Cost-Delay Trade Off of Network Coding in Asymmetric Two-Way Relay Networks

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***Abstract*—In this paper, we analyze and evaluate the cost-delay trade off of opportunistic network coding in a two-way relay network with asymmetric arrival rates. We first formulate the variance of the queue into a Markov chain and derive the transition probability and the stationary distribution. Then we analyze the average power cost and delay, formulate the optimal power-delay trade off, and propose a heuristic discrete solution. After that, we present numerical results on the performance of the discrete solution.**

***Keywords*—Cost-Delay Trade Off, Network Coding, Two-Way Relay Networks**

I. Introduction

Network coding is a very promising technology to efficiently utilize resources of both wired and wireless networks, and has received much attention from both academia and industry recently [1]. Network optimization with limited inter-session network coding has been investigated recently [2], [3]. However, there is a common implicit assumption in these researches that the traffic is perfectly scheduled and network coding is mandatory at nodes appointed by the optimization algorithms. In fact, traffics are often bursty and asymmetric, and we can not always use network coding without waiting. Therefore, there is a trade off between cost and delay when making use of network coding.

The cost-delay trade off has been analyzed in [4]–[6] in a simple two-way relay network. The optimal transmitting probability of the relay node with different queue status is optimized for cost-delay trade off in [4]. Theoretical analysis of cost-delay trade off in [5] shows that the delay will go infinity if the cost is minimized. However, none of them has considered the cost-delay trade off in a real network with asymmetric arrival rates.

In this paper, we analyze the cost-delay trade off of opportunistic network coding in a two-way relay network with asymmetric arrival rates. We first formulate the variance of the queue into a Markov chain and derive the transition probability and the stationary distribution. Then we analyze the average power cost and delay, formulate the optimal power-delay trade off, and propose a heuristic discrete solution. We also show that the performance degradation of the proposed discrete solution is limited. After that, we present numerical results on the performance of the solution.

The rest of the paper is arranged as follows. We give problem formulation and analysis in Section II, and evaluate the performance of the proposed solution in Section III. The whole paper is concluded in Section IV.

II. Problem Formulation and Analysis

A. Problem Formulation

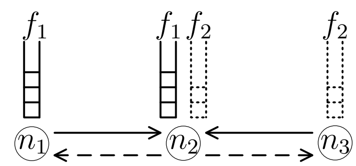

Fig. 1. Two-way relay network coding scenario.

We consider the two-way scenario as shown in Figure 1. We assume the time is slotted, and nodes n_1, n_3 and n_2 transmit in consecutive time slots, which form a frame as shown in Figure 2. We assume the arrival rates of the two flows, f_1 and f_2, follow Bernoulli distribution with probabilities γ_1 and γ_2, respectively. We assume the total queue size at the relay node n_2 is K, the queue lengths of the two flows, f_1 and f_2, are q_1 and q_2, respectively, and there exists $q_1 + q_2 \leq K$.

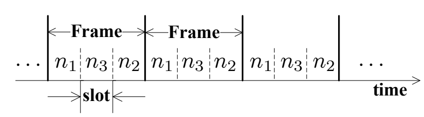

Fig. 2. Frame structure of scheduling in two-way relay networks.

We define the relay strategy at node n_2 as follows. When there are packets coming from both flows, the relay n_2 codes them together and broadcasts it to both n_1 and n_3. When there is only packets from one flow in the queue and the queue size is k_i, in which $i = 1$ or 2 representing indexes of flows, the relay node forwards one uncoded packet with probability g_{k_i}. With the defined relay strategy, we can easily get $q_1 + q_2 = k$ and $q_1 q_2 = 0$ [4] and formulate the variance of the queue length in two consecutive slots into a Markov chain with $2K+1$ states as shown in Figure 3. The negative and positive states represent the packets in the queue belong to flows f_1 or f_2, respectively. The transition probabilities λ_k and μ_k of the Markov chain will be given in the following lemma.

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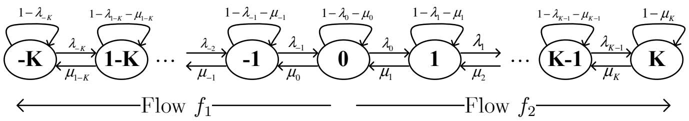

Fig. 3. Markov chain of the variance of the queue size.

