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Reduced Rank Channel Estimation

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ABSTRACT

A space-time wireless communication channels can be decomposed as a set of filters, each consisting of a scalar temporal filter followed by a single spatial signature vector. If only a small number of such filters is necessary to accurately describe the space time channel, we call it a reduced rank channel. We here consider different methods of exploiting this property to improve channel estimation and subsequent space-time equalization performance. Three methods have been studied, a maximum likelihood reduced rank channel estimation method and two different signal subspace projection methods which projects either the channel estimate or the received data samples onto an estimate of the signal subspace, the latter being the new method proposed here. Simulations indicate that even though the maximum likelihood reduced rank method has the smallest channel estimation errors, the BER of the detector based on this model exceeds the BER of the detectors based on the channel models obtained using the two signal subspace projection methods. The best performance is obtained using the proposed method, which also has the lowest complexity.

I. INTRODUCTION

Today, receivers with multiple antenna elements are introduced into cellular communication systems. With such receivers, intersymbol and co-channel interference can be efficiently combatted.

To describe the space-time channel from a transmitter to a multi-antenna receiver, several parameters are required. However, in many cases the temporal channels to different antenna elements will be correlated. One such case when this occurs is when a partial response signal is sent through a channel with very little delay spread. Since all intersymbol interference is caused by the modulation, the time dispersion experienced at the different antenna elements will be highly correlated. When this situation occurs, a *reduced rank* representation of the channel may be used.

The reduced rank property of a channel can be exploited in the channel estimation. One way is to estimate the space-time channel with a maximum likelihood method under the constraint that the resulting channel should be low rank [1].

Another method is to exploit the fact that the vector taps of the channel will lie in the subspace spanned by the signal eigenvectors to the spatial data covariance matrix [2]. We will call this subspace the *spatial signal subspace* or just the *signal subspace*. The channel estimate is here first formed as a straightforward least squares channel estimate. The vector taps in this channel estimate are then projected onto the spatial signal subspace. If co-channel interferers are present, the spatial signal subspace will be spanned by all signal components, desired as well as undesired.

We here propose a third method where the received signal samples are projected directly onto the spatial signal subspace defined above. As well as removing components in the noise subspace, this has the advantage that the dimension of the received signal vector is reduced. This turns out to give better performance and lower complexity.

II. CHANNEL MODEL

Assume that the signal received from user i to antenna element j can be represented as

$$s_{ji}(t) = b_{ji}^0 d_i(t) + \cdots + b_{ji}^{nb_{ji}} d_i(t - nb_{ji}) \triangleq \mathbf{b}_{ji}\mathbf{d}_i(t)$$

where we have defined

$$\begin{aligned} \mathbf{b}_{ji} &\triangleq \begin{bmatrix} b_{ji}^0 & \dots & b_{ji}^{nb_{ji}} \end{bmatrix} \\ \mathbf{d}_i(t) &\triangleq \begin{bmatrix} d_i(t) & \dots & d_i(t-nb_{ji}) \end{bmatrix}^T . \end{aligned}$$

Above, $d_i(t)$ is the signal transmitted from user i, and b_{ji}^m are the parameters used to describe the temporal channel from user i to antenna element j. Let us now combine these temporal channels to form the space-time *channel matrix* for user i:

$$\mathbf{B}_i \triangleq \begin{pmatrix} \mathbf{b}_{1i}^T & \mathbf{b}_{2i}^T & \dots & \mathbf{b}_{Mi}^T \end{pmatrix}^T$$

where M is the number of antenna elements. One realizes that

$$\mathbf{s}_i(t) = \mathbf{B}_i \mathbf{d}_i(t),$$

where $\mathbf{s}_i(t) = \begin{pmatrix} s_{1i}(t) & \dots & s_{Mi}(t) \end{pmatrix}^T$ is the received vector signal caused by the signal transmitted by user i.

