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DISTURBANCE DECOUPLING ADAPTIVE CONTROL

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Abstract. With feedforward and disturbance decoupling control, cancellation of disturbance influences on the output is desired. Several adaptive schemes for implementing this are discussed. It is found that adaptive control based on explicit criterion minimization is a very attractive alternative.
Simulations indicate that the method has excellent robustness properties. For non-minimum phase systems, optimal disturbance rejection is achieved. Self-tuning control and adaptive LQG feedforward for non-minimum phase systems is also discussed.

Keywords. Feedforward, disturbance decoupling, disturbance rejection, adaptive control, explicit criterion minimization, optimization, LQG.

1. INTRODUCTION: IF FEEDFORWARD IS SO GOOD, WHY IS IT NOT USED MORE FREQUENTLY?

When disturbances can be measured, their influence may be cancelled. The usefulness of this idea has been realized for many years in both process control (Shinskey, 1965) and signal processing (Widrow et.al.1975). Despite of this, optimized feedforward links are not very common. In process control, feedforward applications are mostly limited to static gains used to compensate for the static component of a measurable disturbance. The reason seems to be a combination of four main problems, described below. These difficulties may possibly be solved by adaptive control. It will be discussed what requirements the feedforward problem places on an adaptive strategy.

1. The use of an open loop in feedforward control leads to robustness problems. Simple manually tuned static feedforward links sometimes improve control considerably, but in general, design of a good feedforward compensator requires a very accurate process model. The man-hours required for modelling can often not be justified in economic terms. This motivates the use of adaptive control. Robustness of adaptive control becomes of special concern in cases such as this, when the basic control strategy used is not robust.

2. The main process disturbance might not be directly measurable. Instead, auxiliary process variables are used for feedforward. These variables might be corrupted by measurement noise and affected by other variables, especially by the input. If the plant to be controlled is linear, the problem can be formulated as follows:

A system is given in the form

$$\begin{pmatrix} y(t) \\ w(t) \end{pmatrix} = \begin{pmatrix} H_{yu} & H_{yv} \\ H_{wu} & H_{wv} \end{pmatrix} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} + \begin{pmatrix} n(t) \\ m(t) \end{pmatrix} \qquad (1.1)$$

where H_{ij} are discrete or continuous time transfer functions. The main output y(t), the auxiliary measurement w(t) and the input u(t) are all scalar signals. The system is affected by a main disturbance v(t) and possibly by additional disturbances n(t) and m(t). Find transfer functions G_w and G_y for a regulator

$$u(t) = -G_w w(t) - G_y y(t) \qquad (1.2)$$

such that

- o the controlled plant becomes stable and
- o the transfer function from v(t) to y(t) is zero.

If this is possible, the main disturbance influence on the main output can be cancelled completely. We will call cases when $H_{wu}=0$ feedforward design problems. It is desirable that adaptive regulators also can handle cases when $H_{wu}\neq 0$. We then have a scalar version of the disturbance decoupling problem with stability constraints. The multivariable version of this problem has attracted considerable interest in recent years. For solvability conditions and solution methods, see e.g.Özgüler and Eldem (1985). In Kučera (1983), a simple solution for the scalar case is presented. The solvability conditions are restrictive. For example, stability of H_{yu}^{-1} is necessary for feedforward and scalar disturbance decoupling. In other words, the transfer function from u(t) to y(t) must be minimum phase. The problem of what to do if such conditions are not satisfied is rarely discussed in the literature related to disturbance decoupling.

3. Sampling often leads to non-minimum phase sampled systems, even when the continuous time system is minimum phase, cf Åström, Hagander and Sternby (1984). In such cases, complete disturbance decoupling at the sampling instants is not possible with a discrete time regulator. Considerable disturbance rejection may still be possible, however. It is desirable that if perfect disturbance decoupling is possible, an adaptive regulator should converge to this solution. If not, it should minimize the disturbance influence.

4. Feedforward filters often tend to be high-pass filters. They generate large high-frequency input signals and magnify high-frequency components of the measurement noise (m(t) in (1.1)). Since this is unacceptable in most cases, tradeoffs between disturbance rejection and input energy are necessary. Adaptive regulators could achieve this by minimizing a quadratic criterion with an input penalty.

While these are not the only difficulties faced by designers of feedforward regulators, methods to handle them would be highly desirable. In section 2, possible discrete time adaptive control strategies are discussed. Experiences from simulation studies are summarized in section 3. In section 4 it is concluded that there seems to exist at least one method, namely adaptive control based on explicit criterion minimization, which solves the problems mentioned above.

