Abstract—When adaptive modulation is used to counter short-term fading in mobile radio channels, signaling delays create problems with outdated channel state information. The use of channel power prediction will improve the performance of the link adaptation. It is then of interest to take the quality of these predictions into account explicitly when designing an adaptive modulation scheme. We study the optimum design of an adaptive modulation scheme based on uncoded $M$-quadrature amplitude modulation, assisted by channel prediction for the flat Rayleigh fading channel. The data rate, and in some variants the transmit power, are adapted to maximize the spectral efficiency, subject to average power and bit-error rate constraints. The key issues studied here are how a known prediction error variance will affect the optimized transmission properties, such as the signal-to-noise ratio (SNR) boundaries that determine when to apply different modulation rates, and to what extent it affects the spectral efficiency. This investigation is performed by analytical optimization of the link adaptation, using the statistical properties of a particular, but efficient, channel power predictor. Optimum solutions for the rate and transmit power are derived, based on the predicted SNR and the prediction error variance.

Index Terms—Adaptive modulation, channel prediction, flat Rayleigh fading channels.

I. INTRODUCTION

SPECTRALLY efficient communication techniques are of great importance in future wireless communications. Adaptive modulation, or link adaptation, is a powerful technique for improving the spectral efficiency in wireless transmission over fading channels. If complete channel state information (CSI) is known at the transmitter, the Shannon capacity of a fading channel can be approached by optimal adaptation of the signaling parameters such as transmit power, data rate, channel coding rate, or scheme [1], [2]. Adaptive modulation has been extensively studied in [3]–[14] and the references therein.

With adaptive modulation, a high spectral efficiency is attainable at a given bit-error rate (BER) in favorable channel conditions, while a reduction of the throughput is experienced when the channel degrades. The adaptation can also take requirements of different traffic classes and services, such as required BERs, into account.

We consider fast link adaptation, i.e., we strive to adapt to the small-scale fading. The receiver estimates the received power and sends feedback information via a return channel to the transmitter, with the aim of modifying the modulation parameters. Due to the unavoidable delays involved in power estimation, feedback transmission, and modulation adjustment, estimates of the CSI will be based on outdated information. In the so-far-proposed solutions for optimum design of adaptive modulation systems, perfect knowledge of the CSI at the transmitter, as well as error-free channel estimates at the receiver, are common assumptions for the system design and performance evaluation. In real systems, these assumptions are not valid. Due to the time-varying nature of wireless channels, the channel status will change during the time delay between estimation and data transmission. This leads to performance degradation, such as decrease in the throughput.

The impact of the uncertainty in channel estimates on the performance has been discussed in the literature (see, e.g., [5]–[7], [15]–[21]). In [17], the effect of channel estimation errors on the BER performance is illustrated. However, this effect is investigated on the receiver side, not on the choice of link adaptation parameters. In [5] and [6], the impact of time delay on the adaptive modulation performance is characterized. It is shown that systems with low BER requirements are very sensitive to the time delay. They can operate acceptably only if the delay is kept below a critical small value.

The theoretically optimal way of handling the problem would be to optimally estimate the conditional probability density function (pdf) of the BER for a given modulation format, conditioned on all past received signals and other relevant information. The modulation format would then be adjusted based on this pdf. This approach is unfortunately hard to realize, since the BER is a nonlinear function of the channel power gain, which is a quadratic function of the complex channel. We are, therefore, forced to consider a suboptimal two-step approach. The channel power gain is predicted by a (theoretically suboptimum) scheme, and the adaptive modulation system is analyzed, and preferably adjusted, based on the statistics of these predictions.

Adaptive modulation based on prediction has been studied in some previous papers. In [7], a linear predictor is used to estimate the current channel status based on the outdated estimates. The channel status is used to determine the currently appropriate modulation, and the effect of small time delays and the mobile speed on the BER and throughput performance are studied.
However, the signal-to-noise ratio (SNR) thresholds which determine the modulation modes are evaluated based on the simulation results only. Results by Goeckel [15], [16] highlight that to design a practical adaptive modulation system, time variations of the channel which severely degrade the system performance should be taken into account. There, a novel approach to the design of a robust adaptive modulation system based on only a single outdated fading estimate is proposed. In [18]–[20], adaptive modulation schemes based on long-range CSI predictions that assume the use of perfect channel estimates are investigated. A channel predictor based on pilot-symbol-assisted modulation (PSAM) is developed in [21], where the impact of the prediction error on the system performance is analyzed.

The system proposed here uses an unbiased quadratic regression of past noisy channel estimates to predict the signal power at the receiver. This algorithm is developed and analyzed in [22] and [23]. This predictor is not claimed to be optimal, but from our experience, it produces the best performance on measured data among all known channel power predictions, if it uses smoothed noise-reduced complex channel estimates as regressors. Also, closed-form expressions exist for the prediction error statistics when the predictor is applied to Rayleigh fading channels. Starting from the major assumption that the second-order statistics of the channel are known, these expressions will be used in this paper for optimizing the rate adaptation scheme, and for analyzing the resulting BER and spectral efficiency for given prediction error variances.