Lemma 1 *The transition probabilities of queue lengths are*

$$p_{kj} = \begin{cases} \lambda_k, & j = k+1,\ -K \le k \le K-1, \\ \mu_k, & j = k-1,\ -K+1 \le k \le K, \\ 1-\lambda_k-\mu_k, & j = k,\ -K \le k \le K, \\ 1-\lambda_{-K}, & j = k = -K, \\ 1-\mu_K, & j = k = K, \\ 0, & \textit{otherwise}. \end{cases} \tag{1}$$

where

$$\lambda_k = \begin{cases} (1-\gamma_1)\gamma_2(1-g_{k+1}), & 0 \le k < K, \\ (1-\gamma_1)(\gamma_2+(1-\gamma_2)g_k), & -K \le k < 0. \end{cases} \tag{2}$$

$$\mu_k = \begin{cases} (1-\gamma_2)(\gamma_1+(1-\gamma_1)g_k), & 0 < k \le K, \\ (1-\gamma_2)\gamma_1(1-g_{k-1}), & -K < k \le 0. \end{cases} \tag{3}$$

Proof: First, we prove that the difference between queue lengths k and j can not be larger than 1, i.e., $|k-j| \le 1$. Without loss of generality, we assume the packets in the queue is from flow f_1. Since the relay node is only allowed to transmit at most once in each time slot, there exits $j \ge k-1$. Moreover, the queue length is increased only when flow f_2 transmits in the frame and the queue is increased by 1 with probability $1-g_{k+1}$. Thus the queue size can not be increased larger than 1, i.e., $j \le k+1$. Therefore, we have $|k-j| \le 1$.

When $j = k-1$ and $j \ge 0$, we have $p_{kj} = (1-\gamma_2)(1-\gamma_1)g_k + (1-\gamma_2)\gamma_1 = (1-\gamma_2)(\gamma_1+(1-\gamma_1)g_k)$.

Similarly, we can get the probability under other conditions.

We define λ_k and μ_k as in (2) (3), then the transition probability with $-K < j = k < K$ is written as $p_{kk} = 1-\lambda_k-\mu_k$ and the lemma is proved. ■

Theorem 1 *The stationary distribution of the Markov chain is*

$$\pi_k = \begin{cases} \pi_0 \prod_{i=1}^{k} \frac{\lambda_{i-1}}{\mu_i}, & k > 0, \\ \pi_0 \prod_{i=k}^{-1} \frac{\mu_{i+1}}{\lambda_i}, & k < 0. \end{cases} \tag{4}$$

where

$$\pi_0 = \left(1 + \sum_{k=-K}^{-1} \prod_{i=k}^{-1} \frac{\mu_{i+1}}{\lambda_i} + \sum_{k=1}^{K} \prod_{i=1}^{k} \frac{\lambda_{i-1}}{\mu_i}\right)^{-1} \tag{5}$$

Proof: According to the property of the stationary distribution, we have

$$\pi_{-K} = (1-\lambda_{-K})\pi_{-K} + \mu_{1-K}\pi_{1-K}$$
$$\pi_K = (1-\mu_K)\pi_K + \lambda_K\pi_{K-1}$$
$$\pi_k = (1-\lambda_k-\mu_k)\pi_k + \mu_{k+1}\pi_{k+1} + \lambda_{k-1}\pi_{k-1},\ |k| < K$$

With transformations and iterations, we obtain

$$\mu_k\pi_k = \lambda_{k-1}\pi_{k-1},\ -K < k \le K \tag{6}$$

Through which we can easily get the stationary distribution. ■

B. Transmission Power Cost and Delay

Theorem 2 *The normalized transmission power cost is*

$$P = \gamma_1 \sum_{k=-1}^{-K} \pi_k + (\gamma_1+\gamma_2-\gamma_1\gamma_2)\pi_0 + \gamma_2 \sum_{k=1}^{K} \pi_k \tag{7}$$

Proof: The normalized transmission power cost is

$$P = \sum_{k=-K}^{K} p_k \pi_k$$

Since there exists $\mu_k\pi_k = \lambda_{k-1}\pi_{k-1},\ -K < k \le K$, with iterated cancellation, we can write the power cost as

$$P = \gamma_1 \sum_{k=-1}^{-K} \pi_k + (\gamma_1+\gamma_2-\gamma_1\gamma_2)\pi_0 + \gamma_2 \sum_{k=1}^{K} \pi_k$$

■

Theorem 3 *The average delay of the two flows is*

$$D = \frac{1}{\gamma_1} \sum_{k=1}^{K} \pi_{-k} \sum_{k=1}^{K} k\pi_{-k} + \frac{1}{\gamma_2} \sum_{k=1}^{K} \pi_k \sum_{k=1}^{K} k\pi_k \tag{8}$$