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2. ALTERNATIVES FOR ADAPTIVE CONTROL

The system (1.1) is considered in discrete time. For the moment m(t)=0 is assumed. If the time delay in H_{wv} is not larger than the time delay in H_{yv}, (1.1) can be rewritten as a causal model with w(t) as an input.

$$y(t) = (H_{yu}-H_{yv}H_{wv}^{-1}H_{wu})u(t)+H_{yv}H_{wv}^{-1}w(t)+n(t)$$

$$w(t) = H_{wu}u(t)+H_{wv}v(t)$$

The following polynomial parametrization will be used:

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t)+q^{-d}D(q^{-1})w(t)+C(q^{-1})e(t) \quad (2.1)$$

$$H(q^{-1})w(t) = q^{-n}N(q^{-1})u(t)+G(q^{-1})v(t) \quad (2.2)$$

where A,B,... are polynomials in the backward shift operator q^{-1} with degrees na,nb,... Nonzero time delays k, d and n are assumed. Consider the disturbances e(t) and v(t) to be mutually uncorrelated zero mean white stationary random sequences. (Control performance when v(t) is nonzero mean or nonstationary is discussed in section 3.) The polynomials A, C, H and G are assumed monic. Since the control problem is restricted to control of y(t), H is required to be stable. In addition, stability of C and G is assumed.

2.1 Self-tuning regulators

Self-tuners are (mostly implicit) adaptive regulators designed to minimize a sliding short-time criterion, see e.g.Åström and Wittenmark (1985), Clarke and Gawthrop (1975). Their main output feedback is easily complemented with a feedforward term, measuring w(t) in (2.1). Self-tuning regulators employing feedforward, for example Novatune from ASEA, are available commercially, cf Bengtsson and Egardt (1984). Several successful applications have been reported, e.g. by Allidina et.al.(1981). However, considering the problems discussed in section 1, self-tuners have two drawbacks:

- Presence of an input influence on the feedforward signal ($N(q^{-1})\neq 0$ in (2.2)) might lead to instability.
- Unstable B-polynomials in (2.1) may give problems.

Consider the basic minimum variance self-tuner with feedforward, applied on the system (2.1),(2.2). For simplicity, assume d≥k and $C(q^{-1})=1$. Introduce polynomials F (degree k-1) and S (degree na-1) such that

$$1 = A(q^{-1})F(q^{-1})+q^{-k}S(q^{-1}) \quad (2.3)$$

Use of (2.3) and (2.1) gives, omitting the argument q^{-1},

$$y(t+k) = AFy(t+k)+q^{-k}Sy(t+k) =$$

$$= BFu(t)+DFw(t-d+k)+Fe(t+k)+Sy(t)$$

At time t, the best estimate of Fe(t+k) is its mean value, zero. Thus, an optimal output predictor is given by:

$$\hat{y}(t+k|t) = BFu(t)+DFw(t-d+k)+Sy(t)$$

The polynomials BF, DF and S may be estimated recursively on line by minimizing the prediction error. The self-tuning minimum variance regulator calculates u(t) such that the predicted output equals the reference value. For reference zero, the controller is

$$\widehat{BF}u(t) = -\hat{S}y(t)-\widehat{DF}w(t-d+k) \quad (2.4)$$

If the estimated regulator polynomials are true, i.e. if $\widehat{BF}$=BF, $\hat{S}$=S and $\widehat{DF}$=DF, the closed system is given by

$$By(t) = BFe(t) \quad (2.5)$$

$$\tilde{B}w(t) = q^{-n}NSe(t)+GBv(t) \quad (2.6)$$

where

$$q^{-b}\tilde{B} \triangleq q^{-k}BH+q^{-n-d}ND \quad ; \quad b=\min(k,n+d) \quad (2.7)$$

Disturbance decoupling (v does not affect y) and stability is achieved if B and $\tilde{B}$ are stable. Unstable B-polynomials will give problems. Note that even when B is stable, $\tilde{B}$ might be unstable if the input affects w(t) (N≠0 in (2.7)). Use of an input penalty, increase of the sampling period or extension of the prediction horizon often leads to stable control of non-minimum phase systems, cf Åström and Wittenmark (1985). These modifications do however fail in some cases. Neither good control nor even stability is guaranteed. The performance of self-tuners is compared with regulation using explicit criterion minimization in an example in section 3.3.

2.2 An LQG approach

An attractive way of avoiding problems with non-minimum phase systems is to minimize an infinite horizon quadratic criterion, instead of a sliding short-time criterion. LQG theory, either in a state space or a polynomial formulation, achieves this. A polynomial formulation will be discussed briefly in this section.