We restrict our attention to link adaptation with uncoded $M$-quadrature amplitude modulation (QAM). With no coding, the two remaining degrees of freedom (DOFs) are the choice of modulation formats in different SNR regions, and the possibility of using transmit power control within these regions. Exploitation of the statistical information about the prediction errors will be shown to improve the overall system performance, and adjusts the link adaptation better to the true channel conditions. As a result, the BER constraints will be fulfilled also in the presence of prediction errors. This will not be the case if the prediction errors are neglected in the design of the link adaptation. It turns out to be quite important to take into account the prediction errors that are realistically encountered in long-range prediction (by $0.1$–$0.5$ wavelengths) of flat Rayleigh fading channels. Also, low-rate region boundaries that are active in fading dips will be shown to be affected much more by the prediction error than higher rate boundaries.

This paper is organized as follows. Section II describes the system model and the notations which are used throughout. The channel prediction is explained in Section III. The BER is evaluated as a function of predicted instantaneous SNR in Section IV, and optimal rate and power adaptation are derived under different constraints in Section V. Analytical results are presented in Section VI, while Section VII summarizes the results.

II. SYSTEM MODEL

In the adaptive modulation, $M$-QAM schemes with different constellation sizes are provided at the transmitter. For each transmission, the modulation scheme, and possibly also the transmit power, are adjusted to maximize the spectral efficiency, under BER and average power constraints, based on the instantaneous predicted SNR. The channel is modeled by a flat Rayleigh fading channel. At the receiver, demodulation is performed using channel estimates. The discrete model of the system is depicted in Fig. 1. All the signals are sampled at the symbol rate, except for the cases when the index $n$ represents the index of the channel sample at time $nT_s$, where $T_s$ is the symbol period. Here, $g_n$ is the zero-mean, complex channel gain with circular Gaussian distribution, where the power $|g_n|^2$ is $\chi^2(2)$, or exponentially distributed. The autocorrelation function of the complex channel gain is denoted by

$$r_g(m) = E(g_n g_{n-m}^*) .$$

(1)

In the following, $r_g$ will denote the average channel power gain $E|g_n|^2 = r_g(0)$. Moreover, $w_n$ is a sample from a complex white Gaussian noise process, with zero mean and time-invariant variance $\sigma_w^2$. The estimate $\hat{g}_{n-L}$ is the noisy observation of $g_{n-L}$ at the receiver, obtained by a pilot-aided channel estimator.

A time series of these estimates is used at the receiver to predict the channel power gain $|g_n|^2$, which is proportional to the instantaneous received SNR, denoted by $\gamma_n$. The appropriate rate and transmission power levels are fed back to the transmitter, where an error-free feedback channel is assumed. In practice, the feedback information is quantized to limit the return channel bandwidth. This added source of error is not taken into account in the analysis.

Based on the predicted SNR denoted by $\hat{\gamma}_{n-L}$ or $\hat{\gamma}_n$, a modulation scheme with constellation size $M(\hat{\gamma})$ (out of $N$ constellations available at the transmitter), which transmits $k(\hat{\gamma}) = \log_2 M(\hat{\gamma})$ bits per symbol, and a transmit power $S(\hat{\gamma})$, are selected. Each block of $k(\hat{\gamma})$ data bits denoted by $b_{k(\hat{\gamma})}$ is Gray encoded and mapped to a symbol in the signal constellation denoted by $s_{k(\hat{\gamma})}$, which is transmitted over the flat Rayleigh fading channel. The received sample, $r_n$, is used to estimate the channel gain $\tilde{g}_n$, which, in turn, is used to detect the transmitted bits denoted by $\tilde{b}_{k(\hat{\gamma})}$. Since the estimation error in $\tilde{g}_n$ is believed to have a minor effect on the performance, compared with the prediction error, perfect channel estimation is here assumed for the coherent demodulation.
In this paper, the following notations, similar to those of [14], are used. Let $\mathbf{S}$ denote the average transmit signal power. The average received SNR is then given by

$$\gamma = \frac{\mathbf{S}}{\sigma^2}.$$  

(2)

For a constant transmit power $\mathbf{S}$, the instantaneous received SNR is

$$\gamma_n = \frac{\gamma_n P_n}{r_g}.$$  

(3)

where $P_n = |g_n|^2$ is the instantaneous channel power gain. Correspondingly, the instantaneous predicted received SNR is

$$\hat{\gamma} = \hat{\gamma}_n |_{|L} = \frac{\hat{P}_n |_{|L}}{r_g}.$$  

(4)

where $\hat{P}_n |_{|L}$ is the predicted instantaneous channel power gain $|g_n|^2$. For the transmit power $S(\hat{\gamma})$, the instantaneous received SNR is given by $\gamma_n(S(\hat{\gamma})) / \mathbf{S}$, with $\gamma_n$ given by (3).

The region rate boundaries, defined as the ranges of $\hat{\gamma}$ values over which the different constellations are used by the transmitter, are denoted by $\{\hat{\gamma}_i\}_{i=1}^{N-1}$. When the predicted instantaneous SNR belongs to a given rate region, i.e., $\hat{\gamma} \in (\hat{\gamma}_i, \hat{\gamma}_{i+1})$, the corresponding constellation of size $M(\hat{\gamma}) = M_i$ with $k(\hat{\gamma}) = k_i$ bits per symbol is transmitted, where $\hat{\gamma}_N = \infty$. There is no transmission if $\hat{\gamma} < \hat{\gamma}_0$, meaning that $\hat{\gamma}_0$ is the cutoff SNR.

