INDIRECT SPATIO-TEMPORAL EQUALIZATION AND
ADAPTIVE INTERFERENCE CANCELLATION FOR
MULTIPATH ENVIRONMENTS IN MOBILE RADIO
APPLICATIONS\(^1\)

Erik Lindskog

Systems and Control Group, Uppsala University,
P.O. Box 27, S-751 03 Uppsala, Sweden
tel: +46-18-183071 fax: +46-18-503611 e-mail: el@syscon.uu.se

ABSTRACT

Combined spatial and temporal equalization with a multi-antenna decision feedback equalizer is
considered for a TDMA mobile radio system. Unless the time frame allocation is synchronized for
all surrounding base stations, the co-channel interferers may start to interfere at arbitrary time
instances during a base stations reception. A co-channel interferer may thus be present during
the data sequence of the time frame while it is not present during the training sequence. In order
to handle this situation, adaptation to changing interference environment is introduced. Two
versions of the equalizer are considered. The covariance matrix estimates needed for tuning the
equalizer parameters are partitioned into a signal part and a noise plus interference part. The
signal part is derived from identified signal channels, while the noise plus interference part is
constructed by only considering the spatial correlations of the residuals from the identification
procedure. The first algorithm is adapted by updating the noise plus interference part of the
covariance matrix estimate, combined with a retuning of the equalizer at appropriate intervals.
The second algorithm performs the adaptation by tuning a number of separate beamformers.
This adaptation requires considerably less computations, at the price of a potential performance
degradation. For the investigated scenario, the performance degradation is however minor.

1. INTRODUCTION

In digital mobile radio communications the signaling suffers from intersymbol interference due to
multipath propagation. The mobiles also disturb each other as co-channel interferers. With the
use of antenna arrays one can both improve the equalization of the intersymbol interference and
the cancellation of co-channel interferers.

In this paper, a time division multiple access system (TDMA) is considered. It is assumed that
data is transmitted in bursts, in which the first \( N_{\text{seq}} \) symbols constitute a known training sequence,

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$d(t) = \pm 1, \ t = 1, 2, \ldots, N_{\text{seq}}$, and the $N_{\text{data}}$ following symbols are unknown data. The channel from the transmitter to the receiver is assumed to be approximately constant over the duration of a burst, while co-channel interferers may commence interfering at any time during the burst. This is for instance the case in the GSM systems operating at present. As a result, although we tune an adaptive antenna to suppress the co-channel interferers present during the training sequence, a new co-channel interferer may appear during the data sequence from a direction that is not nulled out by the adaptive antenna. This can have a considerable effect on the performance of the receiver.

2. ALGORITHMS

We consider a situation with multiple antennas at the receiver. These antennas are connected to a multi-antenna decision feedback equalizer (DFE) with FIR filters. The structure of this equalizer is shown in Figure 1. The $y_i(t)$:s are the complex baseband signals from the respective antenna elements. The FIR filters are represented as complex polynomials, $S_i(q^{-1}) = s_{i0} + s_{i1}q^{-1} + \ldots + s_{i,nq}q^{-nq}$ and $Q(q^{-1}) = Q_0 + Q_1q^{-1} + \ldots + Q_{nq}q^{-nq}$, in the delay operator $q^{-1}$ ($q^{-1}x(t) = x(t-1)$).

In the filtering there is a smoothing lag $m$. A binary modulation is considered. The output of the decision device, $\hat{d}(t - m | t)$, is either 1 or -1 depending on the sign of the real part of $d(t - m | t)$.