The infinite horizon criterion

$$J=E \lim_{N\to\infty}\frac{1}{2N}\sum_{t=k}^{N}\left((y(t)-r(t-k))^2+\rho(\tilde{\Delta}(q^{-1})u(t-k))^2\right) \quad (2.8)$$

is to be minimized with respect to regulator parameters. Here, r(t-k) is the setpoint for y(t), ρ is an input penalty and $\tilde{\Delta}(q^{-1})$ is a polynomial. To avoid static errors for nonzero setpoints, differential input penalty $\tilde{\Delta}(q^{-1})=1-q^{-1}$ is used. In other cases, $\tilde{\Delta}=1$ is the normal choice. Now, consider the following two regulator structures.

1. Feedforward/disturbance decoupling:

$$u(t) = -\frac{Q}{P}\left(w(t)-\frac{\hat{N}}{\hat{H}}u(t-n)\right)+Tr(t) \quad (2.9)$$

(Stability of A in (2.1) is assumed whenever (2.9) is used.)

2. Combined feedforward and feedback:

$$Ru(t) = -\frac{Q}{\hat{G}}\left(w(t)-\frac{\hat{N}}{\hat{H}}u(t-n)\right)-Sy(t)+Tr(t) \quad (2.10)$$

The criterion (2.8) and the regulators (2.9),(2.10) are also used in section 2.3.

In (2.9) and (2.10), $\hat{N}$, $\hat{H}$ and $\hat{G}$ are estimates of N, H and G in (2.2). These regulators are designed to subtract an estimate of the input influence on w(t) from the feedforward signal. With a correct model, this reduces disturbance decoupling problems (N≠0) to feedforward problems (N=0). The polynomials H, $\hat{H}$, G and $\hat{G}$ are assumed stable. Regulators structures similar to (2.9) have also been discussed by Brosilow and Tong (1978).

In Sternad (1985), minimization of (2.8) for regulator problems with r(t)=0 is considered. One of the conclusions is that optimal feedforward and feedback filters should have a common denominator (R in (2.10)). In addition, the disturbance numerator G should be cancelled by the feedforward filter. This is done explicitely in (2.10). Methods for optimizing P and Q in (2.9) or R, S and Q in (2.10) are presented. As an example of these results, let us discuss optimal feedforward control.

Assume that

- the system (2.1),(2.2) is exactly known and stable
- the regulator (2.9) is to be used and $N(q^{-1})=0$, i.e. we have a feedforward problem.
- B(z) has no zeros on the unit circle.

Introduce β(z) as the stable spectral factor from

$$r\beta(z)\beta(z^{-1}) = B(z)B(z^{-1})+\rho A(z)\tilde{\Delta}(z)\tilde{\Delta}(z^{-1})A(z^{-1}) \quad (2.11)$$

where r is a scalar and β(z) is monic.

Introduce polynomials $Q(z^{-1})$ and $L(z)$ with degrees, cf (2.1),(2.2)

$$\deg L(z) = \max\{\deg\beta-1, nb-d+k-1\} \quad (2.12)$$

$$\deg Q(z^{-1}) = \max\{na+nh-1, nd+ng+d-k\} \quad (2.13)$$

as a solution to the polynomial equation

$$z^{-d+k}B(z)D(z^{-1})G(z^{-1}) = r\beta(z)Q(z^{-1})+A(z^{-1})H(z^{-1})zL(z) \quad (2.14)$$

Then the feedforward regulator

$$u(t) = \frac{-Q(q^{-1})}{\beta(q^{-1})G(q^{-1})} w(t) \quad (2.15)$$

attains the global minimum point of (2.8).

For a proof, see Sternad (1985).

If $\rho=0$ and perfect feedforward is possible, this will of course be attained by (2.15). However, (2.15) also handles problems with input penalties, too small feedforward time delays (d<k) and unstable B-polynomials. In Sternad (1985), the above result is generalized to the disturbance decoupling case and to regulators (2.10) using both feedforward and feedback. It is straightforward to generalize to CARIMA models and regulators in differential form for nonstationary disturbances.

Implementation of these regulators in an explicit adaptive form is straightforward.

"LQG-algorithm" (2.16)

Repeat the following steps at each sampling period:

1. Update recursive estimates of models (2.1) and (2.2) using e.g. ELS.
2. Solve approximately for β in (2.11) by taking a few steps in some recursive algorithm for spectral factorization. See e.g. Kučera (1979).
3. Solve the polynomial equation (2.14) or generalizations thereof.
4. Compute u(t), using (2.9) or (2.10).

For similar algorithms for feedback regulators, see Åström and Zhao-Ying (1982) or Grimble (1984). For cases when the disturbance w(t) is an AR-process (N=0, G=1) Peterka (1984) suggests methods to optimize combined feedback-feedforward regulators for both finite and infinite horizon criteria. Work on an adaptive implementation of the results in Sternad (1985) for regulators of type (2.9) and (2.10) is now proceeding. Results will, it is hoped, be presented at the workshop. However, there is reason for caution. The extreme parameter sensitivity of optimized feedforward filters makes good parameter estimation critical. Use of an explicit adaptive strategy, such as (2.16), may lead to robustness problems. For this reason, it is worthwhile to investigate other methods to compute LQG regulators adaptively. In the next section, a very attractive alternative is presented.

