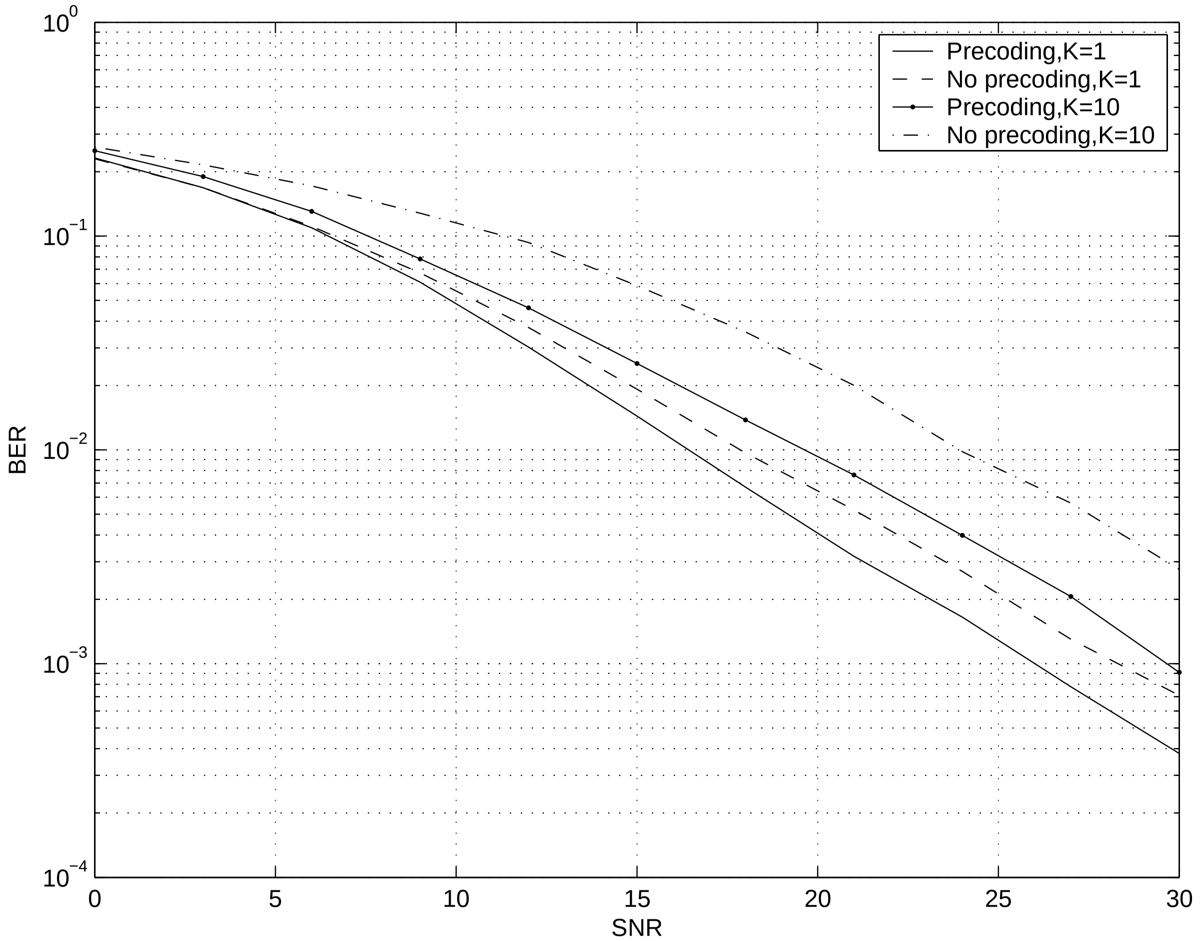
Full LOS case:

- Weights depends on conditioning of LOS part.
- Ill conditioned yields $P_1 = 0.8$ and $P_2 = 0.2$.

Precoder performs smooth transition between spatial and constellation multiplexing!