![Figure 1: Multi-antenna FIR decision feedback equalizer](image)

For a predetermined order of the filters we wish to minimize the expectation of the squared error between the output of the equalizer, before the decision device, $\hat{d}(t - m | t)$, and the transmitted symbols, $d(t - m)$. In other words we want to minimize

$$E[|\hat{d}(t - m | t) - d(t - m)|^2] \tag{1}$$

Let us define the parameter vector, $\theta$, and the corresponding data vector, $Y(t)$,

$$\theta = [s_{10} \ s_{11} \ldots \ s_{1,nq} \ s_{20} \ldots \ s_{M,nq} \ Q_0 \ldots \ Q_{nq}]^T. \tag{2}$$

$$Y(t) = [y_1(t) \ y_1(t - ns) \ y_2(t) \ldots \ y_M(t - ns) \ d(t - m - 1) \ldots \ d(t - m - 1 - nq)]^T. \tag{3}$$

The minimum of (1) is attained by the parameter vector (assuming correct past decisions)

$$\theta = (R_{YY}^\dagger \ R_{Yd})^* \tag{4}$$

where $(\cdot)^*$ represents elementwise complex conjugation. The matrix $R_{YY}$ is the covariance matrix, defined as

$$R_{YY} = E[Y(t)Y^H(t)] \tag{5}$$

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and $R_{Y_d}$ is the cross covariance matrix defined by

$$R_{Y_d} = E[Y(t)d^H(t)].$$

Different strategies can be considered for forming estimates of $R_{YY}$ and $R_{Y_d}$. In [1], however, an indirect scheme considering only the spatial correlations of the interference plus noise showed indications of being a good tuning scheme. This algorithm will therefore be considered here.

![Diagram](image)

**Figure 2: Signal channels.**

Since the training sequence is known, we can use this information for identifying the channels between the transmitter and the respective antenna elements, see Figure 2. These channels are modeled as FIR channels, $B_i(q^{-1}) = b_{i0} + b_{i1}q^{-1} + ... + b_{im}q^{-m}$. Let us define the desired signal to be the part of the signal at the receiver originating from $d(t)$. The channel models can be used for constructing estimates of the parts of the covariance matrices $R_{YY}$ and $R_{Y_d}$ with their origins in the desired signal. By subtracting the desired signal from the received signal, we obtain a residue signal which can be used to construct an estimate of the part of the covariance matrix $R_{YY}$, belonging to the noise and the co-channel interferers.

Consider the use of the residual signal to construct a sample covariance matrix estimate of the noise plus interference part of $R_{YY}$, describing only the spatial correlations of the noise plus interferers.

By constructing the estimates of the covariance matrices $R_{YY}$ and $R_{Y_d}$ with the identified channel models and the spatial correlation of the noise plus interferers we achieve an indirectly tuned DFE, with spatial interference cancellation, denoted IS-DFE in [1].

Because the interferers can begin and end their transmission asynchronously, we wish to achieve adaptive interference cancellation during the data sequence. In this article we will study two adaptive algorithms with the IS-DFE tuning scheme as a base.

### 2.1. Indirect FIR Decision Feedback Equalizer with Adaptive Spatial Interference Cancellation (ISA-DFE)

In order to estimate the covariance matrices $R_{YY}$ and $R_{Y_d}$, we first identify the channels, $B_i(q^{-1}) = b_{i0} + b_{i1}q^{-1} + ... + b_{im}q^{-m}$, from the transmitter to each receiver element $i$, $i = 1, 2, ..., M$, see
Figure 2. The estimates are based on the training data. Based on the estimates of these channels, \( \hat{B}(q^{-1}) \), the received desired signal

\[
s(t) = [s_1(t) \ s_2(t) \ ... \ s_M(t)]^T
\]

is estimated as

\[
\hat{s}(t) = \hat{B}(q^{-1})d(t) \quad t = 1 + nb, ..., N_{\text{seq}}
\]

where

\[
\hat{B}(q^{-1}) = [\hat{B}_1(q^{-1}) \ \hat{B}_2(q^{-1}) \ ... \ \hat{B}_M(q^{-1})]^T.
\]

The undesired signal, \( n(t) = [n_1(t) \ n_2(t) \ ... \ n_M(t)]^T \), consisting of noise and interference, is then estimated from the residuals

\[
\hat{n}(t) = y(t) - \hat{s}(t) \quad t = 1 + nb, ..., N_{\text{seq}}
\]

where

\[
y(t) = [y_1(t) \ y_2(t) \ ... \ y_M(t)]^T.
\]