2.3 Explicit criterion minimization

This control principle has been developed by Trulsson and Ljung (1985), based on earlier ideas by Tzypkin (1971). Let θ_r be a vector of free regulator parameters. The main idea is to see the criterion J (in our case (2.8)) as a function of these regulator parameters. A stationary point $dJ/d\theta_r=0$ is sought with some recursive numerical search method. We will apply this idea to regulators of type (2.9) and (2.10).
Differentiation of (2.8) with respect to θ_r gives:

$$\frac{dJ(\theta_r)}{d\theta_r} = E\left((y(t)-r(t-k))\frac{dy(t)}{d\theta_r} + \rho\tilde{\Delta}u(t-k)\frac{d\tilde{\Delta}u(t-k)}{d\theta_r}\right) \quad (2.17)$$

This expression is used when a stationary point is sought. The signal derivatives $dy(t,\theta_r)/d\theta_r$ and $d\tilde{\Delta}u(t-k,\theta_r)/d\theta_r$ are computed by filtering measurable signals. A model must be identified since model parameters are needed in the filters.

"Adaptive criterion minimization algorithm" (2.18)

Repeat the following steps at each sampling period:

1. Update a recursively estimated model with parameter vector $\theta_m(t)$.
2. Compute approximate signal derivative filters using $\theta_m(t)$ and $\theta_r(t-1)$. Perform the filterings.
3. Update the regulator parameters $\theta_r(t)$ towards a stationary point $dJ/d\theta_r=0$.
4. Compute the control signal u(t), using $\theta_r(t)$.

It has been proven by Trulsson (1983) that

-if all signals stay bounded and
-if the parts of θ_m used in the signal derivative filters converge to their true value w.p.1.

Then, under mild conditions on the updating method for θ_r, the algorithm (2.18) will converge to a local minimum of the criterion w.p.1.

Assume furthermore

- A linear system and a quadratic criterion, such as (2.8).
- A linear regulator with the right number of parameters, making the minimum point of J unique.

Then, the method converges to this unique minimum, which coincides with the LQG solution. It should be noticed that non-minimum phase systems present no special problems. A main motivation for examining the criterion minimization method is the robustness consideration. Thus, it is of prime importance to investigate experimentally how the algorithm behaves if the above conditions are not satisfied. What happens, for example, if θ_m or θ_r have the wrong parameterization? Comparing the algorithm (2.18) with the LQG algorithm (2.16), a recursive regulator update is made instead of a spectral factorization and solutions of polynomial equaitons. LQG design relies, more or less blindly, on the certainty equivalence principle. Model parameters are used as if they were the true ones. Criterion minimization uses both model parameters and measurements of the control performance. If the model is wrong, the regulator adaption may be able to compensate for this, to a large extent. This has in fact been the case in a number of simulation experiments, described in detail in Sternad (1986) and summarized in section 3. The four steps of the control algorithm (2.18) are described below.

Step 4: Computation of u(t). The regulator structure used is (2.9) or (2.10). The parameter vector θ_r is given by

$$\theta_r = (Q_0,\dots,Q_{nQ},P_1,\dots,P_{np},T_0,\dots,T_{nt}) \quad (2.19)$$

for (2.9) and

$$\theta_r = (r_1,\dots,r_{nr},s_0,\dots,s_{ns},Q_0,\dots,Q_{nQ},T_0,\dots,T_{nt}) \quad (2.20)$$

for (2.10).

The P and R polynomials are assumed monic. The polynomials $\hat{N}$, $\hat{H}$ and $\hat{G}$ are the latest estimates in the model of (2.2). For a given model parametrization regulator polynomial orders might be chosen such that optimal LQG control becomes possible. See e.g. (2.13). It has turned out, however, that the exact choice of regulator orders is not critical for control performance.

Step 3: Update of regulator parameters. The criterion (2.8) is to be minimized with respect to θ_r, given by (2.19) or (2.20). Let us rewrite (2.8) in the following way.

$$J = \frac{1}{2} E[(y(t)-r(t-k))^2+\rho(\tilde{\Delta}u(t-k))^2] =$$

$$= \frac{1}{2}\, E(y(t)-r(t-k),\tilde{\Delta}u(t-k))\begin{pmatrix}1 & 0\\ 0 & \rho\end{pmatrix}\begin{pmatrix}y(t)-r(t-k)\\ \tilde{\Delta}u(t-k)\end{pmatrix}$$

$$\triangleq \frac{1}{2}\, E\varepsilon(t)^T\Lambda^{-1}\varepsilon(t) \qquad (2.21)$$

The criterion (2.21) has the same algebraic structure as the type of criterion minimized in recursive prediction error identification for multiple output systems. This suggests that such algorithms could minimize (2.21). An algorithm from Ljung and Söderström (1983) is used. It updates θ_r in an approximate Gauss-Newton direction. Compared to stochastic approximation algorithms, updating θ_r in the gradient direction, this should improve convergence.

