Further Results on Low-Complexity Diversity Combining Schemes

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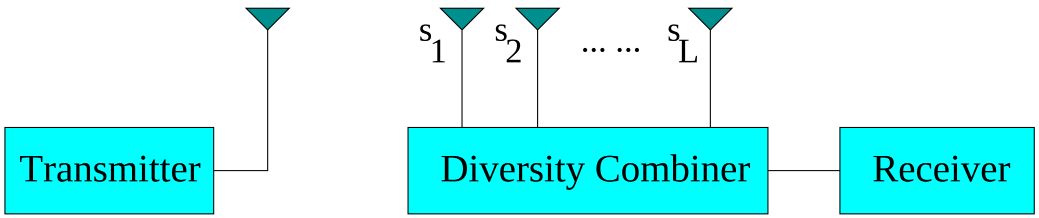
Outline

- Motivation and Goals
- Review of Switched Combining
- Switch-and-Examine Combining (SEC) Schemes
- Generalized SEC for Diversity Rich Environments
- Concluding Remarks

Diversity

- Effective fading mitigation technique.
- Reduce the occurrence of deep fades by
  - Providing the receiver with multiple faded replicas of the same information bearing signal.
  - Taking advantage of the low probability that all diversity paths experience simultaneously a deep fade.
- Antenna reception diversity comes at no cost of spectrum efficiency.



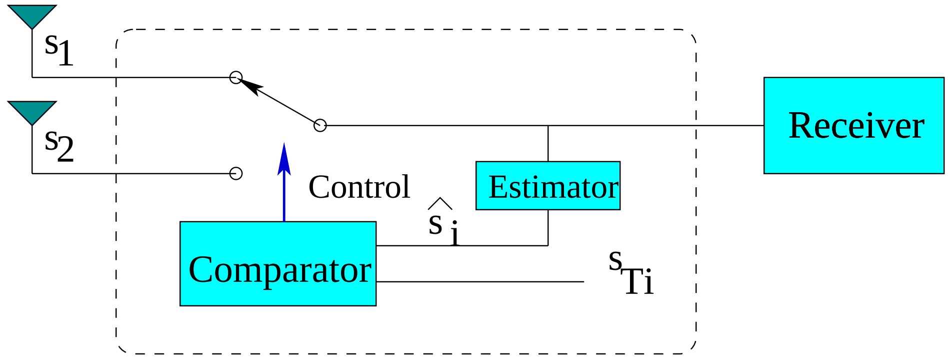
Diversity Combining Schemes

- Maximum ratio combining (MRC).
- Equal gain combining (EGC).
- Selection combining (SC).
- Switched combining.

Tradeoff between performance and complexity!

Switched Combining

- Use current branch and switch when it becomes unacceptable.
- Check branch quality by comparing with a fixed threshold.

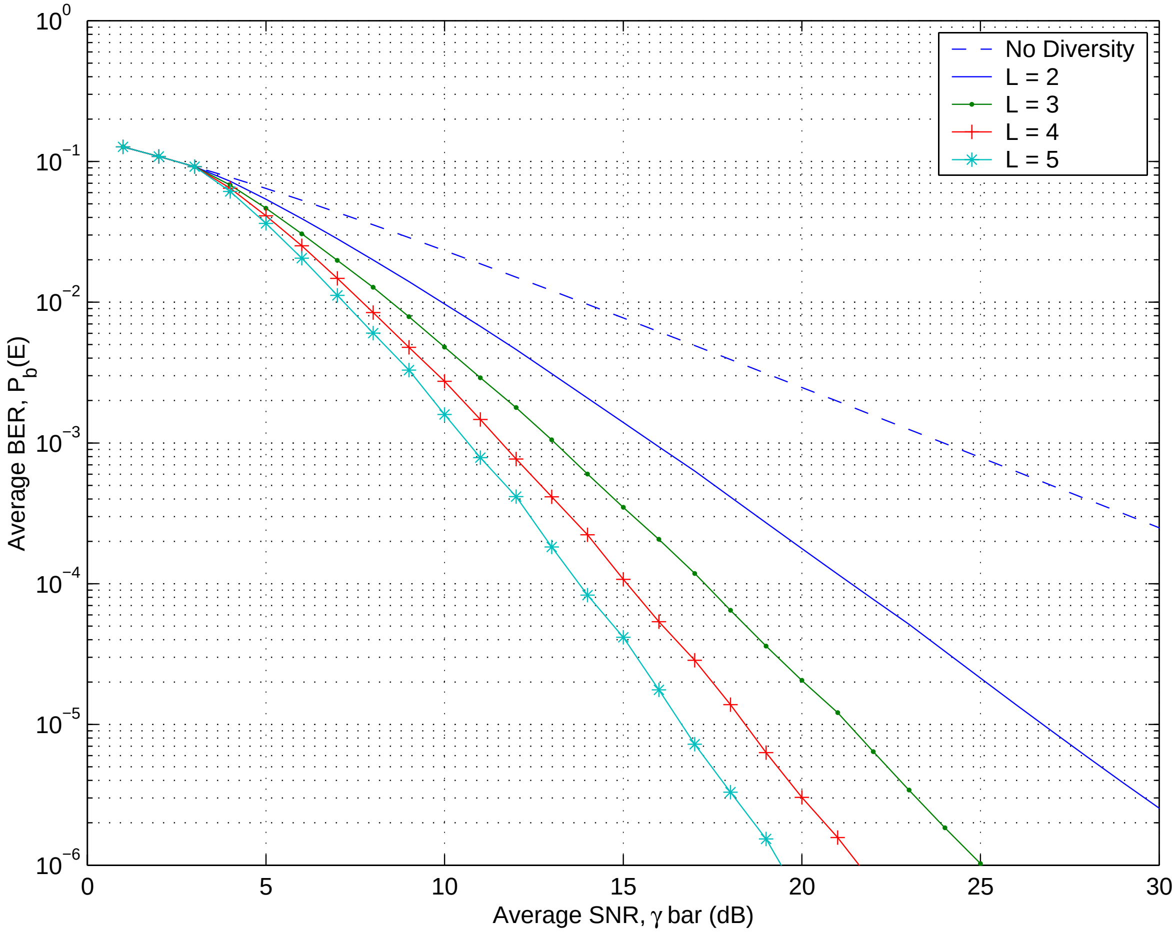


- Complexity savings with respect to SC
  - Only one branch needs to be monitored.
  - Comparison with a fixed threshold.
  - Reduced frequency of branch switching.
- Switch and stay combining (SSC) and switch and examine combining (SEC).