Proof: When the packets in the queue are from flow f_1, the average queue size is

$$Q_1 = \sum_{k=1}^{K} k\pi_{-k}$$

Since the arrival rate of flow f_1 is γ_1, according to Little's Law, the average delay of packets from flow f_1 is

$$D_1 = Q_1/\gamma_1$$

Similarly, for flow f_2, we have

$$Q_2 = \sum_{k=1}^{K} k\pi_k, \qquad D_2 = Q_2/\gamma_2$$

Due to the probabilities of the queue on both sides are $\sum_{k=1}^{K} \pi_{-k}$ and $\sum_{k=1}^{K} \pi_k$, respectively, the average delay is as shown in the theorem. ■

C. Optimal Power Delay Trade off

Lemma 2 *Assuming the normalized maximum transmission power constraint at the relay node is P_{max}, the power-delay trade off can be written as*

$$\textbf{\textit{(P1)}} \quad \min\ \pi_0^2 \left(\frac{1}{\gamma_1} X \sum_{k=1}^{K} k\rho_{-k} + \frac{1}{\gamma_2} Y \sum_{k=1}^{K} k\rho_k\right)$$

$$\textit{s.t.}\ \pi_0[\gamma_1 X + \gamma_2 Y + \gamma_1 + \gamma_2 - \gamma_1\gamma_2] \le P_{max} \tag{9}$$

$$\pi_0 \sum_{k=-K}^{K} \rho_k = 1 \tag{10}$$

$$0 \le \rho_{k+1} \le \rho_k \alpha, \; 0 \le k \le K \tag{11}$$

$$0 \le \rho_{k-1} \le \rho_k / \alpha, \; -K < k \le 0 \tag{12}$$

where

$$\rho_k = \begin{cases} \prod_{i=0}^{k-1} \frac{\lambda_i}{\mu_{i+1}}, & k > 0 \\ 1, & k = 0 \\ \prod_{i=k}^{-1} \frac{\mu_{i+1}}{\lambda_i}, & k < 0 \end{cases} \tag{13}$$

$$X = \sum_{k=1}^{K} \rho_{-k}, \qquad Y = \sum_{k=1}^{K} \rho_k, \qquad \alpha = \frac{1/\gamma_1 - 1}{1/\gamma_2 - 1}$$

Proof: When ρ_k is defined as in (13), there exist

$$\pi_k = \pi_0 \rho_k, \quad \sum_{k=1}^{K} \pi_{-k} = \pi_0 X, \quad \sum_{k=1}^{K} \pi_k = \pi_0 Y$$

Thus the delay is written as

$$\begin{aligned} D =& \frac{1}{\gamma_1} \sum_{k=1}^{K} \pi_{-k} \sum_{k=1}^{K} k \pi_{-k} + \frac{1}{\gamma_2} \sum_{k=1}^{K} \pi_k \sum_{k=1}^{K} k \pi_k \\ =& \pi_0^2 \left(\frac{1}{\gamma_1} X \sum_{k=1}^{K} k \rho_{-k} + \frac{1}{\gamma_2} Y \sum_{k=1}^{K} k \rho_k \right) \end{aligned} \tag{14}$$

Similarly, the transmission power cost can be written as in (9).

Since $\sum_{k=-K}^{K} \pi_k = 1$, there exists $\pi_0 \sum_{k=-K}^{K} \rho_k = 1$ as shown in (10).

When $i \ge 0$, by considering $0 \le g_i \le 1$, we have

$$\frac{\lambda_i}{\mu_{i+1}} = \frac{(1-\gamma_1)\gamma_2(1-g_{i+1})}{(1-\gamma_2)(\gamma_1 + (1-\gamma_1)g_{i+1})} \le \frac{1/\gamma_1 - 1}{1/\gamma_2 - 1} = \alpha \tag{15}$$

Similarly, when $i \le 0$, there exists

$$\frac{\mu_{i+1}}{\lambda_i} \le 1/\alpha \tag{16}$$

Therefore, we have constraints (11)(12). ∎

D. Discrete Solution of Optimal Power Delay Trade Off

In this subsection, we consider how to solve the optimal power-delay trade off problem. Since the problem **(P1)** is neither convex nor concave, it is hard to solve the optimization problem. Therefore, here we propose the discrete solution, which only considers the discrete points of $X = \Phi(k) = \sum_{i=1}^{k} 1/\alpha^k$, $-K \le k < 0$ and $Y = \Xi(k) = \sum_{i=1}^{k} \alpha^k$, $0 < k \le K$. In the following, we first present the optimal distribution of ρ_{-k} and ρ_k with given optimal X and Y, and then show that the performance degradation of this discrete solution is limited.