In a scenario with multiple users, the vector signal $\boldsymbol{y}(t)$ measured at the antenna can be described as

$$\boldsymbol{y}(t) = \mathbf{s}_0(t) + \sum_{k=1}^{N_{co}} \mathbf{s}_k(t) + \boldsymbol{n}(t) = \boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$= \boldsymbol{B}_0 \boldsymbol{d}_0(t) + \sum_{k=1}^{N_{co}} \boldsymbol{B}_k \boldsymbol{d}_k(t) + \boldsymbol{n}(t) . \tag{2}$$

Above, we are interested in the signal from user 0, whereas the signals from users 1 to N_{co} constitute co-channel interference. Furthermore, the term $\boldsymbol{n}(t)$ constitutes noise, which is assumed to be spatially and temporally white.

In general, the channel matrix for user i has rank $\max(M, nb_i{+}1)$. However, in some cases the channel matrices loose rank. When a channel is represented by a rank $N < \max(M, nb_i + 1)$ channel matrix, we call it a *reduced* or *low* rank channel. We will now outline in what scenarios such channels may occur.

Let us consider a multipath propagation model with K "paths" for one of the channel matrices

$$\boldsymbol{B} = \sum_{k=1}^{K} \boldsymbol{a}(\theta_k)\boldsymbol{p}^T(\tau_k)\ . \tag{3}$$

Here $\boldsymbol{a}(\theta_k)$ is the array response for the desired signal traveling along path k and arriving from direction θ_k.The vector $\boldsymbol{p}(\tau_k)$ contains a sampled version of the transmit pulse shaping function $p(t)$ with the sampling offset determined by the relative path delay τ_k

$$\boldsymbol{p}(\tau_k) \stackrel{\triangle}{=} \begin{bmatrix} p(\tau_k) & p(\tau_k - T) & \dots & p(\tau_k - nb_0 T) \end{bmatrix}^T \tag{4}$$

If there is no delay spread in the channel, i.e. all $\tau_k = \tau\ \forall k$, then all $\boldsymbol{p}(\tau_k)$ will be equal, say $\boldsymbol{p}(\tau_k) = \boldsymbol{p}_0,\ \forall k$, and the channel matrix can be written as

$$\boldsymbol{B} = \left(\sum_{k=1}^{K} \boldsymbol{a}(\theta_k)\right)\boldsymbol{p}_0^T = \boldsymbol{a}\boldsymbol{p}_0^T \tag{5}$$

where

$$\boldsymbol{a} = \sum_{k=1}^{K} \boldsymbol{a}(\theta_k)\ . \tag{6}$$

The channel matrix thus has rank one. If there is some delay spread but it is small, so that

$$\boldsymbol{p}(\tau_k) \approx \boldsymbol{p}_0,\ \forall k$$

then the channel matrix will be approximately rank one. To get a channel matrix with an approximative rank larger than one we thus need a "significant" delay spread.

Let us now assume that the paths can be grouped such that the paths within each group have similar propagation delays. Assuming that the spatial signatures for the paths in different groups are different, the approximate rank of the channel model would then be determined by the number of such groups with significant energy. If the number of such groups is small, the channel matrix can be approximated with a low rank matrix.

In the following two sections we present and compare three methods for exploiting the low rank property of the channel matrices in the channel estimation.

III. MAXIMUM-LIKELIHOOD REDUCED-RANK CHANNEL ESTIMATION

Assume the channel matrix $\boldsymbol{B}$ can be decomposed as

$$\boldsymbol{B} = \boldsymbol{A}\boldsymbol{P} \tag{7}$$

where $\boldsymbol{A}$ is an $M \times r$ matrix and $\boldsymbol{P}$ is an $r \times (nb+1)$ matrix. The channel matrix then has a maximum rank of r.