Multi-Branch Switched Diversity

- Multiple antennas $\Rightarrow$ Multi-branch switching.
- SSC: in general does not not benefit for more than two branches.
- Switch and examine combining (SEC)
  - Use current branch and switch only when it becomes unacceptable.
  - Unlike SSC scheme, the combiner examines the channel for the switch-to branch and switches again if unacceptable.
  - The combiner will repeat this process until either an acceptable branch is found or no branch left to be examined.
    - $*$ Three possible termination strategies for SEC (traditional SEC, post-selection (SECps), and scan and wait combining (SWC)).
  - SEC benefits from more than two branches.

Error Performance with SEC



SEC benefits from additional branches!

Model and Mode of Operation of SWC

- Information transmission is done on a time-slot based fashion: Guard period + Data burst.
- Block fading channel model: Data burst is assumed to experience roughly the same fading as that which occurs in the preceding guard period.
- Mode of operation:
  - If the current path is not of acceptable quality then the combiner switches and examines the quality of the next path.
  - Switching and examining process is repeated until either an acceptable path is found or all diversity paths have been examined.
  - In the latter case, the receiver just waits for a one coherence time and then re-start after that period the switching and examining process on all the diversity paths.

Output SNR

- The probability density function of the SWC output SNR can be written as

$$p_{\gamma_{\text{swc}}}(\gamma) = \sum_{n=0}^{\infty} P_L^n \left(\xi_1 p_{\gamma_1}^T(\gamma) + \xi_2 p_{\gamma_2}^T(\gamma) + \cdots + \xi_L p_{\gamma_L}^T(\gamma)\right)$$
$$= \frac{\xi_1 p_{\gamma_1}^T(\gamma) + \xi_2 p_{\gamma_2}^T(\gamma) + \cdots + \xi_L p_{\gamma_L}^T(\gamma)}{1 - P_L},$$

where

  - $p_{\gamma_l}^T(\gamma)$ is the conditional PDF of the truncated (above the threshold γ_{T_l}) random variable (RV) γ_l given that $\gamma_1 < \gamma_{T_1}$, $\gamma_2 < \gamma_{T_2} \cdots, \gamma_{l-1} < \gamma_{T_{l-1}}$.
  - $P_l = P_{\gamma_1,\gamma_2,\cdots\gamma_l}(\gamma_{T_1}, \gamma_{T_2}, \cdots, \gamma_{T_l})$, where $P_{\gamma_1,\gamma_2,\cdots\gamma_l}(\cdot, \cdot, \cdots, \cdot)$ is the joint CDF of γ_1, γ_2, $\cdots$, γ_l.
  - $\xi_l = P[\gamma_1 < \gamma_{T_1},\ \gamma_2 < \gamma_{T_2}, \cdots, \gamma_{l-1} < \gamma_{T_{l-1}}, \gamma_l \geq \gamma_{T_l}] = P_{l-1} - P_l$ for $l = 2, \cdots L$.

Average Probability of Error

- The average BEP $P_b(E)$ in the case where the paths are independent but not necessarily identically distributed is given by

$$P_b(E) = \frac{\sum_{l=1}^{L} \prod_{n=1}^{l-1} P_{\gamma_n}(\gamma_{T_n})(1 - P_{\gamma_l}(\gamma_{T_l}))P_b(E_l)}{1 - \prod_{l=1}^{L} P_{\gamma_l}(\gamma_{T_l})},$$

where

$$P_b(E_l) = Q\left(\sqrt{2\gamma_{T_l}}\right) - \sqrt{\frac{\bar{\gamma}_l}{1+\bar{\gamma}_l}} Q\left(\sqrt{2\gamma_{T_l}\frac{1+\bar{\gamma}_l}{\bar{\gamma}_l}}\right) e^{\gamma_{T_l}/\bar{\gamma}_l},$$

for binary phase-shift-keying (BPSK) operating over Rayleigh fading paths with average SNRs $\bar{\gamma}_l$ $(l = 1, 2, \cdots, L)$.

Delay Statistics

- Average number of coherence time before access

$$\overline{N}_c = \frac{P_L}{1 - P_L},$$

which reduces when the fading is independent across the diversity paths to

$$\overline{N}_c = \frac{\prod_{l=1}^{L} P_{\gamma_l}(\gamma_{T_l})}{1 - \prod_{l=1}^{L} P_{\gamma_l}(\gamma_{T_l})},$$

- Dropping probability

$$P_d = P[N_c > n_{\text{th}}] = P_L^{1+n_{\text{th}}}$$

which reduces when the fading is independent across the diversity paths to

$$P_d = \left(\prod_{l=1}^{L} P_{\gamma_l}(\gamma_{T_l}) \right)^{1+n_{\text{th}}}.$$

Estimation Statistics

- Average number of path estimates before access

$$\overline{N}_e = \frac{1 + \sum_{l=1}^{L-1} P_l}{1 - P_L}$$

which reduces when the fading is independent across the diversity paths to

$$\overline{N}_e = \frac{\sum_{l=0}^{L-1} \prod_{n=1}^{l} P_{\gamma_n}(\gamma_{T_n})}{1 - \prod_{l=1}^{L} P_{\gamma_l}(\gamma_{T_l})}.$$

- Excess estimation

$$\begin{aligned} P_e &= P[N_e > N_{\text{th}}] = 1 - P[N_e \leq N_{\text{th}} = n_{\text{th}} L + l_{\text{th}}] \\ &= P_L^{n_{\text{th}}} P_{N_{\text{th}} - n_{\text{th}} L}. \end{aligned}$$