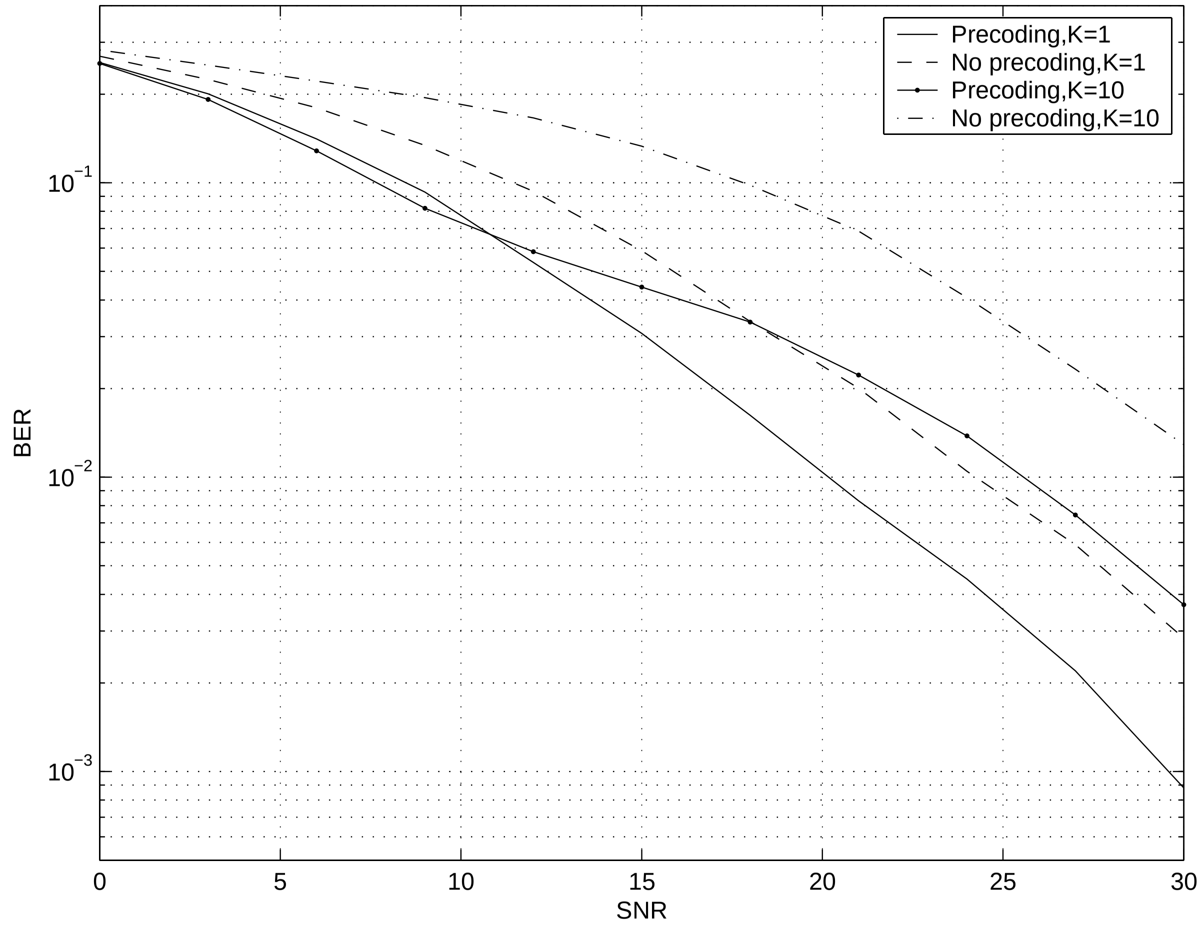
Numerical results

$K = 1, K = 10$, MMSE SIC with/without precoding, no TX correlation.