If the noise vector $\boldsymbol{n}(t)$ is temporally white and Gaussian distributed, then the maximum likelihood rank r estimate of $\boldsymbol{B}$ can be found by generalizing the result in [1] to complex valued signals, as

$$\hat{\boldsymbol{B}}_{ML} = \hat{\boldsymbol{R}}_{\boldsymbol{yd}}\hat{\boldsymbol{R}}_{\boldsymbol{dd}}^{-1/2}\hat{\boldsymbol{S}}\hat{\boldsymbol{S}}^H\hat{\boldsymbol{R}}_{\boldsymbol{dd}}^{-1/2} \tag{8}$$

where

$$\hat{\boldsymbol{R}}_{\boldsymbol{yd}} = \frac{1}{t_{max} - t_{min} + 1}\sum_{t=t_{min}}^{t_{max}} \boldsymbol{y}(t)\boldsymbol{d}^H(t) \tag{9}$$

$$\hat{\boldsymbol{R}}_{\boldsymbol{dd}} = \frac{1}{t_{max} - t_{min} + 1}\sum_{t=t_{min}}^{t_{max}} \boldsymbol{d}(t)\boldsymbol{d}^H(t) \tag{10}$$

and t_{min} and t_{max} are the indices of the first and last sample utilized. Furthermore, the matrix $\hat{\boldsymbol{S}}$ is defined as

$$\hat{\boldsymbol{S}} = \begin{pmatrix} \hat{\boldsymbol{v}_1} & \dots & \hat{\boldsymbol{v}_r} \end{pmatrix} \tag{11}$$

where $\hat{\boldsymbol{v}}_1, \dots, \hat{\boldsymbol{v}}_r$ are the r dominant eigenvectors of the matrix

$$\hat{\boldsymbol{V}} = \hat{\boldsymbol{R}}_{\boldsymbol{dd}}^{-1/2}\hat{\boldsymbol{R}}_{\boldsymbol{yd}}^H\hat{\boldsymbol{R}}_{\boldsymbol{yy}}^{-1}\hat{\boldsymbol{R}}_{\boldsymbol{yd}}\hat{\boldsymbol{R}}_{\boldsymbol{dd}} \tag{12}$$

This estimate will have a rank no larger than r.

IV. SIGNAL SUBSPACE PROJECTION

The maximum likelihood reduced rank channel estimation makes no assumption about the spatial color of the noise $\boldsymbol{n}(t)$ and thus does not use this in the channel estimation. By assuming that the noise $\boldsymbol{n}(t)$ is spatially white it may however be possible to achieve better performance.

We will now use (1) to divide the received signal into a signal part and a noise part. Since the signals $d_i(t)$ are assumed to be uncorrelated with the noise $\boldsymbol{n}(t)$, we can decompose the covariance matrix of the received signal

$$\boldsymbol{R}_{\boldsymbol{yy}} = E[\boldsymbol{y}(t)\boldsymbol{y}^H(t)] \tag{13}$$

as $\boldsymbol{R}_{\boldsymbol{yy}} = \boldsymbol{R}_{\boldsymbol{ss}} + \boldsymbol{R}_{\boldsymbol{nn}}$ with

$$\boldsymbol{R}_{\boldsymbol{ss}} = E\left[\boldsymbol{s}(t)\boldsymbol{s}^H(t)\right] \qquad \boldsymbol{R}_{\boldsymbol{nn}} = E\left[\boldsymbol{n}(t)\boldsymbol{n}^H(t)\right]\ . \tag{14}$$