Comparison of Traditional SEC and SWC

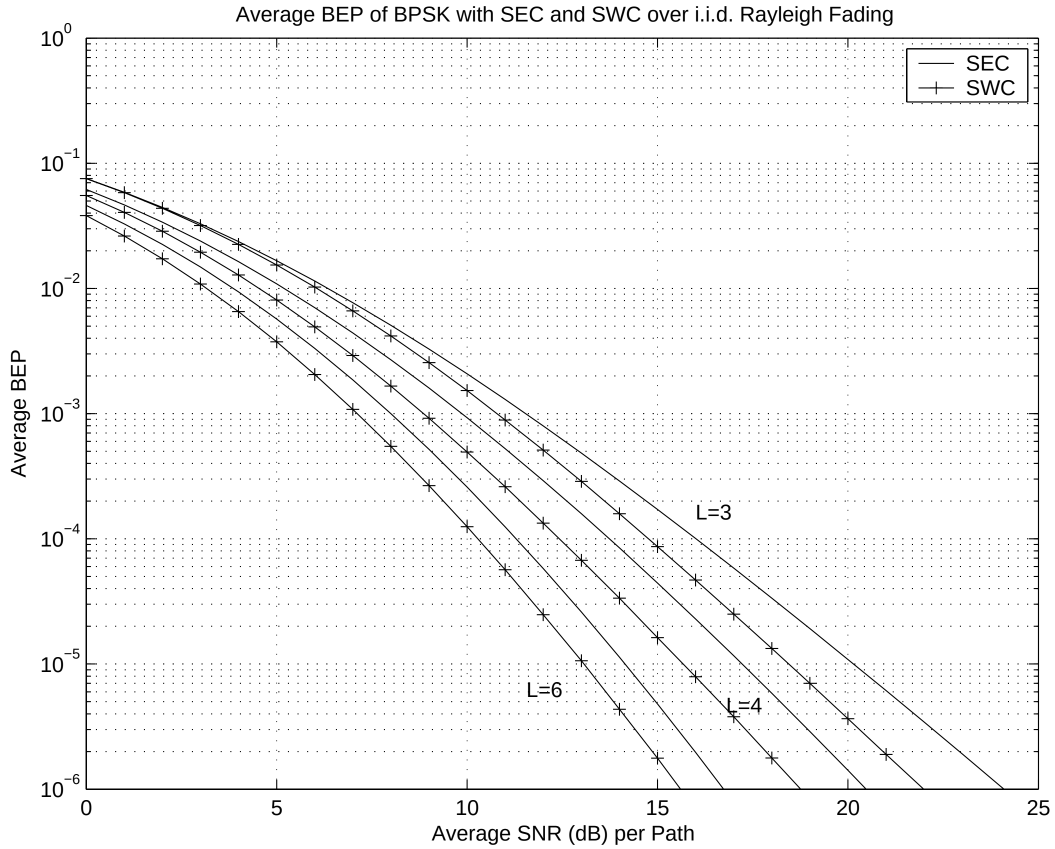


Figure 1: Comparison of the average BEP of BPSK with SEC (using optimal switching threshold) and SWC (using a switching threshold yielding the same average number of path estimations as SEC for a fixed L).

SWC strategy outperforms the traditional SEC strategy !

Average Time Delay for SWC

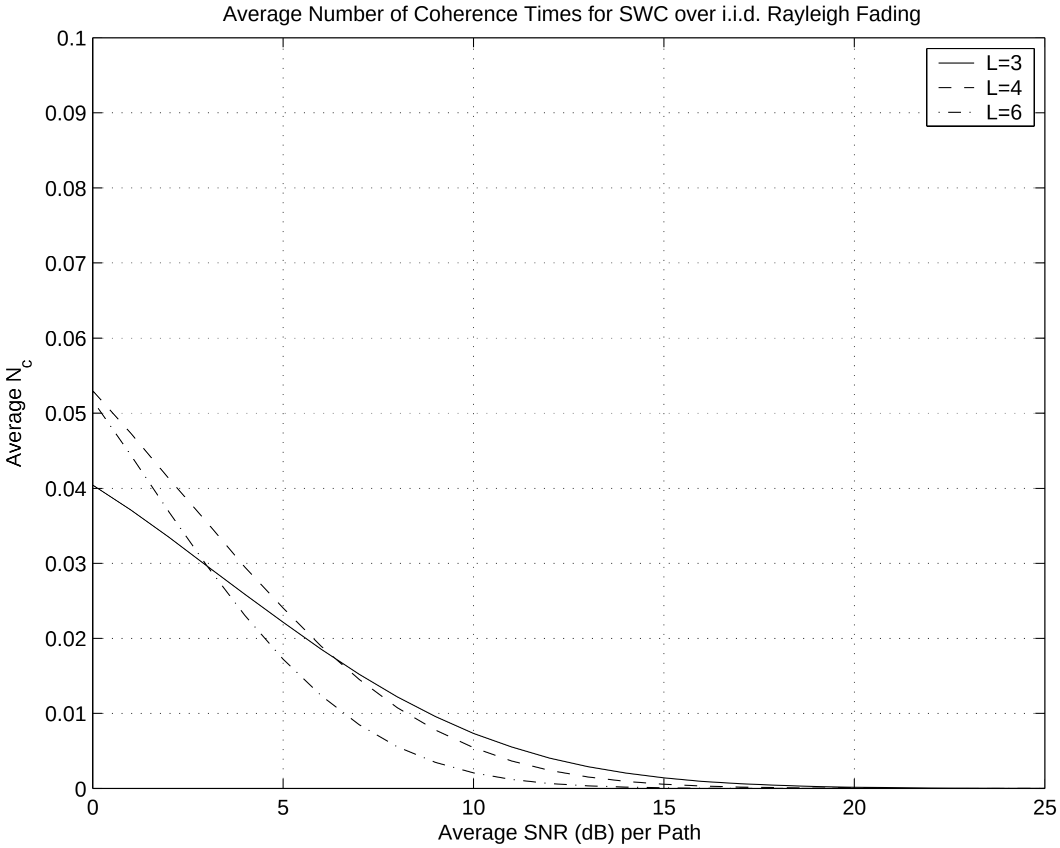


Figure 2: Average number of coherence times required for SWC before channel access as a function of the SNR per path and for various values of L.

Negligible time delay !