We assume that the desired signal, \( s(t) \), and the undesired signal, \( n(t) \), are mutually uncorrelated. Then, \( R_{YY} \) can be divided into two terms

\[
R_{YY} = R_{YsYs} + R_{YnYn}
\]

where \( R_{YsYs} \) is the contribution to \( R_{YY} \) due to the desired signal and, \( R_{YnYn} \) is the contribution due to the noise and interferers. By using the identified channels \( \hat{B}(q^{-1}) \), an estimate of \( R_{YsYs} \) can be obtained as

\[
\hat{R}_{YsYs} = E[\hat{Y}_s(t)\hat{Y}_s^H(t)]
\]

where

\[
\hat{Y}(t) = [\hat{s}_1(t) \ ... \ \hat{s}_1(t - ns) \ \hat{s}_2(t) \ ... \ \hat{s}_M(t - ns) \ d(t - m - 1) \ ... \ d(t - m - 1 - nq)]^T.
\]

Note that the matrix \( \hat{R}_{YsYs} \) can be expressed directly in the coefficients of \( \hat{B}(q^{-1}) \) if (8) is inserted into (14).

By taking only the spatial color of the residuals into consideration an estimate of \( R_{YnYn} \) can be obtained as

\[
\hat{R}_{YnYn} = \frac{1}{N_{\text{seq}} - nb} \sum_{t=1+nb}^{N_{\text{seq}}} N(t)N^H(t)
\]

where (for the case \( ns \geq nq \))

\[
N(t) = [N_1(t) \ ... \ N_M(t) \ Z]^T.
\]

The block \( N_i(t) \) is an \((ns + 1) \times (ns + 1)\) diagonal matrix with \( \hat{n}_i(t) \) in all diagonal elements, and \( Z \) is a zero matrix of dimension \((ns + 1) \times (ns + 1)\).

The cross covariance matrix, \( R_{Yd} \), is estimated by

\[
\hat{R}_{Yd} = E[\hat{Y}_s(t)d^H(t - m)]
\]

This cross covariance estimate can also be expressed directly in the coefficients of \( \hat{B}(q^{-1}) \). An estimate of \( R_{YY} \) can now be formed as

\[
\hat{R}_{YY} = \hat{R}_{YsYs} + \hat{R}_{YnYn}
\]
and the estimated optimal parameter vector is thus

\[ \hat{\theta} = (\hat{R}_{yy}^{-1} \hat{R}_{yd})^*. \]  

(19)

A simple way of introducing adaptation, is to first tune the equalizer parameters during the training sequence with the above scheme. During the data sequence, as decisions are made, these can be used together with the identified channels and the received samples, to compute the residuals at the antenna elements. With these residuals, the matrix \( \hat{R}_{yy} \) can be updated as

\[ \hat{R}_{yy}(t) = \lambda \hat{R}_{yy}(t - 1) + (1 - \lambda)N(t)N^H(t) \]  

(20)

where \( \lambda \) is a forgetting factor close to 1.

At appropriate intervals, the equalizer parameters can be retuned by using equations (18) and (19).

![Diagram of ISAS-DFE](image)

**Figure 3: Structure of the ISAS-DFE**

2.2. Indirect FIR DFE with Separate Adaptive Interference Cancellation for each Beamformer (ISAS-DFE)

Every retuning of the equalizer parameters in the ISA-DFE requires the solution of a system of equations, with \( M(ns + 1) + nq + 1 \) unknowns. Here an algorithm is presented which requires a lower amount of computations for the equalizer retuning. The price to be paid for the reduction in complexity is, however, a potentially degraded performance.