Define the dimθx2 matrix

$$\psi(t,\theta_r) \triangleq -\frac{d\varepsilon(t,\theta_r)^T}{d\theta_r} = \left(-\frac{dy(t,\theta_r)}{d\theta_r}\;,\; -\frac{d\tilde{\Delta}u(t-k,\theta_r)}{d\theta_r}\right) \qquad (2.22)$$

The algorithm for updating θ_r can then be expressed as:

$$S(t) = \lambda(t)\Lambda+\psi^T(t)P(t-1)\psi(t) \quad ; \; S(0) = \lambda(0)\Lambda \qquad (2.23a)$$

$$P(t) = \frac{1}{\lambda(t)}\,[P(t-1)-P(t-1)\psi(t)S(t)^{-1}\psi(t)^TP(t-1)] \qquad (2.23b)$$

$$\theta_r(t) = \theta_r(t-1)+P(t)\psi(t)\begin{pmatrix}y(t)-r(t-k)\\ \rho\tilde{\Delta}u(t-k)\end{pmatrix} \qquad \theta_r(0) = 0 \qquad (2.23c)$$

For updating parameters in an approxmate Newton direction, an approximate inverse of the Hessian is needed. Recursions (2.23a and b) take care of this. The gradient $-\psi(t)\varepsilon(t)$ is also required. Expressions for approximate signal derivatives, to be used in $\psi(t)$ are needed.

Step 2: Signal derivative filterings. If the model equals the system, use of the regulator (2.10) on (2.1),(2.2) gives the closed system

$$\alpha y(t) = \frac{(q^{-d}DHGR-q^{-b}\tilde{B}Q)}{H}\, v(t)+CHRe(t)+q^{-b}\tilde{B}Tr(t) \qquad (2.24)$$

$$\alpha u(t) = -(AQ+q^{-d}DGS)v(t)-CHSe(t)+AHTr(t) \qquad (2.25)$$

where the polynomial $q^{-b}\tilde{B} = q^{-k}BH+q^{-d-n}DN$ has already been encountered in (2.7) and

$$\alpha = AHR+q^{-b}\tilde{B}S \qquad (2.26)$$

In addition, hidden modes corresponding to H and G are present. They are assumed stable and are hence cancelled.

Use of these expressions and differentiation, as explained in Trulsson and Ljung (1985), gives the signal derivative filters. For the combined feedback-feedforward regulator (2.10), with parameter vector (2.20), they are given by:

$$\frac{\partial y(t)}{\partial r_\ell} = \frac{-q^{-b}\tilde{B}}{\alpha}\, u(t-\ell) \quad ; \quad \frac{\partial\tilde{\Delta}u(t-k)}{\partial r_\ell} = -\,\frac{AH\tilde{\Delta}}{\alpha}\, u(t-k-\ell)$$

$$\frac{\partial y(t)}{\partial s_m} = \frac{-q^{-b}\tilde{B}}{\alpha}\, y(t-m) \quad ; \quad \frac{\partial\tilde{\Delta}u(t-k)}{\partial s_m} = -\,\frac{AH\tilde{\Delta}}{\alpha}\, y(t-k-m) \qquad (2.27)$$

$$\frac{\partial y(t)}{\partial Q_j} = \frac{-q^{-b}\tilde{B}}{\alpha G}\, w_1(t-j) \quad ; \quad \frac{\partial\tilde{\Delta}u(t-k)}{\partial Q_j} = -\,\frac{AH\tilde{\Delta}}{\alpha G}\, w_1(t-k-j)$$

$$\frac{\partial y(t)}{\partial T_\tau} = \frac{+q^{-b}\tilde{B}}{\alpha}\, r(t-\tau) \quad ; \quad \frac{\partial\tilde{\Delta}u(t-k)}{\partial T_\tau} = +\,\frac{AH\tilde{\Delta}}{\alpha}\, r(t-k-\tau)$$

where ℓ=1,...,nr, m=0,...,ns, j=0,...,nQ, τ=0,...,nt and

$$w_1(t) = w(t)-q^{-n}\,\frac{N}{H}\, u(t) \qquad (2.28)$$

If the feedforward regulator (2.9), with parameter vector (2.19) is used, the signal derivatives are