Lemma 3 *Assuming the optimal solution is X^* and Y^*, the corresponding values of ρ_{-k} and ρ_k are*

$$\rho_{-k} = \begin{cases} \alpha^k, & k < k_x^* \\ X^* - \Phi(k_x^* - 1), & k = k_x^* \\ 0, & k > k_x^* \end{cases} \tag{17}$$

$$\rho_k = \begin{cases} \alpha^{-k}, & k < k_y^* \\ Y^* - \Xi(k_y^* - 1), & k = k_y^* \\ 0, & k > k_y^* \end{cases} \tag{18}$$

where k_x^* satisfies $\Phi(k_x^* - 1) \le X^* < \Phi(k_x^*)$, k_y^* satisfies $\Xi(k_y^* - 1) \le Y^* < \Xi(k_y^*)$

Proof: Given X^* and Y^*, $\pi_0 = 1/(1 + X^* + Y^*)$ is also fixed, and to obtain the values of ρ_{-k} and ρ_k are equivalent to solve the following two linear programming problems

$$\min \sum_{k=1}^{K} k \rho_{-k}, \qquad \text{s.t.} \sum_{k=1}^{K} \rho_{-k} = X^* \tag{19}$$

$$\min \sum_{k=1}^{K} k \rho_k, \qquad \text{s.t.} \sum_{k=1}^{K} \rho_k = Y^* \tag{20}$$

Since the coefficients of ρ_{-k} and ρ_k increase as k, the minimum is achieved at the extreme point, where ρ_{-k} and ρ_k with k smaller than a threshold are maximized and those with k larger than the threshold are 0. Thus the solution is as presented in this lemma. ∎

Theorem 4 *The delay resulted from the discrete solution is at most larger than the optimal solution by $\max\left(\frac{k_x^*}{\gamma_2}, \frac{k_y^*}{\gamma_1}\right)$, where k_x^* and k_y^* are optimal solutions of power cost delay trade off.*

Proof: Let's define $A(k) = \sum_{i=1}^{k} i \alpha^{-k}$ and $B(k) = \sum_{i=1}^{k} i \alpha^k$, then the minimum delay is

$$\begin{aligned} D^* =& \frac{X^*(A(k_x^*-1) + k_x^*(X^* - \Phi(k_x^*-1))}{\gamma_2(1 + X^* + Y^*)^2} + \\ & \frac{Y^*(B(k_y^*-1) + k_y^*(Y^* - \Xi(k_y^*-1))}{\gamma_1(1 + X^* + Y^*)^2} \end{aligned} \tag{21}$$

In the discrete solution, we only consider the discrete ponits of X and Y, the delay with $X = \Phi(k_x^*)$ and $Y = \Xi(k_y^*)$ is

$$\tilde{D} = \frac{\Phi(k_x^*) A(k_x^*)}{\gamma_2(1 + \Phi(k_x^*) + \Xi(k_y^*))^2} + \frac{\Xi(k_y^*) B(k_y^*)}{\gamma_1(1 + \Phi(k_x^*) + \Xi(k_y^*))^2} \tag{22}$$

The difference between them is

$$\begin{aligned} \tilde{D} - D^* \le & \frac{k_x^* \pi_{k_x^*} \sum_{i=1}^{k_x^*} \pi_i}{\gamma_2} + \frac{k_y^* \pi_{-k_y^*} \sum_{i=1}^{k_y^*} \pi_{-i}}{\gamma_1} \\ \le & \max\left(\frac{k_x^*}{\gamma_2}, \frac{k_y^*}{\gamma_1}\right) \end{aligned}$$

∎

Theorem 5 *With obtained optimal ρ_k by the discrete solution, the transmission probability of uncoded packets, i.e., g_k is*

$$g_k = \begin{cases} 0, & \text{if } 0 < k \le k_x^* \text{ or } k_y^* \le k < 0, \\ 1, & \text{if } k_x^* < k \le K \text{ or } K \le k < k_y^*. \end{cases} \tag{23}$$

Proof: According to Lemma 3, the optimal values of ρ_{-k} and ρ_k of the discrete solution are

$$\rho_{-k} = \begin{cases} 1/\alpha^k, & \text{if } k_x^* \le k < 0, \\ 0, & \text{if } -K \le k < k_x^*. \end{cases} \tag{24}$$

$$\rho_k = \begin{cases} \alpha^k, & \text{if } 0 < k \le k_y^*, \\ 0, & \text{if } k_y^* < k \le K. \end{cases} \tag{25}$$

Taking into account definitions of ρ_k, γ_k, μ_k in (13)(2)(3), respectively, we get g_k presented in this theorem. ■

This theorem shows that when the numbers of packets from flow f_1 and f_2 are less than or equal to k_x^* and k_y^*, we just wait for coding opportunities. Therefore, in the following, we name the optimal k_x^* and k_y^* as stop-waiting queue lengths.

