Systematic Anti-Windup Compensator Design for Multivariable Systems*

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Abstract

The aim of anti-windup compensation is to modify the dynamics of a control loop when control signals saturate, so that a good transient behaviour is attained after desaturation, while avoiding nonlinear oscillations and repeated saturations.

Model-based anti-windup compensation is here considered for multiple-input multiple-output (MIMO) systems. A modified controller structure is proposed, which leaves the nominal closed-loop dynamics unchanged, as long as none of the control signals saturate. The proposed approach is applicable to continuous-time as well as discrete-time systems. Although it is developed for systems in input-output form, it can be used for systems in state-space form as well.

1 Introduction

The problem of finding controllers which have desired properties during or after saturation events has, over the years, resulted in a number of different anti-windup strategies. Many of the proposed methods focus on adjusting the states of the controller during a saturation event. For reasons explained in [6], and more recently also in [3, 4], there is, however, no guarantee that the whole system behaves acceptably during or after a saturation event, when only controller-state windup is prevented. Repeated saturations and even limit cycles might occur. To avoid such effects, the whole linear dynamics around the saturating elements, consisting of nominal controller elements, anti-windup filters and the open-loop plant, has to be taken into account. In the scalar case this can be accomplished in a Nyquist diagram: the loop gain around the saturating element is adjusted so that it stays well away from the function $-1/Y(C)$, where $Y(C)$ is the describing function of the saturation nonlinearity. The aim of the present paper is to introduce a controller structure which makes it possible to generalize this technique for analysis and design to feedback systems with multiple control signals. A key simplification is that the loop gain

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relevant for the saturation behaviour is made diagonal. The diagonal elements can then be adjusted in the same way as for a scalar system.

2 Problem formulation

The aim of anti-windup design, as it will be presented here, is to obtain a good transient behaviour after desaturation. With a good transient behaviour we mean that

1. desaturation transients should have a fast decay;
2. limit cycles and repeated saturations should not occur.

To achieve these aims, a method developed in [8] will be utilized.

Consider a discrete-time\(^1\) multivariable, stable or marginally stable system with \(m\) inputs and \(p\) outputs, parameterized in rational fractional form as

\[
y(k) = \mathbf{B}(q)\mathbf{A}^{-1}(q)v(k) \\
v(k) = \text{sat}[u_W(k)]
\]

Above, \(\mathbf{A}(q)\) is assumed to be a diagonal stable rational matrix, with diagonal elements \(\mathbf{A}_j\). Introduce the controller

\[
u_W(k) = (\mathbf{I} - \mathbf{W}(q)\mathbf{R}(q))v(k) - \mathbf{W}(q)\mathbf{S}(q)y(k) + \mathbf{W}(q)\mathbf{T}(q)r(k)
\]

where \(\mathbf{W}, \mathbf{R}, \mathbf{S}, \mathbf{T}\) are stable rational matrices in \(q\), of appropriate dimension. Here, \(\mathbf{W}\) is the anti-windup filter. The controller structure proposed in (2) is inspired by a similar structure suggested in [3] for scalar systems. It is depicted in Figure 1.

\[\text{Figure 1: A discrete-time MIMO process } y(k) = \mathbf{B}(q)\mathbf{A}^{-1}(q)\text{sat}[u(k)] \text{ with a two degree of freedom controller structure } \{\mathbf{R}(q) \mathbf{S}(q) \mathbf{T}(q)\} \text{ appended with a stable and proper anti-windup transfer-operator matrix } \mathbf{W}(q). \text{ The rational matrix } \mathbf{W}(q) \text{ is to be selected such that the loop gain around the saturations becomes diagonal.}\]

Remark: In the scalar case, the controller structure (2) includes a number of well know anti-windup schemes, of which the following are worth mentioning. (The matrices \(\mathbf{R}, \mathbf{S}\) and \(\mathbf{T}\) are here assumed to be scalar polynomials \(R, S\) and \(T\).)

\(^1\)The anti-windup concept presented here is applicable to both discrete time and continuous time systems. Here we shall however use a discrete time framework, based on the forward shift operator \(q\), \((qy(k) = y(k + 1))\).
1. The observer-based method of Åström and Wittenmark [7] is obtained if $\mathbf{W} = F^{-1}$, where $F$ is the characteristic polynomial of the anti-windup observer.

2. The conditioning technique of Hanus [2] is obtained if $\mathbf{W} = t_0 T^{-1}$, where $t_0$ is the leading element of $T$.

For more details see [4, 5].