III. CHANNEL PREDICTION

The absolute square, i.e., the power, of the time series $g_n$ is to be predicted based on the observations $y_n$ that are assumed to be affected by an additive estimation error $e_n$. Thus, $y_n = g_n + e_n$. The channel estimator which produces $\hat{y}_n$ is assumed to be unbiased and to operate linearly on the received baseband signal. A reasonable assumption on $e_n$, used in the following, is that it is a zero-mean complex circular Gaussian random variable, which is independent of $g_n$. Based on a finite number of past observations of $y_n$, the complex channel at time $n$ is predicted with a prediction horizon $L$ by a linear finite impulse response (FIR) filter

$$\hat{y}_n |_{|L} = \varphi^H_{|L} \theta$$  

(5)

where $\theta$ is a column vector containing $K$ complex-valued predictor coefficients, and

$$\varphi^H_{|L} = [y_{n-L}, y_{n-L-m}, \ldots, y_{n-L-(K-1)m}]$$  

(6)

is the regressor vector, where $H$ represents a Hermitian transpose. A Wiener adjustment of $\theta$ provides the optimal linear predictor in the mean-square error (MSE) sense.

To provide the best prediction performance on fading channels for a limited, prespecified number $K$ of parameters, the delay spacing $m$ (the time delay between consecutive channel samples in the regressor) should be selected in a way appropriate for the fading rate. The predictor performance will be improved by the use of good channel estimates within (6), obtained by the application of noise-reducing smoothing [22]–[24].

The adjustment of an adaptive modulation scheme is determined not by the complex channel gain $g_n$, but by the SNR at the time of transmission. If we, for simplicity, assume that the variance of the noise and disturbance $\epsilon_n$ in Fig. 1 is constant, the channel power $P_n = |g_n|^2$ will have to be predicted. However, the use of the squared magnitude of the linear prediction $\hat{g}_n |_{|L}$ as a predictor of the channel power would, on average, underestimate the true power, and result in a biased estimate. The reason is that the predictability of $g_n$ decreases with $L$. The average power of $\hat{g}_n |_{|L}$ will, therefore, decrease with an increasing prediction horizon $L$ and be lower than the average power of $g_n$. We instead use a recently developed quadratic power predictor [22], which eliminates this bias and enables the attainment of a lower MSE of the estimated power. It is constructed by adding the true average power $r_g$ and subtracting $E(|\hat{g}_n |_{|L}|^2)$ from the estimate $|\hat{g}_n |_{|L}|^2$.

$$\hat{P}_n |_{|L} = \theta^H \varphi_{n-L} \varphi^H_{n-L} \theta + r_g - \theta^H \mathbf{R}_\varphi \theta.$$  

(7)

Here, $\mathbf{R}_\varphi = E(\varphi_{n-L} \varphi_{n-L}^H)$ is the $K \times K$ correlation matrix for the regressors. Note that $E(|\hat{g}_n |_{|L}|^2) = r_g$ for all $L$. The unbiased quadratic predictor that minimizes the power prediction MSE is derived in [22], assuming second-order statistics of $g_n$ are known. It is shown there that the predictor coefficient vector $\theta$ that provides an MSE optimal channel predictor (5) will also result in an MSE optimal power predictor, when used in (7). The optimal adjustment for both of these problems is given by

$$\theta = \mathbf{R}_\varphi^{-1} r_{\varphi \varphi}$$  

(8)

$$r_{\varphi \varphi} = E\{g_n |_{|L}\}$$  

$$= [r_g(L), r_g(L + m), \ldots, r_g(L + (K - 1)m)]^T.$$  

(9)

If $\theta$ is perfectly adjusted, the minimum mean-square value of the channel gain prediction error $\epsilon_n = g_n - \hat{g}_n |_{|L}$ and the power prediction error $\epsilon_{p_n} = P_n - \hat{P}_n |_{|L}$ will be given by

$$\sigma^2_{\epsilon} = r_g - r_{\varphi \varphi}^H \mathbf{R}_\varphi^{-1} r_{\varphi \varphi}$$  

(10)

$$\sigma^2_{\epsilon_{p_n}} = r_g - r_{\varphi \varphi}^H \mathbf{R}_\varphi^{-1} r_{\varphi \varphi}^2$$  

(11)

respectively. Thus, by (8) and (7), the optimum unbiased quadratic power prediction can be expressed in terms of the MSE-optimal linear FIR channel prediction as

$$\hat{P}_n |_{|L} = |\hat{g}_n |_{|L}|^2 + \sigma^2_{\epsilon_{p_n}}.$$  

(12)

