Transmitting Composite Gaussian Sources over Rayleigh Fading Channels

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Problem formulation

Find performance limits and design practical systems for transmitting composite sources or sources with memory over power and bandlimited channels of different types. Signals and channel noise are Gaussian.

JSC - Joint Source Channel, E - Encoding, D - Decoding

Time discrete signaling

Performance criterion: SNR=Signal variance/Noise variance(RMS)

Outline

- Repetition of the concept of OPTA
- RD theory for correlated sources
- Channel capacity for flat fading channels
- OPTA for flat fading channels
- Image RD performance
- Image transmission over fading channels

OPTA

OPTA=optimal performance theoretically attainable:

Set rate of a source equal to the channel capacity, and derive SNR as function of CSNR

Assume:

- Bandwidth W $\rightarrow$ Sampling frequency: $F_{s_x} \geq 2W$, Rate per source symbol: R $\rightarrow$ Minimum rate per second: $2WR$
- Channel bandwidth B $\rightarrow$ Maximum signaling speed: $F_{s_y} = 2B$, Capacity per channel use: C $\rightarrow$ Channel capacity per second: $2BC$

OPTA: $2WR = 2BC$

$$\boxed{R = \frac{B}{W}C = \frac{F_{S_y}}{F_{s_x}}C = rC}$$

r number of channel samples per source sample

Simplest case

RD-function of memoryless, Gaussian source:

$$R = \frac{1}{2}\log\left(\frac{\sigma_X^2}{\sigma_D^2}\right) = \frac{1}{2}\log(SNR)$$

Channel capacity of memoryless, Gaussian channel:

$$C = \frac{1}{2}\log\left(1 + \frac{\sigma_C^2}{\sigma_N^2}\right) = \frac{1}{2}\log(1 + CSNR)$$

$R = rC \rightarrow$

$$\frac{1}{2}\log(SNR) = r\frac{1}{2}\log(1 + CSNR)$$

Solve with respect to SNR:

$$\boxed{SNR = (1 + CSNR)^r}$$

RD-function for correlated source

Gaussian source with power spectral density $S_{XX}(F)$ and bandwidth W: parametric evaluation of rate and distortion

$$R_W(\mu) = \int_0^W \max\left\{0, \log \frac{S_{XX}(F)}{\mu}\right\} dF$$

and

$$D_W(\mu) = 2\int_0^W \min\{\mu, S_{XX}(F)\}\, dF$$

By selecting a value of μ a rate and a corresponding distortion result

High rate case

Assume $\mu < S_{XX}(F)$ for all F

Then $D_W(\mu) = 2W\mu$, or $\sigma_D^2(\mu) = \frac{D_W(\mu)}{2W} = \mu$

Resulting rate:

$$R_W = \int_0^W \log\left(S_{XX}(F)\right) dF - W \log(\sigma_D^2).$$

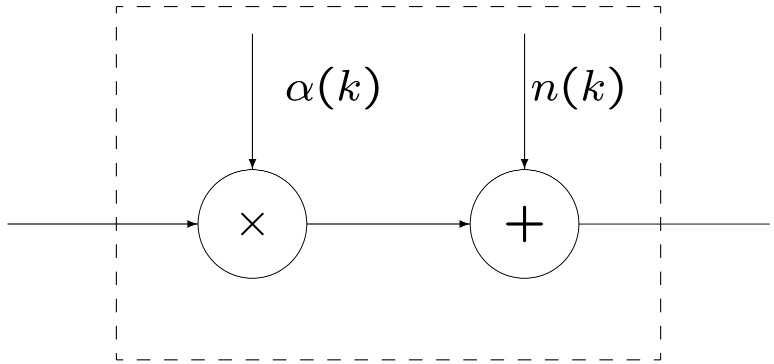
Introduce the *spectral flatness measure*

$$\beta_X^2 = \frac{\exp\left\{\frac{1}{W}\int_0^W \log\left(S_{XX}(F)\right) dF\right\}}{\frac{1}{W}\int_0^W S_{XX}(F) dF}$$

$\longrightarrow$

$$\boxed{R_W = W \log\left(\frac{\sigma_X^2}{\sigma_D^2}\,\beta_X^2\right)}$$

Capacity of fading channel, channel model



Introduce $\gamma = \bar{P}\alpha/N_0 B$, where $\bar{P}$ is the average power into the channel

Let the instantaneous power vary → Instantaneous CSNR: $\gamma P/\bar{P}$

Channel capacity per channel use for this CSNR:

$$C = \frac{1}{2}\log\left(1 + \gamma\frac{P}{\bar{P}}\right)$$

Taking the probability $f_\Gamma(\gamma)$ of being in the channel state γ into consideration, the average channel capacity per second is

$$C_B = B\int_0^\infty f_\Gamma(\gamma)\log\left(1 + \gamma\frac{P}{\bar{P}}\right)$$