First, tune the equalizer parameters during the training sequence with the algorithm described in Section 2.1. Then, reorganize the equalizer to the structure shown in Figure 3. Compute the channels from the transmitter to the signals, \( z_i, i=1,...,M \), located after the beamformers. We call these channels the “resulting channels” and denote them, \( C_i(q^{-1}), i=0,1,...,ns \). The resulting channels are obtained by using the identified channels \( \hat{B}(q^{-1}) \) and the beamformer coefficients,
\[ w_{ij} \]

\[ C_i(q^{-1}) = \sum_{j=1}^{M} \hat{B}_j(q^{-1})w_{ij}, \quad i = 0, 1, ..., n_s. \quad (21) \]

We define a computed desired signal, \( \hat{z}_i(t) \), after each beamformer, to be the decided symbols filtered through the respective resulting channel, \( C_i(q^{-1}) \)

\[ \hat{z}_i(t) = C_i(q^{-1})\hat{d}(t), \quad i = 0, 1, ..., M \quad (22) \]

The idea is now to adapt each of the beamformers, \( i=0,1,...,n_s \), during the data sequence in such a way that the desired signal is filtered in a similar way as during the training sequence, while possibly new interferers are adaptively suppressed. Each beamformer can be adapted independently with the recursive least squares algorithm in order to minimize the MSE between each beamformer output, \( z_i(t) \), and its computed desired signal, \( \hat{z}_i(t) \).

In order to adapt the beamformer coefficients we need an initial estimate of the inverse of the covariance matrix for the received signals, \( R_{yy}^{-1} = \left( E[y(t)y^H(t)] \right)^{-1} \). For this purpose we can use the inverse of the sample covariance matrix estimate based on the training sequence

\[ \hat{R}_{yy}^{-1} = \left( \frac{1}{N_{\text{seq}}} \sum_{t=1}^{N_{\text{seq}}} y(t)y^H(t) \right)^{-1}. \quad (23) \]

Let us define the \( i \)th beamformer coefficient vector as

\[ w_i = [w_{i1} \; w_{i2} \; ... \; w_{iM}]^T, \quad i = 1, 2, ..., M \quad (24) \]

During the data sequence the individual beamformers can now be adapted as

\[ \hat{d}(t-m|t) = \text{sign}(\text{real}(\sum_{i=0}^{n_s} w_i(t-1)^Tw(t-i) - Q(q^{-1})\hat{d}(t-m-1|t-1))) \]

\[ \hat{z}_i(t-m) = C_i(q^{-1})\hat{d}(t-m|t) \]

\[ \Phi(t) = \hat{R}_{yy}^{-1}(t-1)y(t-m) \]

\[ K(t) = \Phi(t)/[\lambda + y^H(t-m)\Phi(t)] \]

\[ \hat{R}_{yy}^{-1}(t) = [\hat{R}_{yy}^{-1}(t-1) - K(t)\Phi^H(t)]/\lambda \]

\[ \epsilon_i(t) = \hat{z}_i(t-m) - y^H(t-m)w_i(t-1), \quad i = 0, 1, ..., n_s \]

\[ w_i(t) = w_i(t-1) + K(t)\epsilon_i(t), \quad i = 0, 1, ..., n_s \]

where \( \lambda \leq 1 \) is a forgetting factor.

3. SIMULATIONS

In the simulations, a circular array consisting of eight antennas, as shown in Figure 4, was used. Filter orders of \( n_s=3 \) and \( nq=2 \), and a smoothing lag of \( m=3 \), were used in all algorithms. To simplify the simulations, the decided data used in the feedback filter were set equal to the true transmitted symbols. In the simulations, the BER as a function of the training sequence length was investigated.
3.1. Scenario 1:

The desired signal is impinging on the array from the directions $\alpha = 0$, 30, -60 and 180 degrees, through the channels $B(q^{-1}) = 1 + 0.5q^{-1}$, $0.5q^{-1} + 0.8q^{-2}$, $0.5q^{-2} + 0.2q^{-3}$ and $0.2q^{-3} + 0.3q^{-4}$ respectively. A co-channel interferer is impinging on the array from the direction $\alpha_{\text{co}} = 135$ degrees, having a constant channel $B_{\text{co}}(q^{-1}) = b_{\text{co}}$. The constant $b_{\text{co}}$ was selected such that the SIR, averaged over all the antenna elements, became 3 dB. Independent white noise giving a SNR of 4dB, averaged over the antenna elements, was also added.