Following [4], let us regard the difference between the actual and the saturated control signal as an exogenous disturbance

$$\delta(k) \triangleq v(k) - u_W(k) .$$

By omitting the argument $q$, and combining (1)-(3), the closed-loop system is then obtained as

$$y(k) = y_{\text{nom}}(k) + y_0(k) = \mathcal{H}_{\text{nom}} r(k) + \mathcal{H}_\delta \delta(k)$$

where

$$\mathcal{H}_{\text{nom}} = \mathcal{B} (\mathcal{R} \mathcal{A} + \mathcal{S} \mathcal{B})^{-1} \mathcal{T} ; \quad \mathcal{H}_\delta = \mathcal{B} (\mathcal{R} \mathcal{A} + \mathcal{S} \mathcal{B})^{-1} \mathcal{W}^{-1} .$$

Above $\mathcal{H}_{\text{nom}}$ constitutes the nominal closed-loop system which is obtained in (4) when $\delta = 0$, i.e. when the control signals do not saturate. When a control signal exits saturation, $\mathcal{H}_\delta$ will determine the resulting transient behaviour. According to the specified requirements above, the dynamics of $\mathcal{H}_\delta$ should be fast. This can be achieved by appropriate choices of $\mathcal{W}$. However, if the dynamics of $\mathcal{H}_\delta$ is made too fast, then repeated saturations and limit cycles may occur. Thus, the requirements 1. and 2. above are often contradictory. It is therefore essential that an anti-windup design includes a trade-off between a fast transient and a small influence of nonlinear effects. A design method is presented next, which utilizes simple scalar tools for attaining such a trade-off.

### 3 Systematic anti-windup design

For the design of the anti-windup filter $\mathcal{W}$ in (2), the criterion

$$J = \| \mathcal{H}_\delta \|_2^2 + \| \mathcal{Q} ((\mathcal{L}_v + \mathcal{I})^{-1} - \mathcal{I}) \|_2^2$$

is introduced. This criterion is a generalization of a criterion suggested for the scalar case by Stenstad and Rönnebäck in [5]. In (6), $\mathcal{Q}$ is a diagonal penalty matrix whereas $\mathcal{L}_v$ represents the loop gain around the saturation nonlinearity. Now, by choosing $\mathcal{W}$ as

$$\mathcal{W} = \mathcal{P} (\mathcal{R} \mathcal{A} + \mathcal{S} \mathcal{B})^{-1} ,$$

where $\mathcal{P}$ is a stable and rational matrix to be determined, the rational matrices $\mathcal{H}_\delta$ and $\mathcal{L}_v$ are given by

$$\mathcal{H}_\delta = \mathcal{B} \mathcal{P}^{-1} ; \quad \mathcal{L}_v = \mathcal{P} \mathcal{A}^{-1} - \mathcal{I}$$
respectively. If \( \mathcal{P} \) is chosen diagonal, then \( \mathcal{L}_v \) will be diagonal. By insertion of (8) into (6), the criterion can be rewritten as

\[
J = \| B \mathcal{P}^{-1} \|^2_2 + \| Q (A \mathcal{P}^{-1} - I) \|^2_2 .
\]  

Minimizing (9), with respect to \( \mathcal{P} \), for a given penalty matrix \( Q \), is shown in [8] to be equivalent to the solution of \( m \) separate scalar spectral factorization equations

\[
r_j \mathcal{P}_j \mathcal{P}_j^* = \sum_{i=1}^p B_{ij} B_{ij}^* + \rho_j A_j A_j^* ; \quad \mathcal{P} = \text{diag}(\mathcal{P}_j) .
\]

Here, (10) has to be solved for \( j = 1, 2, \ldots, m \), where \( m \) is the number of process inputs, \( p \) is the number of process outputs, \( r_j \) is a scale factor and \( \rho_j \) is the \( j \)th diagonal element of \( Q \). The design of a multivariable anti-windup compensator is thus reduced to \( m \) scalar designs, in which the \( m \) elements of the diagonal loop gain matrix \( \mathcal{L}_v \) are systematically adjusted. Note that if \( \rho_j \to \infty \), then \( \mathcal{P}_j \to A_j \). The \( j \)th loop gain \( \mathcal{L}_j \) will then contract and stay well away from the negative real axis. As a result, repeated saturations will not occur in that loop. However, the desaturation transients may then show an unsatisfactory behaviour, since the common denominator of the \( j \)th column of \( \mathcal{H}_j \) goes towards the plant dynamics \( A_j \). On the other hand if \( \rho_j \) is selected small, the dynamics of the \( j \)th column of \( \mathcal{H}_j \) will become fast, while the \( j \)th loop gain may become large. This, in turn, may generate repeated saturations and limit cycles. The user must therefore select the values of \( \rho_j \) properly to obtain an appropriate trade-off.
4 Example

The controller in (2) is used for two different choices of $\mathbf{W}$. In both the cases, the nominal controller, $\mathbf{R}, \mathbf{S}, \mathbf{T}$, originates from an observer-based state-feedback LQ-control law, expressed in input-output form. The input penalty is 0.01 for both inputs. The model used for simulation describes a Heavy Oil Fractionator [1], with two inputs and two outputs. The model used for controller- and anti-windup filter design, is obtained by subspace-identification. In the first example, we select $\mathbf{W} = \mathbf{I}$, which simply means that the observer is fed with saturated control signals. The result of the simulation is shown in the four upper diagrams, ($D 1.1 - D 1.4$). In the other example, the method proposed in this paper was used for the design of $\mathbf{W}$. The result, after adjustment of the penalties $\rho_1, \rho_2$, is shown in the six lower diagrams, ($D 2.1 - D 2.6$). The two bottom diagrams show the diagonal elements of the loop gain $\mathcal{L}$, and the functions $-1/Y(C)$.

References