Dropping Probability for SWC

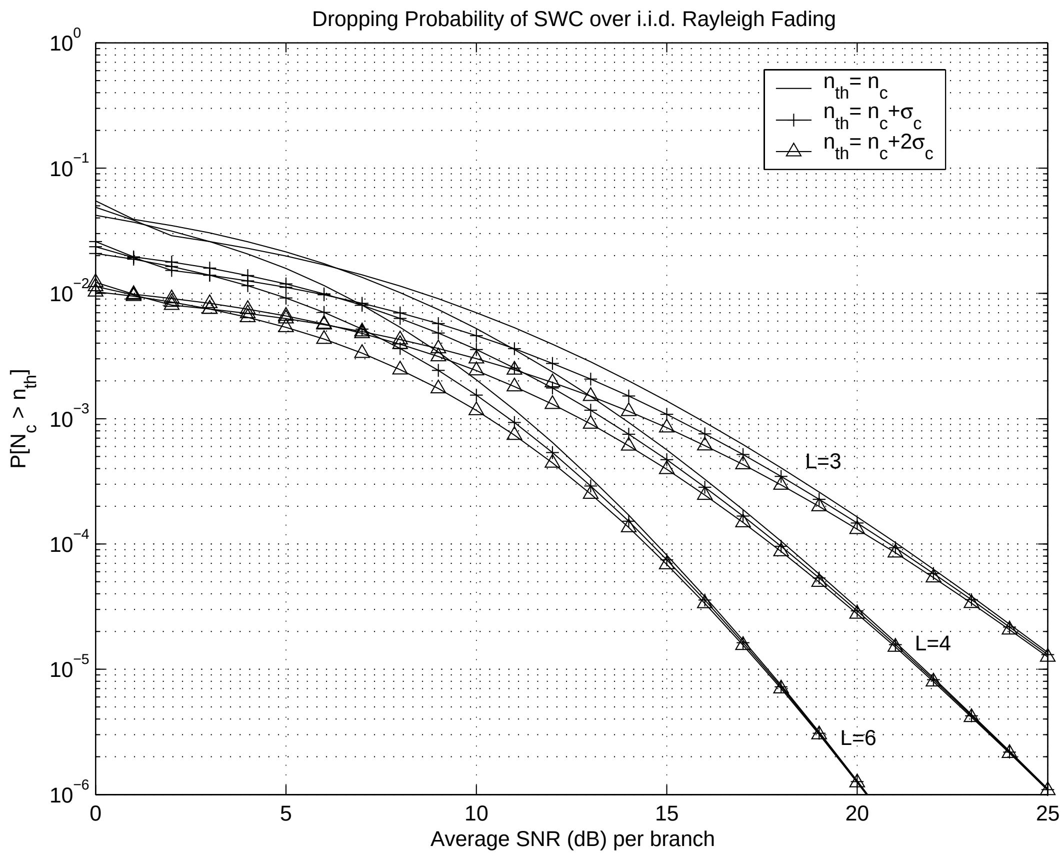


Figure 3: Dropping probability of SWC as a function of the SNR per path and for various values of L.

Negligible dropping probability !

Comparison with SC and MRC

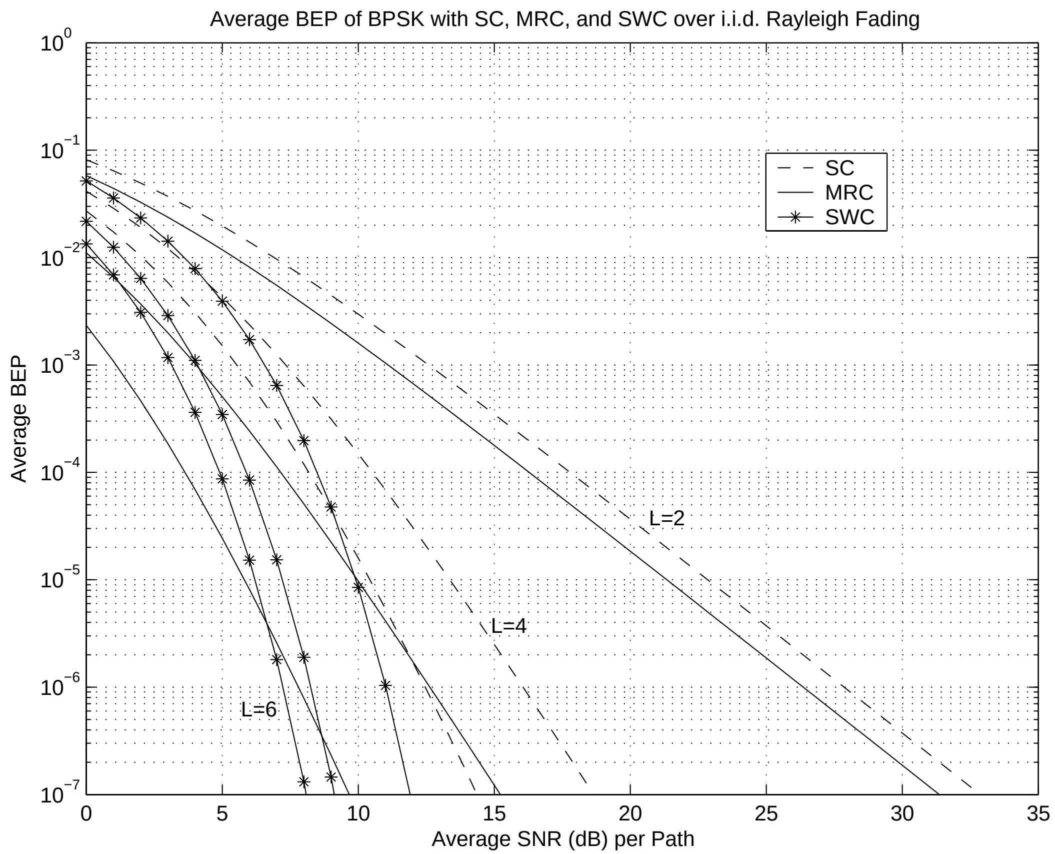


Figure 4: Comparison of the average BEP of BPSK with MRC, SC, and SWC over i.i.d. Rayleigh fading paths as a function of the SNR per path and for various values of L.

SWC can outperform MRC and SC !