In other words, the squared magnitude of the optimal FIR channel prediction (5), with $\theta$ by (8), is modified simply by adding the variance (10) of the channel prediction. The bias compensation will reduce the total prediction MSE. It also provides superior performance, as compared with the use of linear power predictors that are based on channel power samples $|\{y_n|^2|$ as regressors [23]. For a given prediction $\hat{P}_n |_{|L}$ by (12), the conditional power-prediction error variance, denoted by $\sigma^2_{\epsilon_{p_n}} |_{|L}$, is given by [23, Eq. (7.48)]

$$\sigma^2_{\epsilon_{p_n}} |_{|L} = \sigma^2_{\epsilon} |_{|L} = 2r_g - \sigma^2_{\epsilon_{p_n}}.$$  

(13)

If we average over the predicted power in (13), we obtain

$$\sigma^2_{\epsilon_{p_n}} = \sigma^2_{\epsilon} [2r_g - \sigma^2_{\epsilon}]$$  

(14)

since, with the unbiased predictor, $E(\hat{P}_n |_{|L}) = E(P_n) = r_g$

This expression can also be obtained by substituting (10) into (11).
An important indication of the predictor performance is the relative standard deviation of the conditional power prediction error. Using (13), this measure is given by

\[ \frac{\sigma_{\hat{h}_n}}{\hat{p}_{h_{n-L}}} = \sigma_e \sqrt{\frac{2 \hat{p}_{h_{n-L}} - \sigma_e^2}{\hat{p}_{h_{n-L}}}}. \]  

(15)

For a given \( \sigma_e \), (15) increases as \( \hat{p}_{h_{n-L}} \) becomes small, i.e., \( \hat{p}_{h_{n-L}}^2 \) by (17), and that on an additive white Gaussian noise (AWGN) channel. More specifically, where the time index \( n \) is chosen, the instantaneous true SNR, \( \gamma_n \), conditioned on \( \gamma_{h_{n-L}} \) will be given by

\[ f_{\hat{\gamma}}(\gamma|\gamma_{h_{n-L}}) = \frac{U(\gamma)U(\gamma - \frac{\sigma_e^2}{r_g})}{\frac{2 \sigma_e^2}{r_g} \sqrt{\gamma}} \exp \left[ -\frac{\gamma - \frac{\sigma_e^2}{r_g}}{\frac{2 \sigma_e^2}{r_g}} \right] I_0 \left( \sqrt{\frac{2 \sigma_e^2}{r_g}(\gamma - \frac{\sigma_e^2}{r_g})} \right) \]  

(16)

where \( U(\gamma) \) is the Heaviside’s step function, \( \hat{\gamma} \) is given by (2), and \( I_0(\gamma) \) is the zeroth-order modified Bessel function. The time index \( n \) was dropped in the pdf expressions, since \( \gamma_n \) and \( \gamma_{h_{n-L}} \) are both stationary random processes. The pdf of \( \hat{\gamma} \) will be given by

\[ f_{\hat{\gamma}}(\gamma) = \frac{U(\gamma - \frac{\sigma_e^2}{r_g})}{\gamma(1 - \frac{\sigma_e^2}{r_g})} \exp \left[ -\frac{\gamma - \frac{\sigma_e^2}{r_g}}{\gamma(1 - \frac{\sigma_e^2}{r_g})} \right]. \]  

(17)

This is a shifted \( \chi^2(2) \) distribution, with the shift \( \gamma \sigma_e^2/r_g \) caused by the bias compensation term in (7) and (12).

IV. M-QAM BER PERFORMANCE

The transmitter adjusts the constellation size, and possibly also the transmit power, based on the instantaneous predicted SNR \( \gamma_{h_{n-L}} \), where the time index \( n \) will be dropped in the following. Evaluation of the optimal power and constellation size (or rate) adjustments, which maximize the spectral efficiency and satisfy the BER requirement, requires an invertible expression for the BER as a function of \( \gamma \). Assuming a square M-QAM with Gray-encoded bits, constellation size \( M_t \), and transmit power \( S(\gamma) \), the instantaneous BER (I-BER) as a function of \( \gamma \) and \( \gamma_{h_{n-L}} \) on an additive white Gaussian noise (AWGN) channel, is approximated by [14]

\[ \text{BER}(\gamma, \gamma_{h_{n-L}}) \approx 0.2 \exp \left( \frac{-1.6 \gamma S(\gamma)}{M_t - 1} \right) \]  

(18)

which is tight within 1 dB for \( M_t \geq 4 \) and BER \( \leq 10^{-3} \). Moreover, the I-BER as a function of the instantaneous predicted SNR, \( \hat{\gamma} \), is obtained as

\[ \text{BER}(\hat{\gamma}) = \int_0^\infty \text{BER}(\gamma, \gamma_{h_{n-L}}) f_{\hat{\gamma}}(\gamma|\gamma_{h_{n-L}}) d\gamma, \]  

(19)

Using (16) to average (18) over the whole range of the instantaneous true SNR, \( \gamma \), results in

\[ \text{BER}(\hat{\gamma}) \approx 0.2 \beta(\gamma) \exp \left[ (1-x(\gamma))(1-z(\gamma)) \right] \]  

(20)

where

\[ x(\gamma) = \frac{\gamma}{\sigma_e^2/r_g}, \]  

(21)

\[ z(\gamma) = \frac{1}{1 + A_t S(\gamma)} \]  