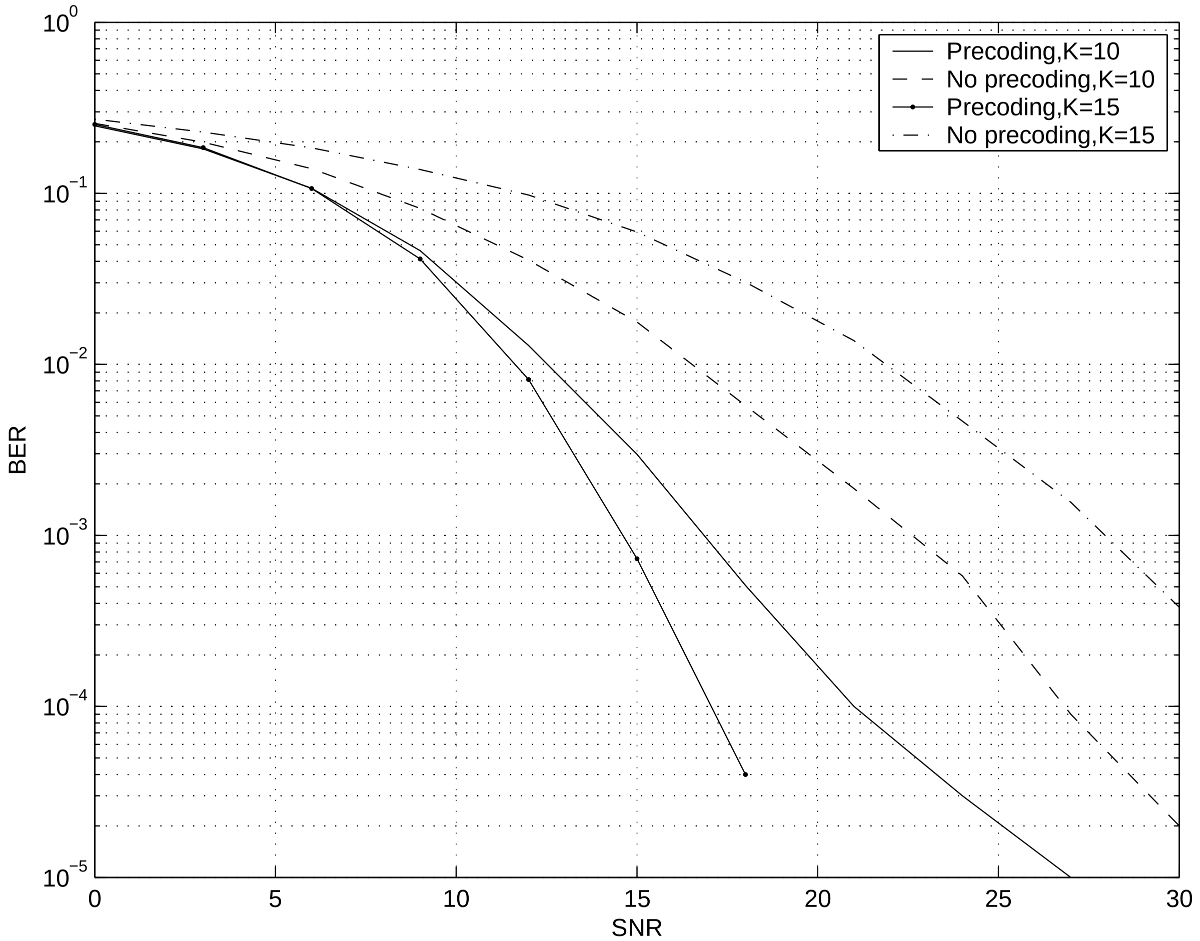
Numerical results

$K = 1, K = 10$, MMSE SIC with/without precoding, TX correlation is 0.8.