Non IID Environment

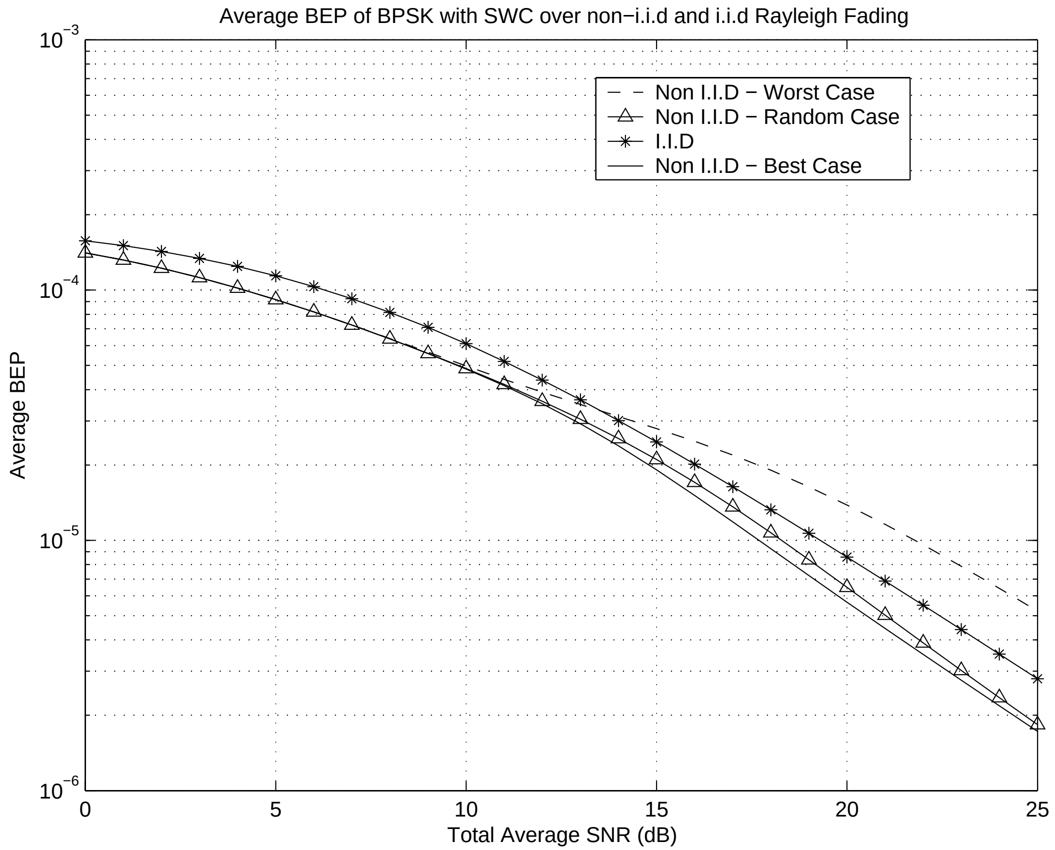


Figure 5: Comparison of the average BEP of BPSK with SWC ($\gamma_T = 8$ dB and $L = 5$) over an i.i.d. Rayleigh fading environment and a non-i.i.d Rayleigh fading environment (exponentially decaying power delay profile with $\delta = 0.3$).

Statistical information helps !

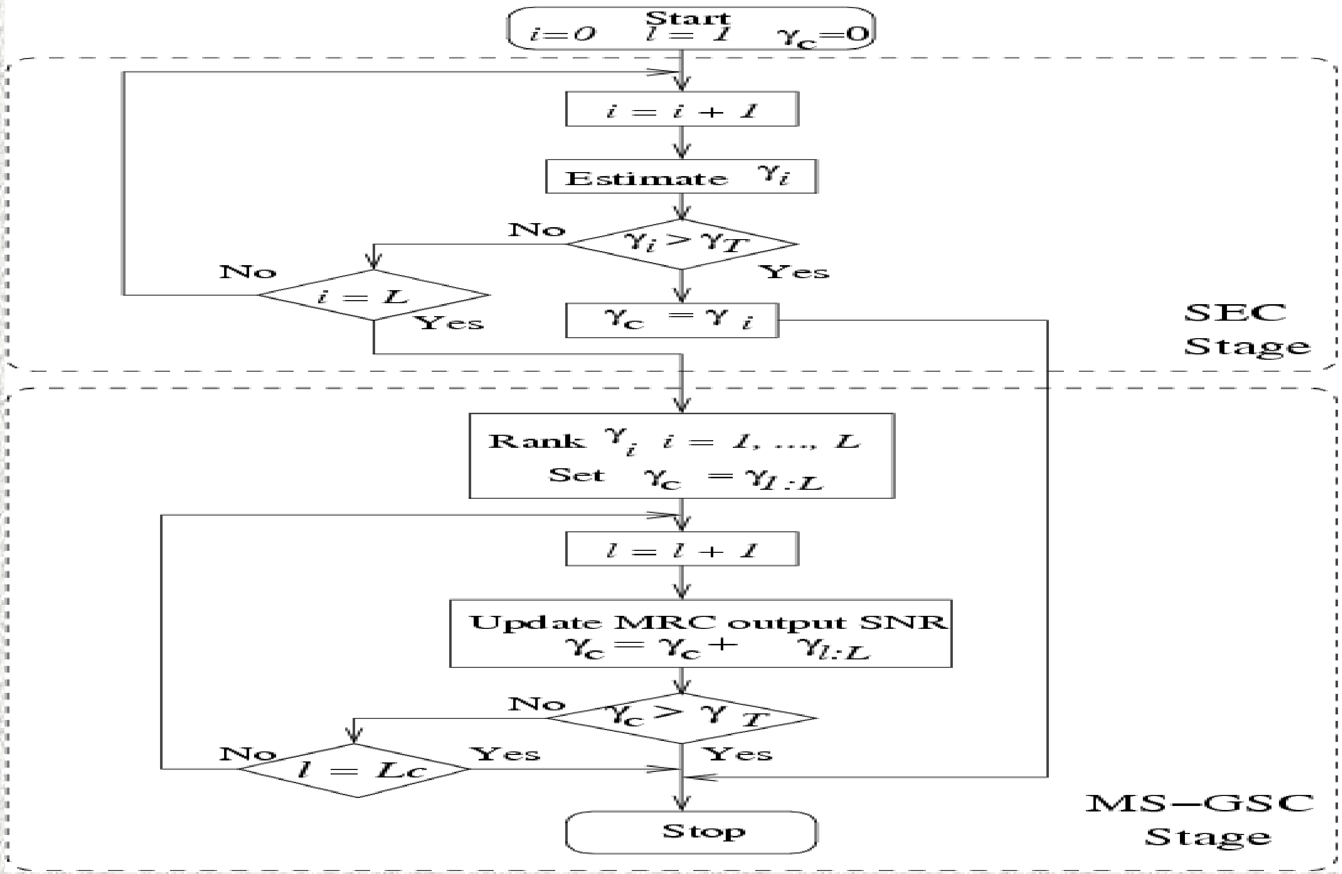
Combining in Diversity Rich Environments

- Performance of diversity combining schemes improve with additional combined diversity paths.
- Emerging and proposed wireless systems will operate in diversity rich environments (Examples: Ultra-wideband, millimeter-wave, and MIMO systems).
- For best performance: MRC
  - Requires one RF chain for each diversity path.
  - Mandates complete knowledge of channel conditions.
  - Sensitive to channel estimation errors.
- To reduce complexity and be less sensitive to channel estimation errors: Only ``good'' diversity paths are MRC combined.