(22)

\[ A_t = \frac{1.6}{M_t - 1} \]  

(23)

Note that \( x(\gamma) \geq 1 \), since \( \gamma \geq \sigma_e^2/r_g \) by (17), and that \( 0 \leq z(\gamma) < 1 \). Finally, similar to [14], the average BER (A-BER) is given by

\[ \text{BER} = \frac{\sum_{i=0}^{N-1} k_i f_{\gamma_{h_{n-L}}}^i \text{BER}(\gamma) f_{\hat{\gamma}}(\gamma) d\gamma}{\sum_{i=0}^{N-1} k_i f_{\gamma_{h_{n-L}}}^i f_{\hat{\gamma}}(\gamma) d\gamma}. \]  

(24)

In [25], it is shown that the use of (18), which gives the analytical expression (20) for the I-BER, results in a small approximation error, as compared with integrating (8) in [14] (which is a more accurate approximation for BER) in (19).

V. OPTIMAL RATE AND POWER ADAPTATION

The spectral efficiency of a modulation scheme is given by the average data rate per unit bandwidth \( (R/B) \), where \( R \) is the data rate and \( B \) is the transmitted signal bandwidth. When a modulation with constellation size \( M_t \) is chosen, the instantaneous data rate is \( k_i / T_s \) b/s. Assuming the Nyquist data pulses \( (B = 1/T_s) \), the spectral efficiency is given by

\[ \eta = \sum_{i=0}^{N-1} k_i \int_{\gamma_{h_{n-L}}}^{\gamma_{h_{n-L}}+1} f_{\gamma_{h_{n-L}}}^i f_{\hat{\gamma}}(\gamma) d\gamma \]  

b/s/Hz.  

(25)

As explained in Section II, a transmission rate is assigned to each rate region boundary. The rate region boundaries will be adjusted to maximize the spectral efficiency, subject to various constraints. For SNR levels between the rate boundaries, the transmit power may furthermore be adjusted. We have an optimization problem with two possible DOFs: the rate region boundaries and the transmit power.

In this paper, we consider the following scenarios. First, we intend to maximize the spectral efficiency where both the average power and I-BER are constrained. The transmit power, as well as the rate, are adapted to satisfy the requirements. Then, we study a case where constant transmit power is presumed. The motivation is that the data rate adaptation has the major effect in increasing the spectral efficiency, as compared with the power adaptation, as shown by [5] and [12]. Also, transmission with variable power complicates the practical implementation: transmission of only integers \( k_i \) requires less feedback bandwidth, compared with using fast adaptive modulation combined with fast power control. We thereafter relax the BER constraint by constraining the A-BER (24) instead of the I-BER (19), which results in an increase in the spectral efficiency.

We illustrate the derivation of the optimum rate region boundaries, and possibly also transmit power adjustment, of these
and $\mathcal{S} = 1$ are assumed. It is shown that the approximation error in (28) results in negligible errors.

As it is clear from above, the optimization problem can be simplified to a search for the optimal rate region boundaries. For this purpose, we form the Lagrangian function from the spectral efficiency criterion (25) and the power constraint (26), which is here treated as an equality constraint. It is given by

$$J(\gamma_0, \gamma_1, \cdots, \gamma_{N-1}) = \sum_{i=0}^{N-1} k_i \int_{\gamma_i}^{\gamma_{i+1}} f_s(\gamma) d\gamma + \lambda \left( \sum_{i=0}^{N-1} S_i(\gamma_i) f_s(\gamma_i) d\gamma - \mathcal{S} \right)$$  

where $\lambda \neq 0$ is the Lagrangian multiplier. Solving

$$\frac{\partial J}{\partial \gamma_i} = 0, \quad 0 \leq i \leq N - 1$$

results in

$$S_{i-1}(\gamma_i) - S_i(\gamma_i) = \frac{k_i}{\lambda} - \frac{k_{i-1}}{\lambda}, \quad 0 \leq i \leq N - 1$$  

where $k_{-1} = 0$ and $S_{-1}(\gamma) = 0$. From (29) and (33), we obtain

$$\gamma_i = \ln \left( \frac{0.2}{\text{TBER}} \right) \left( \frac{\sigma_e^2}{r_g} \frac{\mathcal{S}}{1.6} \frac{M_i - M_{i-1}}{k_i - k_{i-1}} \right),$$

$$0 \leq i \leq N - 1.$$  

The Lagrange multiplier $\lambda$ is numerically evaluated based on the power constraint (26). Given (29) and (34), the average power constraint (26) can be written as

$$\sum_{i=0}^{N-1} \int_{\gamma_i}^{\gamma_{i+1}} S_i(\gamma) f_s(\gamma) d\gamma = \rho \exp \left( \frac{\sigma_e^2}{r_g} \ln \left( \frac{0.2}{\text{TBER}} \right) \right)$$

$$\times \sum_{i=0}^{N-2} (M_i - 1) (\text{Ei}(\rho \Delta_i) - \text{Ei}(\rho \Delta_{i+1}))$$

$$+ (M_{N-1} - 1) \text{Ei}(\rho \Delta_{N-1}) \leq \mathcal{S}$$

where $\text{Ei}(\gamma)$ is the Exponential Integral, and for convenience, the notations

$$\rho = -\ln \left( \frac{0.2}{\text{TBER}} \right) \frac{\mathcal{S}}{1.6} \frac{1}{\gamma} \left( 1 - \frac{\sigma_e^2}{r_g} \right)$$

$$\Delta_i = \frac{M_i - M_{i-1}}{k_{i-1} - k_i}, \quad 0 \leq i \leq N - 1$$  

are used. Here, the average power constraint will be fulfilled if $\lambda < 0$. A bisection method is used to numerically search for $\lambda$, which meets the power constraint with equality.