$$\frac{\partial y(t)}{\partial Q_j} = \frac{-q^{-b}\tilde{B}}{\alpha P}\, w_1(t-j) \quad ; \quad \frac{\partial\tilde{\Delta}u(t-k)}{\partial Q_j} = -\,\frac{\tilde{\Delta}}{P}\, w_1(t-k-j)$$

$$\frac{\partial y(t)}{\partial P_i} = \frac{+q^{-b}\tilde{B}Q}{\alpha P^2}\, w_1(t-i) \quad ; \quad \frac{\partial\tilde{\Delta}u(t-k)}{\partial P_i} = +\,\frac{Q\tilde{\Delta}}{P^2}\, w_1(t-k-i) \qquad (2.29)$$

where j=0,...,nQ and i=1,...,np. Derivatives with respect to $T(q^{-1})$ are the same as in (2.27).

The filters in (2.27)-(2.29) depend on the true system. If parameters from the model are used, this gives approximate signal derivatives, which can be used in $\psi(t)$.

Step 1: System identification. Models with the parametrization (2.1) and (2.2) are updated using two recursive prediction error algorithms for single output systems, cf Ljung and Söderström (1983). Stability monitoring of C, H and G is provided. Normally, the system is identified in open loop for the first 20 samples. This period is also used for filling the signal vectors, to be used in the signal derivative filterings. This start-up procedure improves the transient behaviour of the regulator parameters.

Remarks: In the criterion (2.8), the equation (2.23c) and the signal derivative components (2.27)/(2.29), the input u is delayed with respect to the output. From simulations, this has been found to be important. If the criterion

$$J = \frac{1}{2}E[(y(t)-r(t-k))^2+\rho(\tilde{\Delta}u(t))^2]$$

is used (giving k=0 in (2.27)/(2.29) and (2.23c)), the algorithm diverges if large input penalties ρ are used. If $k\geq 1$ is used, no problems occur. The time delay need not equal the dealy in the true system. Under-estimation of the delay creates no problem (unless the estimate is zero). Since $\alpha(q^{-1})$ from (2.26) is a denominator polynomial of both the signal derivative filters and the closed model (2.24),(2.25), stability of α must be monitored. As suggested in Trulsson (1983), stability is checked both before the filterings (2.27)/(2.29) are performed and when a new regulator update has been made. These checks are necessary for assuring a good behaviour of the algorithm.

3. SIMULATIONS

The examples of section 3.1 illustrate the behaviour of systems controlled by adaptive criterion minimization. Experience from simulations with this algorithm are summarized in section 3.2. Performance is compared with self-tuning control in section 3.3.

3.1 Disturbance decoupling using explicit criterion minimization

Example 1. Consider the following system, where exact disturbance decoupling is possible:

$$(1-0.5q^{-1}-0.25q^{-2}+0.125q^{-3})y(t) =$$
$$= q^{-1}(1-1.7q^{-1}-0.1q^{-2})u(t)+q^{-1}(2-2q^{-1}+0.5q^{-2})w(t)$$
$$(1-q^{-1}+0.25q^{-2})w(t) = q^{-1}u(t)+(1+0.5q^{-1})v(t) \qquad (3.1)$$

The disturbance v(t) is white noise with standard deviation 0.1. A regulator problem with r(t)=0 is considered. The goal is to cancel the influence of v(t) on the main output y(t). An adaptive criterion minimizing regulator of type (2.10) is used. Both model and regulator (nr=2, ns=0, nQ=2) have the right parametrization. Figure 1 illustrates the uncontrolled output. Figure 2 shows a typical behaviour of the controlled output. Both model and regulator parameters converge quickly. The regulator parameters are shown in Figure 3. The first 20 samples are used for identification only.

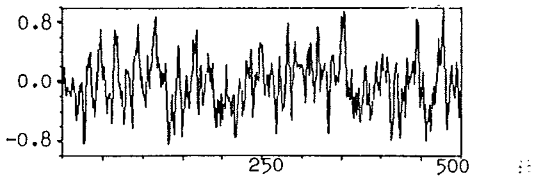

Figure 1. Output y(t) of the open system (3.1), with standard deviation σy=0.33.

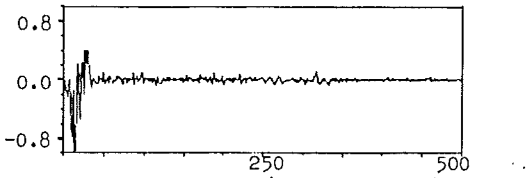

Figure 2. Output of the adaptively controlled system. The vertical scale is the same as in Fig. 1. At time 500, σy is 0.008 and decreasing.