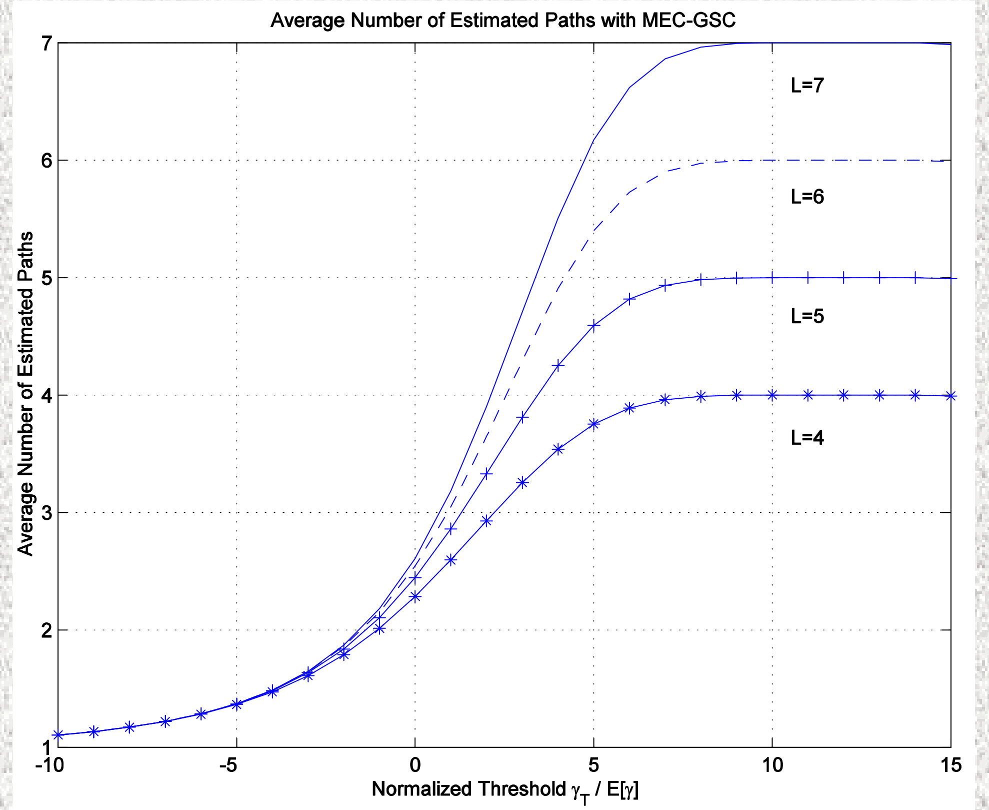
Generalized Selection Combining

- Hybrid scheme which bridges between the two extreme combining techniques offered by SC and MRC [Kong and Milstein, ICUPC'95].
- Combine the Lc strongest paths among the L available ones.
- Performance analysis of GSC received a great deal of attention over the last couple of years.
- Variant of GSC was proposed recently:
    - Minimum Selection GSC [Kim et al., ISCAS'03 and Gupta et al. ICC'04]
    - Same hardware complexity and same number of channel estimates as GSC but less combined paths in average.