We now make the critical assumption that the noise vector $\boldsymbol{n}(t)$ consists only of white noise with variance σ_n^2:

$$\boldsymbol{R}_{\boldsymbol{nn}} = \sigma_n^2\boldsymbol{I}\ .$$

The M-dimensional space containing the received signal vectors can now be divided up into two subspaces, the *signal subspace* and the *noise subspace*. The signal subspace is the subspace spanned by the eigenvectors of the signal covariance matrix $\boldsymbol{R}_{\boldsymbol{ss}}$

$$\text{Signal subspace} = \text{span}(\boldsymbol{v}_1^s, \dots, \boldsymbol{v}_r^s) \tag{15}$$

where $\boldsymbol{v}_1^s, \ldots, \boldsymbol{v}_r^s$ are the r largest eigenvectors of the signal covariance matrix. The noise subspace is the orthogonal complement of the signal subspace

$$\text{Noise subspace} = \text{Signal subspace}^{\perp} \tag{16}$$

We note that an eigenvector $\boldsymbol{v}_i$ of the signal covariance matrix $\boldsymbol{R_{ss}}$ with eigenvalue λ_i^s is also an eigenvector to the received signal covariance matrix $\boldsymbol{R_{yy}}$ but with eigenvalue $\lambda_i^s + \sigma_n^2$ since

$$\boldsymbol{R_{yy}} \boldsymbol{v}_i^s = \left(\boldsymbol{R_{ss}} + \sigma_n^2 \boldsymbol{I}\right) \boldsymbol{v}_i^s = \lambda_i^s \boldsymbol{v}_i^s + \sigma_n^2 \boldsymbol{v}_i^s = (\lambda_i^s + \sigma_n^2) \boldsymbol{v}_i^s \ . \tag{17}$$

Furthermore, since the vectors in the noise subspace are orthogonal to the vectors in the signal subspace, they are orthogonal to all columns in the signal covariance matrix. Every vector in the noise subspace is then an eigenvector to $\boldsymbol{R_{yy}}$ with eigenvalue equal to σ_n^2, since for any vector $\boldsymbol{v}^n$ in the noise subspace

$$\boldsymbol{R_{yy}} \boldsymbol{v}^n = (\boldsymbol{R_{ss}} + \sigma_n^2 \boldsymbol{I}) \boldsymbol{v}^n = \sigma_n^2 \boldsymbol{v}^n \ . \tag{18}$$

The noise subspace will thus be spanned by the eigenvectors of $\boldsymbol{R_{yy}}$ with eigenvalues equal to σ_n^2 and the signal subspace will be spanned by the eigenvectors of $\boldsymbol{R_{yy}}$ with eigenvalues strictly greater than σ_n^2. A base of vectors spanning the signal subspace can thus be constructed by selecting the eigenvectors of $\boldsymbol{R_{yy}}$ with eigenvalues above the noise level σ_n^2.

The signal part of the spatial covariance matrix can now be expressed as

$$\boldsymbol{R_{ss}} = \boldsymbol{B}_0 \boldsymbol{B}_0^H + \sum_{i=1}^{N_{co}} \boldsymbol{B}_i \boldsymbol{B}_i^H \ . \tag{19}$$

When any of the channel matrices has full rank, or when many co-channel interferers are present, the rank of $\boldsymbol{R_{ss}}$ will be full. However, when the channel of the desired user has low rank, and when only a few dominant co-channel interferers with low rank channels are present, $\boldsymbol{R_{ss}}$ may loose rank.

A. Channel signal-subspace projection

As noted in [2], the standard least squares channel estimate $\hat{\boldsymbol{B}}_{LS}$ can be improved by exploiting that all columns of the true channel matrix $\boldsymbol{B}$ lie in the signal subspace (15). To utilize this property, we will project the least squares estimate $\hat{\boldsymbol{B}}_{LS}$ onto an estimate of the signal subspace:

$$\hat{\boldsymbol{V}}_s \triangleq \left[\hat{\boldsymbol{v}}_1^s \quad \ldots \quad \hat{\boldsymbol{v}}_r^s\right] \ , \tag{20}$$

where $\hat{\boldsymbol{v}}_i^s, \ i = 1, \ldots, r$ are the r largest eigenvectors to the sample covariance matrix

$$\hat{\boldsymbol{R}}_{\boldsymbol{yy}} \triangleq \frac{1}{t_{max} - t_{min} + 1} \sum_{t=t_{min}}^{t_{max}} \boldsymbol{y}(t) \boldsymbol{y}^H(t) \ . \tag{21}$$

Note that no training sequence is needed to estimate $\boldsymbol{R_{yy}}$.