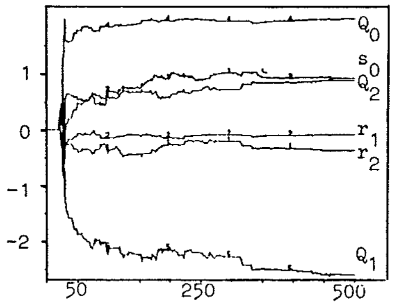

Figure 3. The regulator parameters.

Example 2. The robustness of the criterion minimization method, when applied to the non-minimum phase system

$$(1-0.95q^{-1})y(t) = (1+2q^{-1})u(t-2)+w(t-3)+(1+0.7q^{-1})e(t)$$
$$w(t) = v(t) \qquad (3.2)$$

is investigated. For a regulator of type (2.10) the optimal regulator degrees are nr=2, ns=0, nQ=2. When v(t) and e(t) are uncorrelated white disturbances, the regulator quickly converges to a minimum variance regulator for non-minimum phase systems, combined with an optimal feedforward.

In the following simulations, v(t) and e(t) are white noises with standard deviation 0.1 until time 100. At that point, v(t)=w(t) becomes a square wave with amplitude 1 and period 60. (An open system disturbance of amplitude 17 would result from this.) In Figure 4 the system is controlled adaptively. An input penalty $\rho=1$, $\tilde{\Delta}(q^{-1})=1$ is used in this and the following simulations.

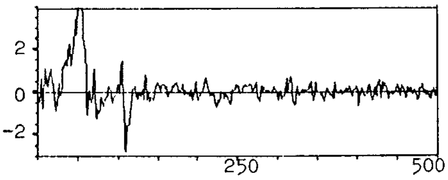

Figure 4. Output for the adaptively controlled system. Model and regulator have correct parametrization.

We now investigate the sensitivity to under and overparametrization. In the simulation presented in Fig. 5.

- An auxiliary output model (2.2) is used. H, N and G are assumed to be a first order. This might create problems for the identification
- The delay k is underestimated in the filterings (2.27): k=1, while the delay of the true system is 2.
- The regulator is under-parametrized (nr=1,ns=0,nQ=1).

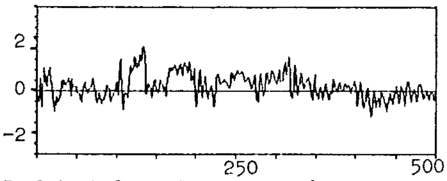

Figure 5. Output for under-parametrized regulator and underestimated time delay.

In Figure 6 another variant is tested:

- The main output model (2.1) is over-parametrized. Polynomial orders na=2, nb=4, nd=4 and nc=2 are used.
- An auxiliary output model is used, as above.
- The delay k is overestimated in (2.27): k=3.
- The regulator is over-parametrized (nr=3, ns=2, nQ=3).

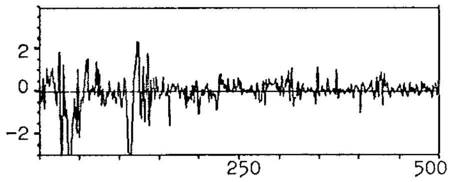

Figure 6. Output for over-parametrized regulator and model and overestimated time delay.

In this example at least, the robustness seems satisfactory.

Example 3. A test is made with an extreme example:

The unstable non-minimum phase system

$$(1-2q^{-1}+1.5q^{-2})y(t) = q^{-1}(1+2q^{-1}+2q^{-2})u(t)$$
$$+q^{-2}(1+0.5q^{-1})w(t)+e(t)$$
$$w(t) = v(t) \qquad (3.3)$$

with poles in $1\pm0.71i$ and zeros in $-1\pm i$. Let both v(t) and e(t) be white disturbances with standard deviation 0.1. The result of using the regulator (2.10), with nr=2, ns=1, nQ=2 and ρ=1 is shown in Figure 7. A model with the right parametrization is used. The standard deviation is 0.25 in the interval [300,1000].

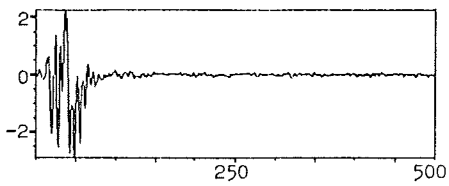

Figure 7. Controlled output of the open loop unstable non-minimum phase system (3.3).

3.2 Summary of simulation results using adaptive criterion minimization

A significant number of simulation experiments are described in Sternad (1986). The following preliminary conclusions have been drawn from them. Compare with the requirements discussed in section 1.