Power distribution

Minimize $O = C_B + \lambda \bar{P}$ with respect to P where

$$\bar{P} = \int_0^\infty f_\Gamma(\gamma) P(\gamma) d\gamma$$

Optimal distribution of power:

$$\boxed{P(\gamma) = \bar{P}\left[\frac{1}{\gamma_0} - \frac{1}{\gamma}\right],}$$

where

$$\frac{1}{\gamma_0} = 1 + \int_{\gamma_0}^\infty \frac{1}{\xi} f_\Gamma(\xi) d\xi$$

"Water filling"

The power must always be positive, which implies that we can transmit signals over the channel only if $\gamma > \gamma_0$

Rayleigh fading channel

$$f_\Gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp(-\gamma/\bar{\gamma})$$

Can find relation between outage level and mean CSNR ($\bar{\gamma}$):

$$\bar{\gamma} = \frac{\bar{\gamma}}{\gamma_0} - E_1(\frac{\gamma_0}{\bar{\gamma}})$$

where the exponential integral is defined by

$$E_1(\beta) = \int_\beta^\infty \frac{1}{\alpha} \exp(-\alpha) d\alpha$$

Capacity of Rayleigh fading channel:

$$\boxed{C_B = B\bar{\gamma}\left(\frac{1}{\gamma_0} - 1\right) = BE_1(\frac{\gamma_0}{\bar{\gamma}})}$$

High CSNR OPTA for fading channel

Setting $R_W = C_B$:

$$W \log\left(\frac{\sigma_X^2}{\sigma_D^2}\,\beta_X^2\right) = B\int_{\gamma_0}^{\infty} f_\Gamma(\gamma)\log\left(\frac{\gamma}{\gamma_0}\right)d\gamma$$

It is possible to solve for the obtainable SNR:

$$SNR = \frac{\sigma_X^2}{\sigma_D^2} = \beta_X^{-2}\exp\left\{r\int_{\gamma_0}^{\infty} f_\Gamma(\gamma)\log\left(\frac{\gamma}{\gamma_0}\right)d\gamma\right\}$$

Introduce the average $CSNR = \bar{P}/\bar{P}\bar{\gamma} = \bar{\gamma}$

From $P(\gamma) = \bar{P}\left[1/\gamma_0 - 1/\gamma\right]$ we find: $1 + CSNR = \bar{\gamma}/\gamma_0 \rightarrow$

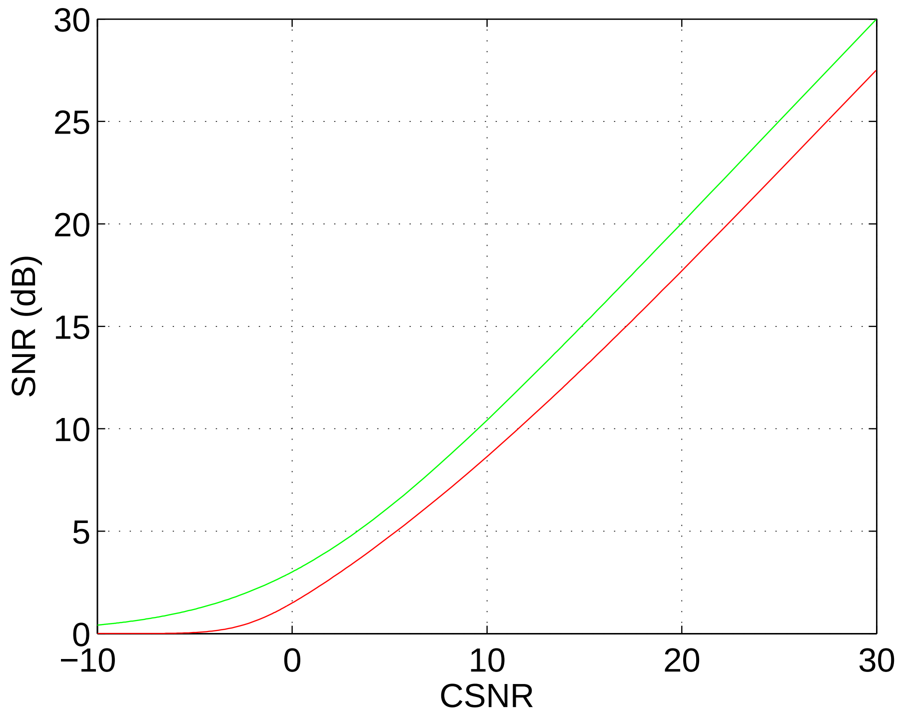
$$\boxed{SNR = (1 + CSNR)^r \eta^r \beta_X^{-2}}$$

where we have introduced the fading loss factor

$$\eta = \frac{\exp\left\{\int_{\gamma_0}^{\infty} f_\Gamma(\gamma)\log\left(\frac{\gamma}{\gamma_0}\right)d\gamma\right\}}{\frac{\bar{\gamma}}{\gamma_0}} \text{ (= 1 without fading)}$$