3.2. Scenario 2:

Exactly the same as Scenario 1, except that a second co-channel interferer has been added impinging from -30 degrees with the same power as the first co-channel interferer.

3.3. Simulations

In the simulations we assume the following: during the training sequence only the co-channel interferer of Scenario 1 is present, whereas during the data sequence the second co-channel interferer of Scenario 2 is also present. Two versions of the ISA-DFE was used. The first one was tuned for Scenario 1 and then evaluated for Scenario 2. This illustrates the performance when the ISA-DFE is working without adaptation, for the setup considered. The second version of the ISA-DFE
was tuned for Scenario 2 and evaluated for Scenario 2. The improvement in performance of this equalizer as compared to the former one, shows how much can be gained by adapting to the new scenario. The BER as a function of the training sequence length can give us a feeling for how fast the algorithm adapts. How fast a real implementation of the ISA-DFE adapts will naturally also depend on the selected forgetting factor.

![Figure 6: BER for the DFE:s. The ISA-DFE adapted to interference Scenario 2 (solid), ISAS-DFE with separate beamformer adaptation (dashed), ISA-DFE not adapted to interference Scenario 2 (dashed-dotted). Correct past decisions were assumed in the simulations.](image)

The ISAS-DFE was initially tuned by using the training sequence data from Scenario 1. It was then recomputed with the batch least squares algorithm corresponding to equations (25) – (31), with the exception that correct past decisions were assumed. The number of used data was equal to the number of training symbols.

As can be seen in Figure 6, the attainable improvement for the ISA-DFE after adaptation is considerable. The actual reduction in BER due to the adaptation will, of course, differ from scenario to scenario. It should however be obvious that adapting to changes in interfering environment is advantageous. It can also be noted that the performance degradation of ISAS-DFE as compared to the ISA-DFE is minor in the scenario considered.

Other algorithms for beamforming and equalization have also been investigated, see for example [2], [3] and [4], but they tend to either have difficulties working with short training sequences or have worse performance for the scenarios considered. See [5].

### 4. COMPUTATIONAL COMPLEXITY

For relative comparisons, we here define a complexity unit, \( cu \). One \( cu \) is defined to be the amount of computations required for one complex multiplication. We assume that a real division requires 18 time as long time as a real multiplication\(^1\). A complex division will then correspond to 6.5 complex multiplications, and is counted as 6.5 \( cu \). Additions are neglected in the complexity estimations, but their number is roughly be proportional to the number of multiplications (which dominate). All the complexity computations are only approximate estimates.

When computing the complexity for solving systems of linear equations, use has been made of the fact that the matrices involved are positive definite. The fact that many of the involved matrices

\(^1\)This is the case for the signal processor TMS320C50
are complex conjugate symmetric has also been used. For instance the computation of

\[ \hat{R}_{yy} N_{\text{seq}} = \sum_{l=1}^{n_{\text{seq}}} y(l)y^H(l) \]

only requires \( \frac{N_{\text{seq}}}{2}(M^2 + M) \) complex multiplications, where \( M \) is the number of elements in \( y(l) \).