The resulting estimate of the channel will then be given by

$$\hat{\boldsymbol{B}}_{ss} = \hat{\boldsymbol{V}}_s \hat{\boldsymbol{V}}_s^H \hat{\boldsymbol{B}}_{LS} \tag{22}$$

This projection will remove components outside the (estimated) signal subspace. If the signal subspace is reasonably well estimated this will improve the channel estimate as some noise-induced estimation errors will be removed. This method was suggested in [2] and will be called *channel signal-subspace projection*.

B. Data signal-subspace projection

We can also project the received signal onto the estimated signal subspace (20) *before* any other processing is performed. We thus form the new data vectors

$$\boldsymbol{y}_{ss}(t) = \hat{\boldsymbol{V}}_s^H \boldsymbol{y}(t) \tag{23}$$

and estimate the channel and noise plus interference covariance matrix of this new data set. The channel estimate will be exactly the same as for the channel signal subspace projection method (after re-transformation to the full space) but the quality of the estimate of the noise-plus-interference spatial covariance matrix improves.

Another very important feature of this method is that since the dimension of the projected signal vector $\boldsymbol{y}_{ss}(t)$ is lower than the dimension of the original signal vector $\boldsymbol{y}(t)$, all processing including the equalizer tuning and execution is reduced in complexity.

A requirement for both of these methods is of course that the number of antennas, M, is strictly greater than the rank of the channel. The larger the difference between the number of antennas and the rank of the channel, the better. A larger difference means a larger noise subspace meaning that more components of the estimated channel or received signal only created by noise are removed by the projection.

V. ESTIMATION OF THE SPATIAL NOISE-PLUS-INTERFERENCE COVARIANCE MATRIX

Another important quantity of the channel for space-time equalization is the spatial noise plus interferer covariance matrix

$$\boldsymbol{R}_{n+i} = E[(\sum_{i=1}^{K} \boldsymbol{B}_i \boldsymbol{d}_i(t) + \boldsymbol{n}(t))(\sum_{i=1}^{K} \boldsymbol{B}_i \boldsymbol{d}_i(t) + \boldsymbol{n}(t))^H] \ . \tag{24}$$

In all the three methods described above, this matrix is estimated from the residuals of the channel estimation. Note however, that for the novel data signal-subspace method, the dimension of the covariance matrix is r, whereas it for the other two methods has dimension M.

VI. SIMULATIONS

For the simulations we used a circular array with radius $1.25\,\lambda$ and ten antenna elements. We simulated a scenario where the desired signal arrives along two paths at angles 0 and 45 degrees. The channels (i.e. pulse shaping) in the two paths were $0.44 + q^{-1} + 0.44q^{-2}$ and $0.44q^{-1} + q^{-2} + 0.44q^{-3}$ respectively. Two different co-channel interferers impinged on the array from directions -45 and 90 degrees, each with the channels $0.44 + q^{-1} + 0.44q^{-2}$.

Spatially and temporally white noise was added giving a signal to noise ratio of 3dB. The transmitted signal was BPSK modulated. A multi-channel MLSE [3] was used for the equalization.

We applied the three methods to this scenario. For the maximum likelihood reduced rank estimation, a rank two channel was estimated, whereas for the two projection algorithms, the respective quantities were projected onto a rank four estimate of the signal subspace.[1]