OPTA: nonwhite source on Rayleigh channel

$$SNR = \beta_X^{-2} \exp\left\{\left(\frac{1}{\gamma_0} - 1\right)\bar{\gamma}\, r\right\} = \beta_X^{-2} \exp\left\{E_1\!\left(\frac{\gamma_0}{\bar{\gamma}}\right) r\right\}$$



$r = 1.$ OPTA without fading (green), OPTA with fading (red)

RD-function for multiple Gaussian sources

Assume a continuum of sources with variances distributed according to a probability density function $f_\Sigma(\sigma_X^2)$.

The rate and the distortion per source sample are respectively given as

$$R(\mu) = \frac{1}{2}\int_\mu^\infty f_\Sigma(\sigma_X^2)\log\left(\frac{\sigma_X^2}{\mu}\right)d\sigma_X^2,$$

and

$$\sigma_D^2 = \mu\int_\mu^\infty f_\Sigma(\sigma_X^2)d\sigma_X^2 + \int_0^\mu \sigma_X^2 f_\Sigma(\sigma_X^2)d\sigma_X^2$$

The first term in σ_D^2 is the distortion for the part of the signal set that is coded, while the last term is the noise resulting from signal components that are skipped

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Modelling multi-source systems

Source example: image

Decorrelation: Split image by 2D filterbank or transform into subimages belonging to different frequency bands $\rightarrow$ uncorrelated bands

Different parts of the image have different local statistics represented by local variances $\rightarrow$ split subimages into blocks where the RMS value of each block will be estimates of the variance

The blocks are normalized by their estimated standard deviations

The individual blocks are well modelled by a Gaussian pdfs with variances $\sigma_{X_i}^2,\ i \in 1, 2, 3, \ldots, N$

Calculated RD for images

Assume N sources with variances $\sigma^2_{X_n}$

The total rate expressed in terms of the parameter μ is given by

$$R_N(\mu) = \frac{1}{2} \sum_{n \in \mathcal{N}} \log \left(\frac{\sigma^2_{X_n}}{\mu} \right),$$

while the total distortion is given by

$$D_N(\mu) = (N - N_{\mathcal{N}})\, \mu + \sum_{n \notin \mathcal{N}} \sigma^2_{X_n}$$

$\mathcal{N}$ is the set of sources for which the variances $\sigma^2_{X_n} > \mu$, and $N_{\mathcal{N}}$ is the number of sources in this set

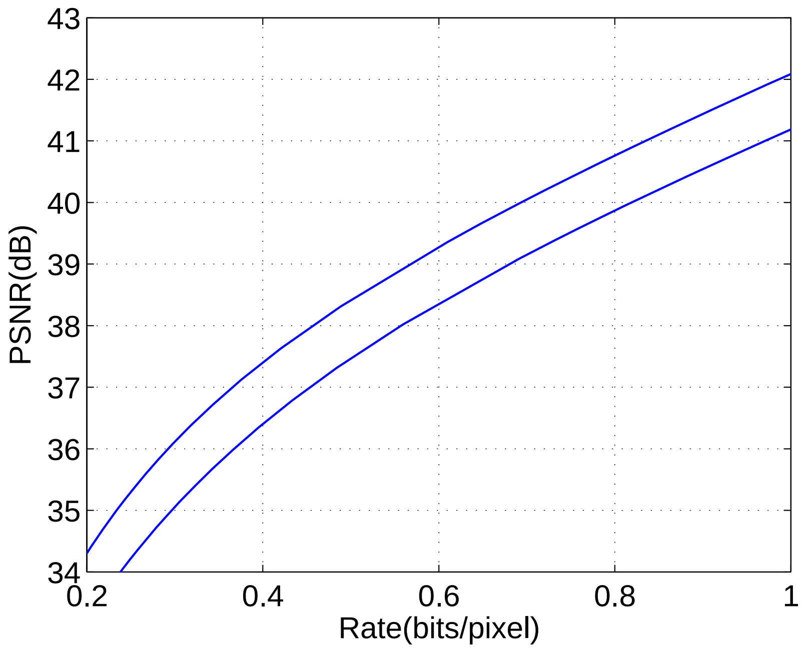
RD result for Lena

Original: 512 × 512 Lena

Split by separable filterbank: first 8 × 8 uniform bands

Then the lowpass band is further split using a three stage two-band split

Resulting subbands are subdivided into 4 × 4 (upper curve) and 8 × 8 (lower curve)