Minimum-Estimation-Combining (MEC) GSC

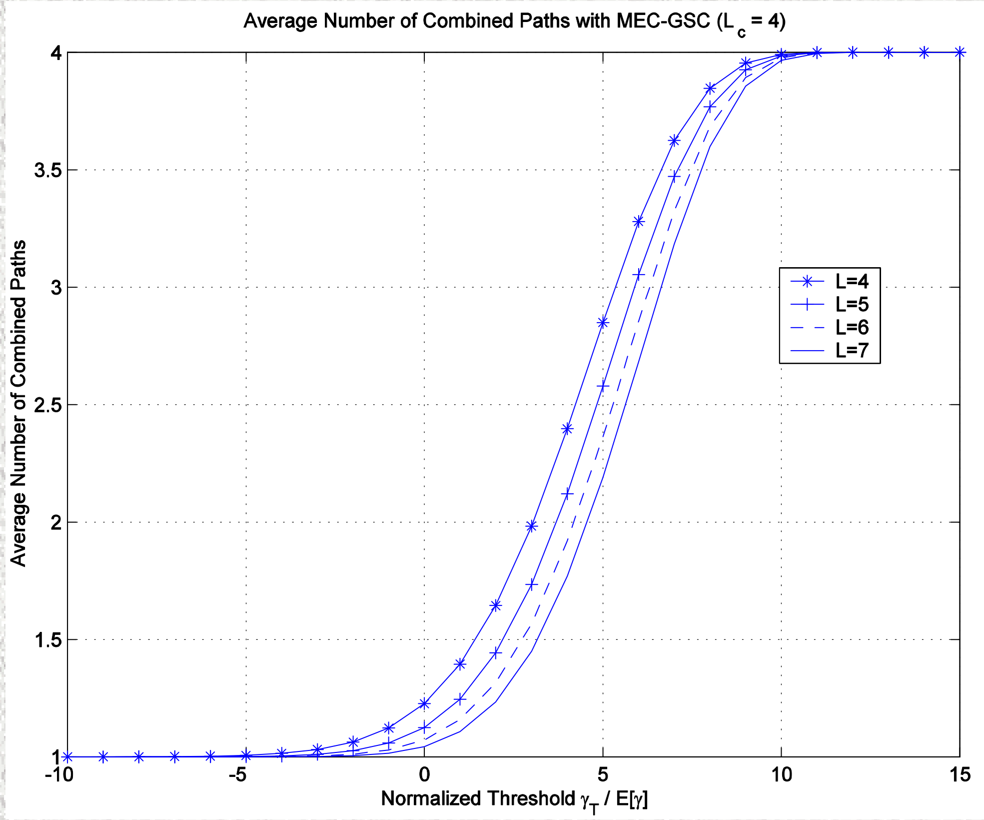


Average Number of Channel Estimates

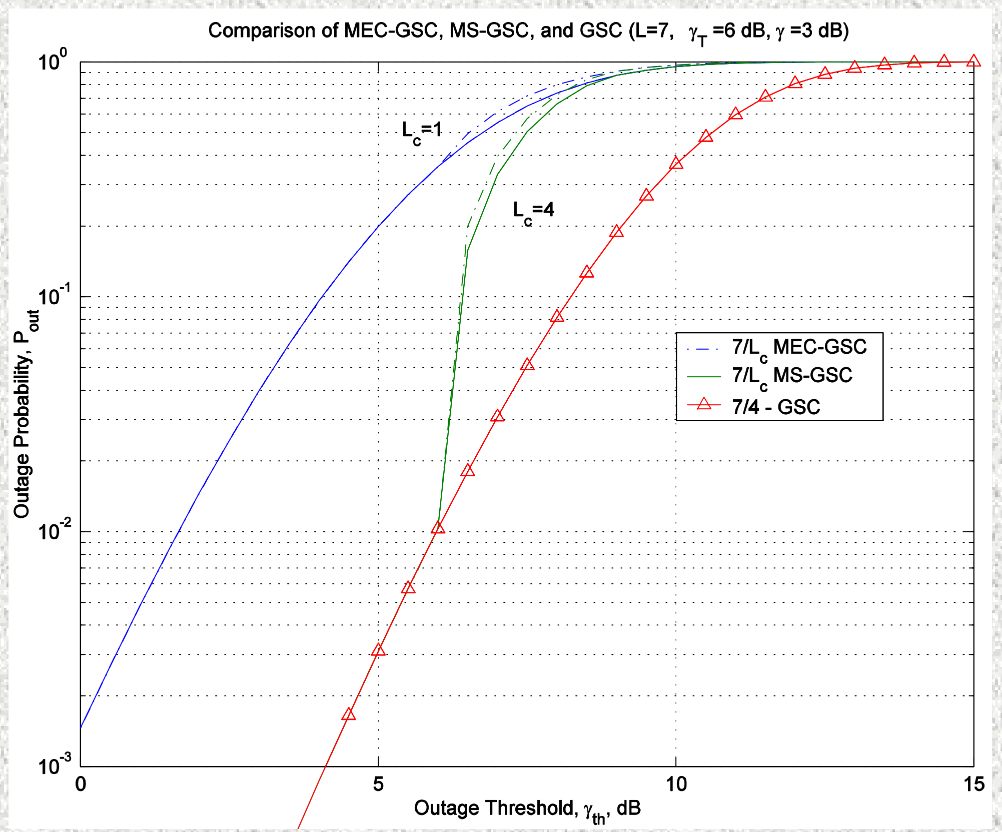


Student Version of MATLAB

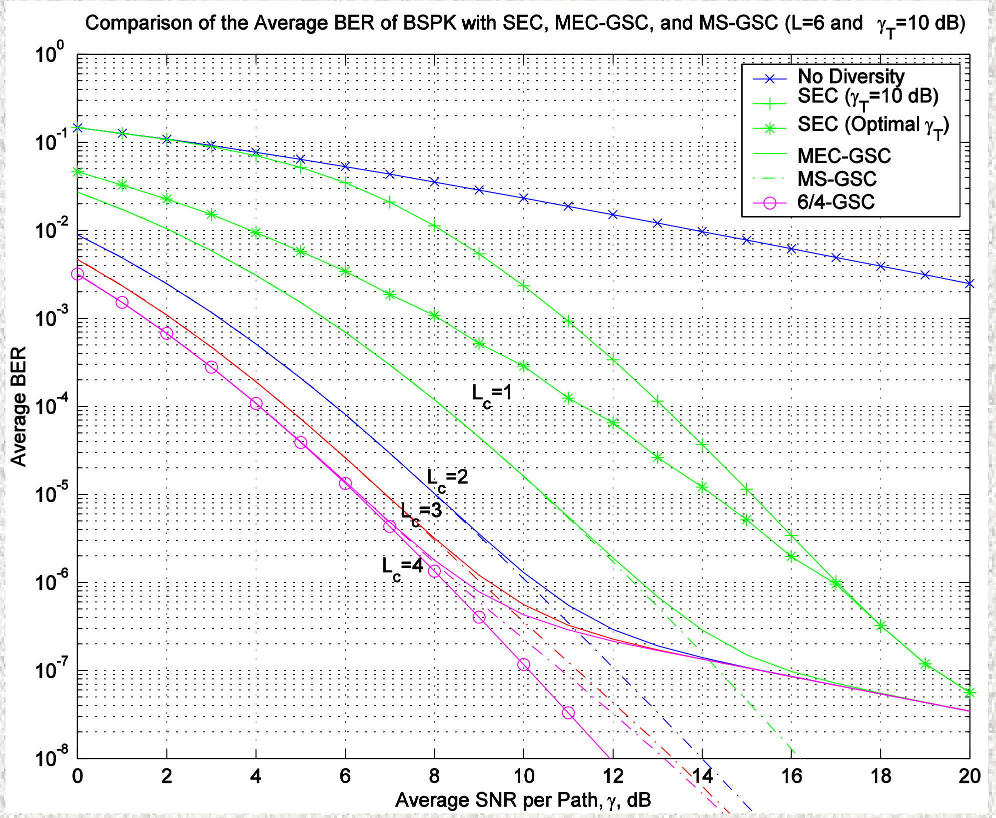
Average Number of Combined Paths



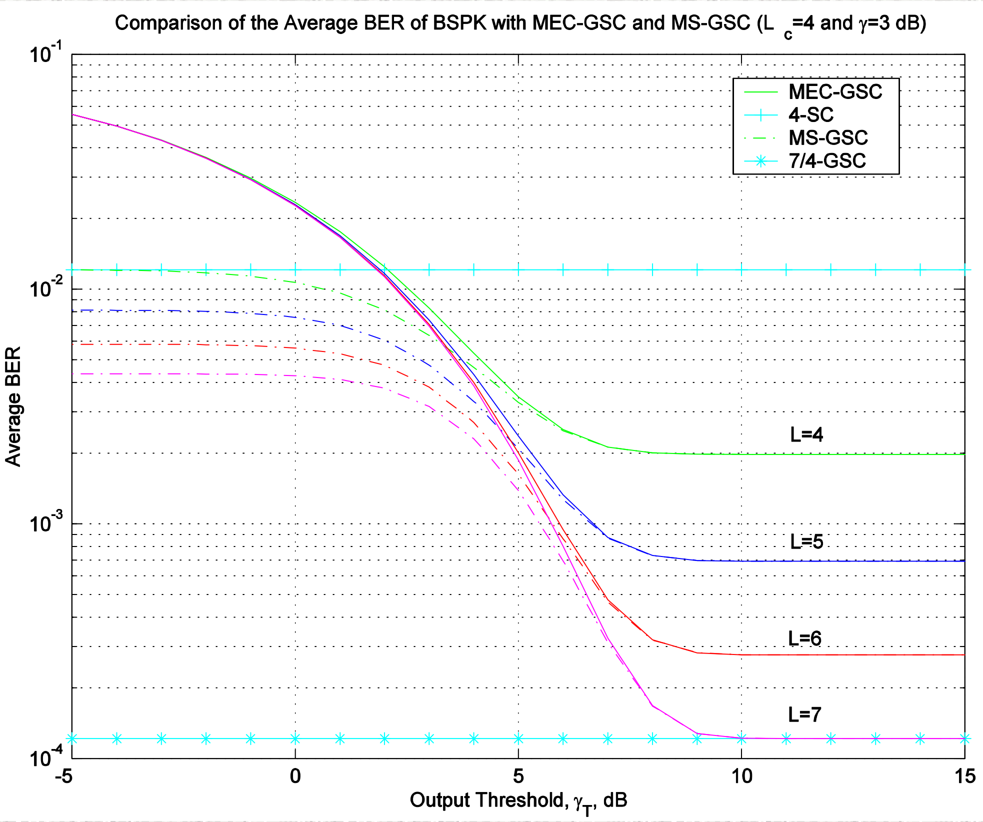
Outage Probability Comparison



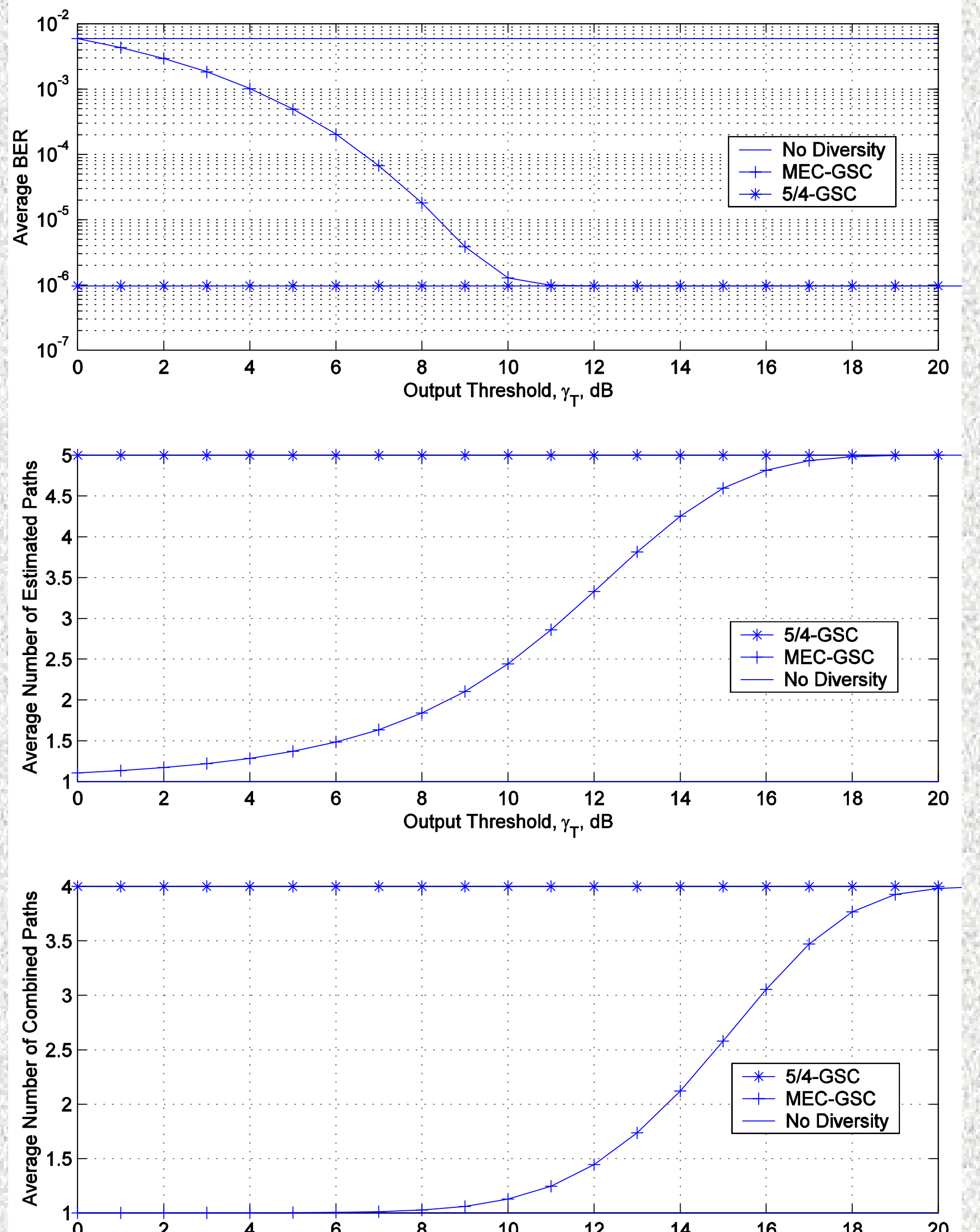
Average BER Comparison



Average BER Comparison



Tradeoff



Average BER
Output Threshold, γ_T, dB
No Diversity
MEC-GSC
5/4-GSC
Average Number of Estimated Paths
Output Threshold, γ_T, dB
5/4-GSC
MEC-GSC
No Diversity
Average Number of Combined Paths
5/4-GSC
MEC-GSC
No Diversity

Concluding Remarks

- Switched-based diversity schemes offer adaptive low-complexity solutions for fading mitigation.
- Switch/scan and wait lead to tremendous performance gain at the expense of negligible time delay.
- MEC-GSC minimizes the average number of channel estimates and average number of combined paths while still approaching the performance of GSC.
- Applications of these schemes in multiuser diversity and multiuser OFDM (OFDMA).