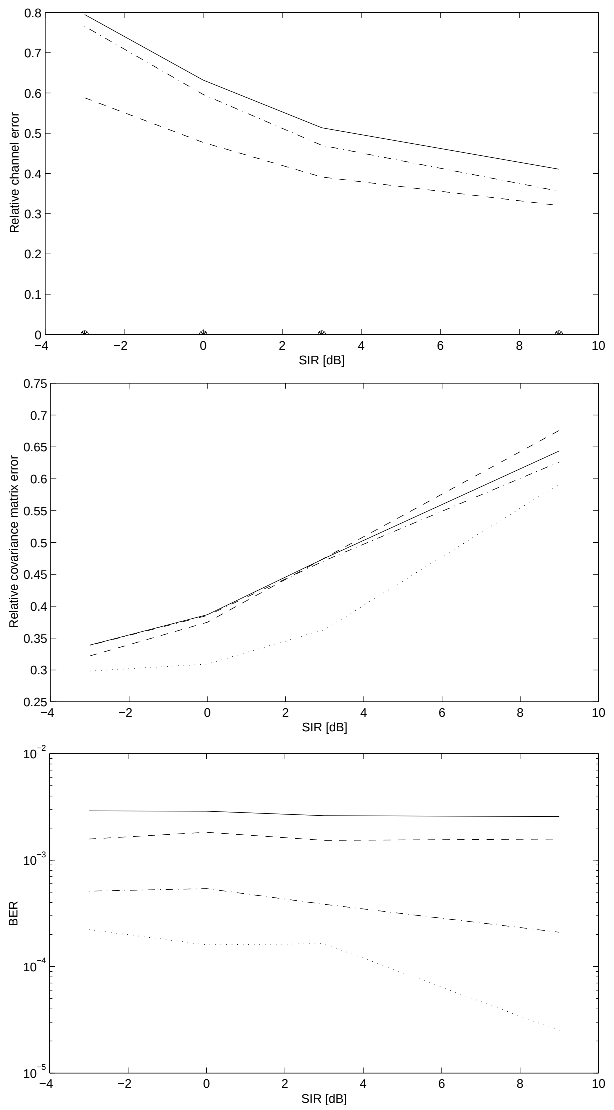

Figure 1: Relative channel error, covariance matrix error and BER as a function of SIR with a SNR of 3dB using the FIR least squares (solid), maximum likelihood reduced rank (dashed), channel signal subspace projection (dash-dotted) and data signal subspace projection (BER and covariance matrix error dotted, channel error dashed-dotted) channel estimation methods.

In Figure 1 the relative channel estimation error and relative covariance matrix errors as well as the resulting BER for the equalizer can be seen. Note that the maximum likelihood reduced rank estimation method (MLRR) has the smallest relative channel error. However, the BER of the detector based on the model estimated with MLRR is not the lowest. On the other hand, both the channel signal subspace projection method (CSSP) and the data signal subspace projection method (DSSP) have significant improvements in terms of equalizer BER even though their relative channel estimation errors are larger than for the MLRR method. Note that the DSSP method has the best noise plus interferer spatial covariance matrix estimate and the best BER performance.

[1] We only used the data received during the training sequence to estimate the signal subspace to get a fair comparison with the maximum likelihood reduced rank channel estimation method.

VII. SUMMARY

We have here considered different methods of exploiting the reduced rank property of a space-time channel in wireless communications. Three methods have been studied, a maximum likelihood reduced rank channel estimation method which finds the channel of a given rank in a maximum likelihood sense [1], and two signal subspace projection methods which projects either the channel estimate [2] or, as for the here proposed method, the received signal samples onto an estimate of the spatial signal subspace.

Even though the model estimated using the maximum likelihood reduced rank method provides the best accuracy, a detector based on this model does not achieve the lowest BER. The BER from detectors based on the signal subspace projection methods are lower, despite the fact that the accuracy of the estimated channel models is worse. Apparently, the signal subspace projection methods find channel estimates that are better suited for the purpose of space-time equalization.

The here proposed method of projecting the received signal samples directly on the signal subspace gives the best equalizer performance. This improvement is contributed to a better estimated spatial noise plus interference covariance matrix. Since this method also has lower complexity than the channel signal subspace projection method, it is clearly preferable.

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