B. I-BER and Constant Power (C-POW)

We now consider the use of an I-BER constraint and of a constant transmit power $S(\gamma) = \mathcal{S}$ that is adjusted to satisfy the average power constraint (26) with equality. The BER expression (20) then becomes

$$\text{BER}(\gamma) = \frac{0.2}{1 + A_1^*} \exp \left[ \frac{A_1^* S}{1 + A_1^* S} (1 - x(\gamma)) \right].$$
The average power constraint (26) implies that the cutoff SNR \( \tilde{\gamma}_0 \) should satisfy
\[
\frac{S}{S_0} = \frac{1}{\int_{\tilde{\gamma}_0}^{\infty} f_s(\tilde{\gamma})d\tilde{\gamma}} \tag{38}
\]
which implies that the transmit power used when transmission does occur will be higher than \( S_0 \), and it is given by
\[
S = S_0 \exp \left[ \frac{-\tilde{\gamma}_0 - \frac{\sigma^2_r}{r_y}}{\tilde{\gamma}_0 \left(1 - \frac{\sigma^2_r}{r_y}\right)} \right]. \tag{39}
\]
Moreover, the I-BER constraint must be fulfilled at all the rate region boundaries, such that
\[
\text{BER}(\tilde{\gamma}) \leq \text{BER}(\tilde{\gamma}_i) = \text{TBER} \quad \gamma_i \in [\gamma_i, \gamma_{i+1}), \quad 0 \leq i \leq N - 1 \tag{40}
\]
which by (37) and (21) results in
\[
\tilde{\gamma}_i = \frac{\sigma^2_r}{r_y} \left[ 1 - \frac{1 + A_i S}{A_i S} \ln \left( \frac{\text{TBER}}{0.2} \left(1 + A_i S\right)\right) \right], \quad 0 \leq i \leq N - 1. \tag{41}
\]
Thus, the cutoff SNR \( \tilde{\gamma}_0 \) and the transmit power \( S \) are found through (39) and (41). Once it is done, \( \{\tilde{\gamma}_i\}_{i=0}^{N-1} \) are easily obtained from (41).

C. A-BER and Constant Power (C-POW)

Finally, we investigate the case concerning the A-BER constraint with constant transmit power. Similar to Section V-B, the transmit power must satisfy (38). The A-BER constraint is given by
\[
\text{BER} \leq \text{TBER} \tag{42}
\]
where (37) is used for the I-BER in (24). Forming the Lagrangian function from the criterion (25) and the constraint (42), here treated as an equality constraint, gives
\[
J(\gamma_0, \gamma_1, \ldots, \gamma_{N-1}) = \sum_{i=0}^{N-1} k_i \int_{\gamma_i}^{\gamma_{i+1}} f_s(\gamma) d\gamma + \lambda \left[ \sum_{i=0}^{N-1} k_i \int_{\gamma_i}^{\gamma_{i+1}} (\text{BER}(\gamma) - \text{TBER}) f_s(\gamma) d\gamma \right]. \tag{43}
\]
The optimum rate region boundaries are found through solving
\[
\frac{\partial J}{\partial \gamma_i} = 0, \quad 0 \leq i \leq N - 1 \tag{44}
\]
which results in
\[
\text{BER}(\tilde{\gamma}_i) = \text{TBER} - \frac{1}{\lambda}, \quad 0 \leq i \leq N - 1. \tag{45}
\]
Similar to the previous case, we have
\[
\tilde{\gamma}_i = \frac{\sigma^2_r}{r_y} \left[ 1 - \frac{1 + A_i S}{A_i S} \ln \left( \frac{\text{TBER} - \frac{1}{\lambda}}{0.2} \left(1 + A_i S\right)\right) \right], \quad 0 \leq i \leq N - 1. \tag{46}
\]

We can evaluate the optimal rate region boundaries and transmit power through (39) and (46), based on \( \lambda \) that satisfies the A-BER constraint. As shown in Appendix A, the optimal solution that fulfills the A-BER constraint with equality exists when TBER < \((0.2/(1 + A_0 S))\). Otherwise, the solution results in a lower A-BER than required, with a consequent reduction in the spectral efficiency. For more details, see Appendix A.

VI. RESULTS AND DISCUSSION

We assume that six different \( M \)-QAM signal constellations corresponding to 4-QAM, 16-QAM, 64-QAM, 256-QAM, 1024-QAM, and 4096-QAM are available at the transmitter. Although the use of very large constellations is questionable from the practical point of view, it is included here to illustrate the effect of prediction errors of various magnitudes. The results presented here are evaluated for flat Rayleigh fading channels with \( r_y = 1 \) and for \( S = 1 \), where \( \tilde{\gamma} \) given by (2) is adjusted by varying the noise variance \( \sigma^2_n \).