III. Performance Evaluation

In this section, we present numerical results of the proposed discrete solution for opportunistic network coding.

A. Achieved Minimum Delay

We first fix the arrival rate of flow f_2 as $\gamma_2 = 0.6$ and that of f_1 as $\gamma_1 = 0.1$, 0.3 and 0.5. The achieved minimum delay with different normalized power constraints are shown in Figure 4. We can find that the minimum of P_{max} for all γ_1 are 0.6, while the zero-delay power increases with γ_1. When the power constraint is larger than the zero-delay power constraint, the achieved minimum delay decreases with the power constraint.

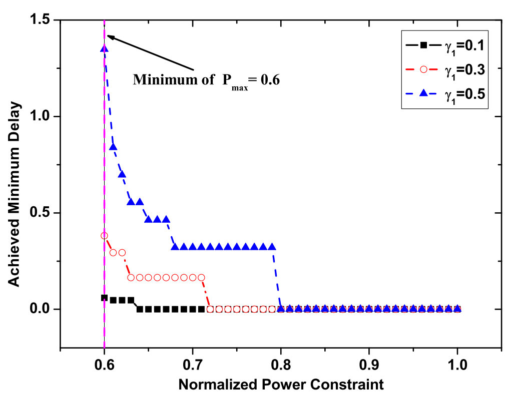

Fig. 4. Minimum delay with $\gamma_2 = 0.6$ and $\gamma_1 = 0.1$, 0.3 and 0.5.

B. Stop-Waiting Queue Lengths

The stop-waiting queue lengths of flows f_1 and f_2 are shown in Figures 5 and 6, respectively. Since the stop-waiting queue length of flow f_2 with $P_{max} = 0.60$ is much larger than those with $P_{max} \ge 0.61$, they are listed in Table I. Figures 5 and 6 show that the stop-waiting queue lengths decrease with the power constraints because the delay declines with smaller stop-waiting queue lengths. Comparing figure 5 with figure 6, we can find, with a fixed maximum power constraint, the stop-waiting queue length of flow f_2 with larger arrival rate is larger than that of f_1, which indicates that opportunistic network coding waits for the flow with larger arrival rate.

TABLE I
STOP-WAITING QUEUE LENGTHS OF FLOW f_2 WITH $P_{max} = 0.6$.

Arrival Rate of Flow f_1	0.1	0.3	0.5
Stop-Waiting Queue Length of Flow f_2	14	29	85

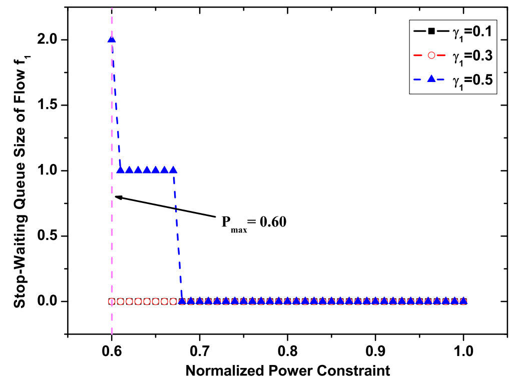

Fig. 5. Stop-waiting queue lengths of flow f_1 with $\gamma_2 = 0.6$ and $\gamma_1 = 0.1$, 0.3 and 0.5.

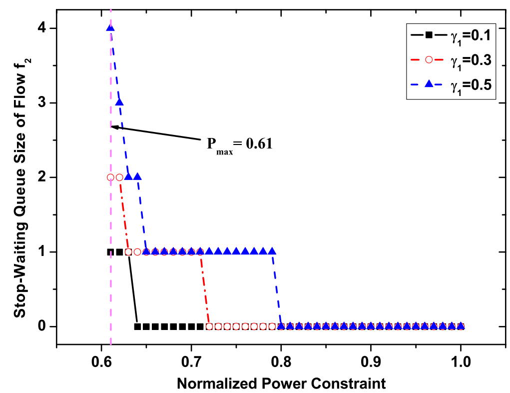

Fig. 6. Stop-waiting queue lengths of flow f_2 with $\gamma_2 = 0.6$ and $\gamma_1 = 0.1$, 0.3 and 0.5.

IV. Conclusion

In this paper, we have analyzed the optimal power cost delay trade off of opportunistic network coding in the two-way relay network with asymmetric arrival rates. We also proposed an approximated discrete solution to solve the optimization problem and evaluated its performance. Results show that with higher transmission power constraints, we can achieve lower delay and smaller stop-waiting queue lengths.

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