- o Feedforward and feedback control of non-minimum phase systems works without problems. When regulator and model have the right parametrization, convergence to the LQG-optimal regulator is almost always achieved.
- o SISO disturbance decoupling (w(t) affected by u(t-n)) is handled without problems.
- o Use of an input penalty improves the output behaviour in the transient phase.
- o Systems with unknown time delays give no problems if the degree of the model B-polynomial is chosen large enough.

- If the auxiliary output w(t) is corrupted by additional measurement noise (m(t) in (1.1)), the regulator compensates by relying less on this signal, as it should.
- The influence of v(t) on y(t) can be effectively eliminated even when v(t) is non-stationary without explicitely using an integrating regulator. (If needed, the regulator converges to an integrating one spontaneously.) The combined feeback-feedforward strategy (2.10) has, so far, shown excellent robustness properties. In many cases, control has been good even when estimation has failed. For example, if $\hat{G}$ in (2.10) is totally wrong (but stable), adaption of R, S and Q may compensate for this. The robustness of feedforward only (2.9), while not bad, is clearly inferior to (2.10). For robustness reasons, (2.10) should be used instead of (2.9).
- Very good control results are achieved with underparametrized regulators. Explicit criterion minimization is an attractive approach to restricted complexity optimal control problems. This has also been observed by Goodwin and Ramdage (1979) and by Stanković and Radenković (1984).
- Over-parametrized regulators present no problems. Unstable common regulator factors are prevented by the stability monitoring of α. Estimator windup in (2.23) can be avoided by turning off the adaption if some P-matrix diagonal element exceeds a bound.
- Over-parametrization of the model raises no problems, if a robust identification method is used. Severe under-parametrization is sometimes dangerous. If, for example, the auxiliary output w(t) is affected by the input, while $N(q^{-1})=0$ in the model, the algorithm often diverges.
- Divergence sometimes occurs when tight bounds are used on the input. The optimization (2.23) then has problems with local minima.

3.3 Comparison between explicit criterion minimization and self-tuning controllers

Example 4. Control performance for the non-minimum phase system

$$(1-.5q^{-1})y(t)=(.5+1.25q^{-1}+.5q^{-2})u(t-1)+(2-1.5q^{-1})w(t-2)$$

$$(1-0.9q^{-1})w(t) = (1-0.3q^{-1})v(t)$$

is investigated, when v(t) is white noise. In Figure 8 ,normalized input and output standard deviations $\sigma u/\sigma v$ and $\sigma y/\sigma v$ are shown. The point (1) corresponds to the open system. The curve from (1) to (2) is the result of adaptive criterion minimization for different ρ, measured in the time interval [500-1000]. The performance is the same for regulators (2.9) and (2.10). It is virtually identical to the result of LQG-optimal feedforward, calculated from (2.11)-(2.15).

Since B is unstable, a minimum variance self-tuner will be unstable. It can be stabilized with an input penalty $\rho>0.17$. The curve from (1) to (3) gives the performance of a self-tuner with input penalty in the interval $[\infty,0.6]$. Performance is clearly worse than with LQG or criterion minimization. The point (4) is achieved by a self-tuner with $\rho=0$ and extenden prediction horizon k=2. It almost achieves minimal σy, but requires a large input. For comparision, the result of minimum variance feedback (point (5)) and optimal static feedforward $u(t) = -0.24w(t)$ (point (6)) is also shown.

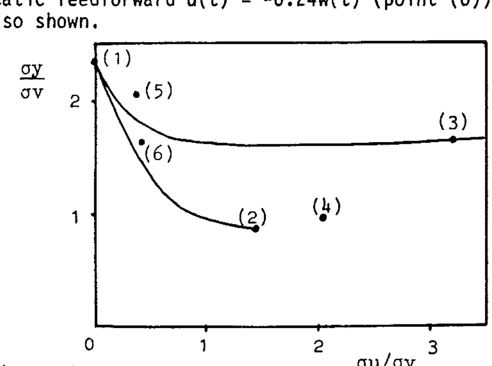

Figure 8.

For some non-minimum phase systems, self-tuners with input penalty almost achieve LQG-optimal performance. This is the case e.g for system (3.2). For others, their performance is markedly worse. In general, self-tuners have slightly better transient performance than explicit criterion minimization, which has to adapt more parameters (both θ_m and θ_r).

4. CONCLUSIONS

Robustness problems and the need of good modelling limits the use of feedforward, despite the promise of this control principle. Use of adaptive algorithms may remove these limitations. Of the adaptive methods discussed, the self-tuners are simple and will work well in many cases. The explicit criterion minimization approach can be applied to a larger class of problems: Optimal feedforward of non-minimum phase systems can be realized easily. Feedforward signals influenced by the input can be used without problems. Simulations indicate that a combination of feedforward with feedback, optimized with adaptive criterion minimization, has good robustness properties.

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