The optimal region boundaries for different policies when the required BER\(^2\) is \( \text{TBER} = 10^{-3} \), the prediction error variances are \( \sigma^2_p = 0.001 \) and 0.1, and the average received SNR is \( \tilde{\gamma} = 20 \text{ dB} \), can be seen from Figs. 3–5. Fig. 3 shows that for the I-BER, V-POW policy, the transmit power follows the inverse waterfilling pattern with respect to (w.r.t.) \( \tilde{\gamma} \) within each rate region. The peak power within each interval increases as the rate increases. Fig. 4 illustrates that under the I-BER, C-POW policy, the I-BER does not exceed the required BER while it reaches the target BER at the boundaries, as intended. Finally, as shown in Fig. 5, the A-BER, C-POW policy results in an I-BER fluctuation around the required A-BER to maintain the TBER, on average.

\(^2\)Corresponding results for TBER = \( 10^{-7} \) are available in [25].

\(^3\)Since \( r_y = 1, \sigma^2_p \) will here represent the normalized power prediction MSE (NMSE). For comparison, the use of the average power \( r_y \) as a power prediction would result in a prediction error variance \( \sigma^2_p = 0. \) for all prediction horizons. For a flat-fading channel with Jakes spectrum, and where the channel estimates \( \hat{g}_n \) have an estimate-to-estimate error ratio of 20 dB, \( \sigma^2_p = 0.1 \) corresponds to a prediction 0.3 wavelengths ahead in space, while \( \sigma^2_p = 0.01 \) is obtained when predicting 0.1 wavelengths ahead [22], [23]. See also [26]. The level \( \sigma^2_p = 0.001 \) is essentially equivalent to perfect prediction of the channel power.
An interesting phenomenon observed in these figures is the effect of the prediction error variance on the SNR thresholds. When a large prediction error variance is taken into account, the relative standard deviation (15) of the conditional prediction error is larger in fading dips \( \hat{\gamma} < \gamma \). Due to the average power constraint, this allows the use of higher transmit power when transmission is allowed. For \( \hat{\gamma} \gg \gamma \), the effect caused by power constraint sometimes dominates, which explains why the SNR limits belonging to the large constellation sizes can be reduced.

Fig. 4. I-BER, C-POW policy: I-BER versus instantaneous predicted SNR of \( M \)-QAM schemes for \( \gamma = 20 \) dB and TBER = \( 10^{-3} \). Solid and dashed lines correspond to \( \sigma^2_{\epsilon_p} = 0.001 \) and 0.1, respectively. Discontinuities of the functions coincide with the rate region boundaries \{\( \gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 \)\}.

Fig. 5. A-BER, C-POW policy: I-BER versus instantaneous predicted SNR of \( M \)-QAM schemes for \( \gamma = 20 \) dB and TBER = \( 10^{-3} \). Solid and dashed lines correspond to \( \sigma^2_{\epsilon_p} = 0.001 \) and 0.1, respectively. Discontinuities of the functions coincide with the rate region boundaries \{\( \gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 \)\}.

Fig. 6. Optimum rate region boundaries, \( \gamma_1 \), of the three considered transmission schemes in terms of the average SNR for TBER = \( 10^{-3} \) and \( \sigma^2_{\epsilon_p} = 0.1 \).

Fig. 7. \( M \)-QAM spectral efficiency versus average received SNR for TBER = \( 10^{-3} \) for the three policies considered here. Lines with and without rings correspond to \( \sigma^2_{\epsilon_p} = 0.1 \) and 0.001, respectively.

Fig. 6 shows how the average SNR affects some of the rate boundaries of the adaptive \( M \)-QAM schemes for TBER = \( 10^{-3} \) and \( \sigma^2_{\epsilon_p} = 0.1 \). As observed, the SNR thresholds below the average SNR are increased more than the ones above the average. This effect increases with the prediction error variance, as can be seen in Figs. 3–5. The increase in the rate boundaries is the largest for the I-BER, V-POW policy, while the A-BER, C-POW policy shows the least sensitivity. The reason for this behavior is that the relative standard deviation (15) of the conditional prediction error is larger in fading dips \( \hat{\gamma} < \gamma \). So the adaptive modulation scheme is forced to act more cautiously there.

The maximum spectral efficiency for TBER = \( 10^{-3} \) and \( \sigma^2_{\epsilon_p} = 0.001 \) and 0.1 are illustrated in Fig. 7 for the three considered policies. The features observed in this figure are as follows. The gain in the spectral efficiency when using good predictors is considerable, as compared with the poor predictors. Comparing different policies from the spectral efficiency point of
For the I-BER, V-POW and I-BER, C-POW policies, where has a for . This condition implies that (48) then

for a required . The results are shown for the channel prediction error expressions (which are invertible and differentiable) in terms of the predicted SNR, to maximize the spectral efficiency while satisfying the BER and average transmit power constraints. Optimum solutions for adjusting the adaptive rate and transmit power are derived. The analytical results show that when the prediction error increases, the rate region boundaries for a given constellation size are raised for the SNRs lower than average SNR, while they are sometimes lowered for the SNRs higher than the average SNR. Moreover, the spectral efficiency decreases as the predictor error variance increases. Also, the gain due to the transmission with varying power is minor, and becomes even negligible, when the prediction quality deteriorates. It is demonstrated that the BER considerably increases when the system is not designed based on realistic assumptions, such as erroneous prediction.