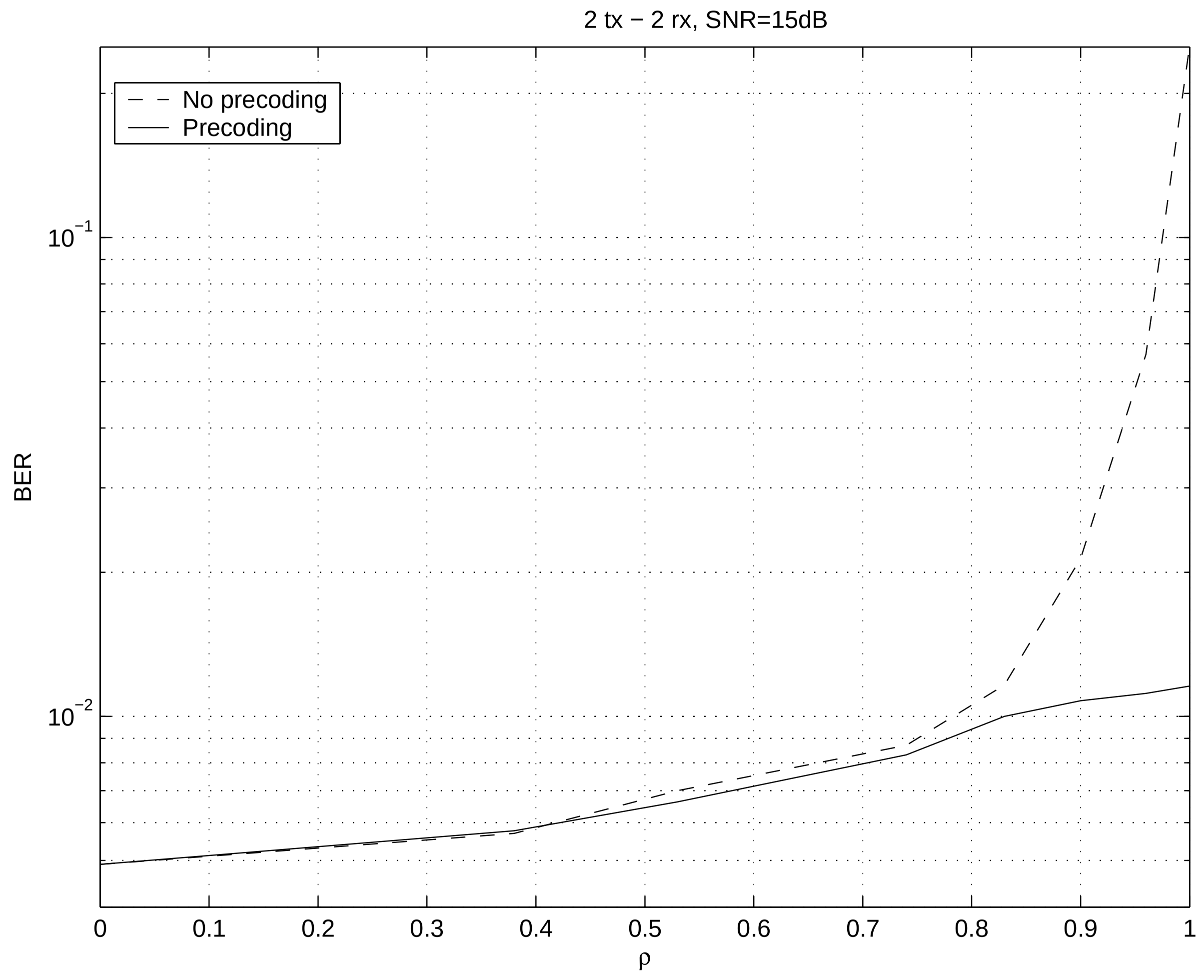
Numerical results

$K = 10, K = 15$, ML decoding, no TX correlation.



Numerical results

$K = 0$. ML with/without precoding, various TX correlation levels.



Conclusions

- Ill-behaved channels are detrimental to MIMO STC performance. A solution is precoding.
- In low-rate STC, gain is limited. Analytical solution difficult, except for macro-diversity case.
- In high-rate STC, gain is large. Analytical (simple) solution is possible. Links spatial muxing to constellation muxing. Can be extended to arbitrary number of antennas (see paper [Akhtar 04] upon request).

Open problems:

- Find closed form precoding for high-rate code, function of RX correlation.
- Find precoder linking STC and SM, function of correlation, Rice (results to appear).