Equate the average rate per source sample to the rate reduction factor times the channel capacity per sample for a Rayleigh fading channel:

$$\frac{1}{2N}\sum_{n\in\mathcal{N}}\log\left(\frac{\sigma^2_{X_n}}{\mu}\right)=r\frac{\bar{\gamma}}{2}\left(\frac{1}{\gamma_0}-1\right)$$

The set $\mathcal{N}$ depends on μ. Assuming that the set is known, μ is given by

$$\mu=\left(\prod_{n\in\mathcal{N}}\sigma^2_{X_n}\right)^{\frac{1}{N_{\mathcal{N}}}}\exp\left(-r\frac{N}{N_{\mathcal{N}}}\left(\frac{1}{\gamma_0}-1\right)\bar{\gamma}\right)$$

Iteratively method: Find $\mathcal{N}$ by gradually skipping the smaller variances and recalculating μ until $\sigma^2_{X_n}>\mu$ for $n\in\mathcal{N}$

The distortion per sample is then given by

$$\sigma^2_D(\mu)=\frac{1}{N}\left((N-N_{\mathcal{N}})\,\mu+\sum_{n\notin\mathcal{N}}\sigma^2_{X_n}\right)$$

RD behavior based on theoretical channel model

RD behavior based on:

- Variances from image subbands blocks
- Channel acts according to the probability law

→ If this were the case, we would need exactly r channel samples on the average to transmit one image sample

Obtained result is upper bound

Practical system design

Assume:

- Dimension changing mappings are available for arbitrary dimension change (Fredrik and Pål Anders)
- Exact channel state information is available at the transmitter
- Channel state remains constant under transmission of each block

Set rate of one signal equal to instantaneous channel capacity times r

$$\frac{1}{2}\log\left(\frac{\sigma_{X_n}^2}{\mu}\right) = \frac{r}{2}\log\left(\frac{\gamma(t_i)}{\gamma_0}\right)$$

Resulting mapping requirement

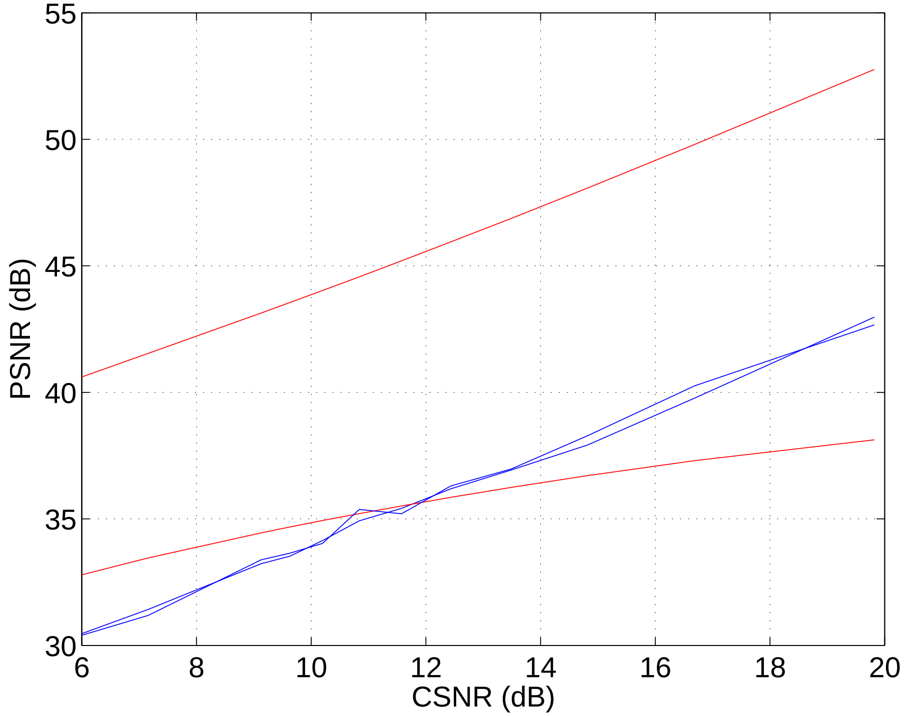
$$r_{n,i} = \frac{\log\left(\frac{\sigma^2_{X_n}}{\mu}\right)}{\log\left(\frac{\gamma(t_i)}{\gamma_0}\right)}$$

Can pick image blocks to make mapping easy and close to optimal when only a finite set of mappings is available

Same result irrespective of combination if mapping can be implemented

Can also find best possible system using only one-to-one mapping (linear) for the case $r = 1$: Largest signal variances should be transmitted when channel is at its best

Resulting performance



Red: Optimal (Upper: $r = 1$, Lower: $r = 0.2$), Blue: Optimal linear ($r = 1$)

Conclusion

Have shown some theoretical limits for JSSC for correlated Gaussian source and flat fading channels

Have pointed to some of the tricks for application in image transmission

Potential: low complexity, low delay, robust system

Greg is going to show more details