The optimization problem that is discussed here can be solved for any family of modulations, as long as accurate BER expressions (which are invertible and differentiable) in terms of the predicted SNR are available. A competitive candidate is trellis-coded modulation (TCM), which will improve the spectral efficiency at a given [27].

VII. CONCLUSION

The optimum design of an adaptive modulation scheme based on uncoded \( M \)-QAM is investigated. The transmitter adjusts the transmission rate, and possibly also power, based on the predicted SNR, to maximize the spectral efficiency while satisfying the BER and average transmit power constraints. Optimum solutions for adjusting the adaptive rate and transmit power are derived. The analytical results show that when the prediction error increases, the rate region boundaries for a given constellation size are raised for the SNRs lower than average SNR, while they are sometimes lowered for the SNRs higher than the average SNR. Moreover, the spectral efficiency decreases as the predictor error variance increases. Also, the gain due to the transmission with varying power is minor, and becomes even negligible, when the prediction quality deteriorates. It is demonstrated that the BER considerably increases when the system is not designed based on realistic assumptions, such as erroneous prediction.

The optimization problem that is discussed here can be solved for any family of modulations, as long as accurate BER expressions (which are invertible and differentiable) in terms of the predicted SNR are available. A competitive candidate is trellis-coded modulation (TCM), which will improve the spectral efficiency at a given [27].

APPENDIX A

Here, we discuss that the optimal rate region boundaries, discussed in Section V-C, exist if the search interval for \( \lambda \) is carefully chosen. For simplicity, we denote \( \alpha = \left( \frac{\tau \sigma_w^2}{\sqrt{\gamma} - \tau \sigma_w^2} \right) \) and \( x(\gamma) = x_i \), where \( x_i \geq 1 \). Based on (39) and (46), we have to solve \( f(x_i) = 0 \) for \( i = 0 \), where

\[
 f(x_i) = 1 - x_i - \frac{1 + A_0 \sigma_w(\gamma^{x_0-1})}{A_0 \sigma_w(\gamma^{x_0-1})} \times \ln \left( \frac{TBER}{0.2} \left( 1 + A_0 \sigma_w(\gamma^{x_0-1}) \right) \right). \tag{47}
\]

It can be easily shown that \( f(x_0) \) is a monotonic function for \( x_0 \geq 1 \). Since \( f(x_0) \to \pm\infty \) \( \to -\infty \), then \( f(x_0) = 0 \) has a solution, if \( f(x_0 = 1) \geq 0 \). This condition implies that

\[
 \begin{align*}
 \text{if } TBER & > \frac{0.2}{1 + A_0 \sigma_w^2}, & \lambda & \in A_3, \\
 \text{if } TBER & < \frac{0.2}{1 + A_0 \sigma_w^2}, & \lambda & \in A_2 \cup A_3
 \end{align*} \tag{48}
\]
where $\Lambda_1$, $\Lambda_2$, and $\Lambda_3$ are sets of real numbers given by

$$
\Lambda_1 = \left( \frac{1}{\text{TBER}}, \frac{1}{\frac{0.2}{1 + \text{TBER}}} \right)
$$

$$
\Lambda_2 = \left( \frac{1}{\text{TBER}}, \frac{0.2}{1 + \text{TBER}} \right)
$$

$$
\Lambda_3 = \left( \frac{1}{\text{TBER}}, \infty \right)
$$

(49)

respectively. Note that $\Lambda_1$ and $\Lambda_3$ are positive sets, while $\Lambda_2$ is negative. Since we aim at $\text{BER} = \text{TBER}$, considering that $\text{BER}(\bar{\gamma})$ decreases as $\bar{\gamma}$ increases, the intuition is to have $\text{BER}(\bar{\gamma}_i) > \text{TBER}$ for $0 \leq i \leq N - 1$, which, based on (45), implies that $\lambda < 0$. The results from (48) show that this condition is satisfied only for $\lambda \in \Lambda_2$.

Therefore, when $\text{TBER} < (0.2/(1 + \text{TBER}))$, a numerical search based on a bisection method for $\lambda \in \Lambda_2$ is used to find a $\lambda$ which meets the A-BER constraint (42) with equality. Otherwise, for $\text{TBER} > (0.2/(1 + \text{TBER}))$, we see from (48) that $\lambda$ can only take positive values. This means that the I-BER is always lower than TBER, and hence, the A-BER never reaches TBER. Therefore, in this case, we choose $\lambda = 1/(\text{TBER} - (0.2)/(1 + \text{TBER}))$ to obtain the closest possible value to the average target BER. By this choice, the rate region boundaries are found through a similar procedure, as explained in Section V-B.

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**REFERENCES**


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