### 4.1. Complexity of the ISA-DFE

If the identification of the channels, \( B_i(q^{-1}) \), is performed with the least squares algorithm this has a computational complexity

\[ C_{\text{ISA, id}} \approx (N_{\text{seq}} - nb)(nb + 1)M \text{ cu.} \]  

(32)

Computing the residuals amounts to

\[ C_{\text{ISA, res}} \approx (nb + 1)N_{\text{seq}}M \text{ cu.} \]  

(33)

The initial tuning of the equalizer parameters requires

\[ C_{\text{ISA, Tune}} \approx \frac{1}{6}(M(ns + 1) + nq + 1)^3 + \frac{19}{4}(M(ns + 1) + nq + 1)^2 + \frac{1}{2}N(M^2 + M) + \frac{1}{2}(nb + 1 - \frac{n}{2})^2(ns + 1)^2M^2 \text{ cu.} \]  

(34)

After the initial tuning a retuning will amount to

\[ C_{\text{ISA, Retune}} \approx \frac{1}{6}(M(ns + 1) + nq + 1)^3 + \frac{19}{4}(M(ns + 1) + nq + 1)^2 \text{ cu} \]  

(35)

assuming that \( \hat{R}_{Y_nY_n} \) has been updated, generating the computational complexity of

\[ C_{\hat{R}_{Y_nY_n, \text{ update}}} \approx M^2 + (nb + \frac{5}{2})M + M(ns + 1) + nq + 1 \text{ cu} \]  

(36)

per time step (including the computations required for estimating of \( \hat{d}(t - m|t) \)).

### 4.2. Complexity of the ISAS-DFE

The initial tuning requires the same amount of computations as for the ISA-DFE except for an additional

\[ C_{\text{ISAS, Initial extra}} \approx \frac{1}{2}(M^2 + M)N_{\text{seq}} + \frac{11}{12}M^3 + \frac{37}{4}M^2 + (ns + 1)(nb + 1)M \text{ cu} \]  

(37)

for the computation of \( \hat{R}_{YY}^{-1} \) and the channels \( C_i(q^{-1}) \). After the initial tuning the complexity of the adaptive retuning of the beamformer weights at each time step is

\[ C_{\text{ISAS, Adaptive step}} \approx 2M^2 + M(3ns + nq + 5) + (nb + 1)(ns + 1) \text{ cu.} \]  

(38)
4.3. Comparison of the ISA- and the ISAS-DFE

The ISA-DFE requires the solution of a large system of linear equations for each retuning. In contrast, the ISAS-DFE divides the retuning problem into \( n_s + 1 \) tunings of individual beamformers. This results in a much lower complexity.

Let us illustrate the difference in complexity by computing the number of time steps the ISAS-DFE can adapt at the same expense as required for the ISA-DFE to perform interference updating at each time step and one retuning. Consider the case corresponding to the simulations of Section 3, with \( M = 8, n_s = 3, n_q = 2, n_b = 4 \).

According the expression (35), one retuning of the ISA-DFE then requires

\[
C_{ISA, \text{ Retune}} \approx 12964 \text{ cu.}
\]  

(39)

The updating of \( \hat{R}_{Y_nY_n} \) requires, according to (36),

\[
C_{\hat{R}_{Y_nY_n}, \text{ update}} \approx 151 \text{ cu}
\]  

(40)

at each time step. Adaptation of one step with the ISAS-DFE requires, according to (38)

\[
C_{ISAS, \text{ Adaptive step}} \approx 276 \text{ cu.}
\]  

(41)

The result is that the ISAS-DFE can adapt continuously for approximately 104 time steps at the same expense as the ISA-DFE can do \( \hat{R}_{Y_nY_n} \)-updating and one retuning.

It can thus be concluded that the ISAS-DFE requires a much lower amount of computations for the adaptation.

5. CONCLUSIONS AND DISCUSSION

Two proposed algorithms, with indirect spatio-temporal channel equalization and adaptive spatial interference cancellation, have been presented.

The simulations show that the utilization of these algorithms can provide considerable improvement of the BER when the interference environment changes.

The ISAS-DFE algorithm requires much less computations as compared to the ISA-DFE at the expense of potentially degraded performance. In the simulations performed, this degradation turned out to be minor. The ISAS-DFE is therefore an interesting alternative to the ISA-DFE when combating variations in the co-channel interference environment.

In the simulations, decision errors where not considered. Further work should be performed in order to investigate their impact on the BER and the adaptation.

6. REFERENCES


