Unbiased Power Prediction of Rayleigh Fading Channels

Torbjörn Ekman
UniK
PO Box 70, N-2027 Kjeller, Norway
Email: torbjorn.ekman@signal.uu.se

Mikael Sternad and Anders Ahlén
Signals and Systems, Uppsala University
PO Box 528, SE-751 20 Uppsala, Sweden
Email: mikael.sternad@signal.uu.se , anders.ahlen@signal.uu.se

***Abstract*— Prediction of the rapidly fading envelope of a mobile radio channel enables a number of capacity improving techniques like fast resource allocation or fast adaptive modulation. Some power prediction algorithms are based on linear prediction of the taps of the complex impulse response and then forming the power prediction as the sum of the absolute square of the prediction of the taps in the impulse response. This will render a biased power predictor, that generally underestimates the power. We propose a bias compensated power predictor and derive the optimal prediction coefficients for the Rayleigh fading channel. The importance of efficient noise reduction of channel samples that are used as inputs to the predictor is also emphasized. A performance evaluation of the prediction algorithm is carried out on measured broadband mobile radio channels.**

I. Introduction

Prediction of the coefficients of mobile radio channel is of interest for a range of applications such as power control, adaptive resource allocation, as well as adaptive coding and modulation [1], [2]. Power control in e.g. WCDMA requires short-term prediction over only a small fraction of the distance between two dips in the short-term fading pattern. Radio resource allocation and planning would require accurate and more long-term prediction, the longer the better. Different methods have been proposed to cope with this problem [3], [4].

Here we will predict the received power of a mobile radio channel by prediction of the individual complex taps in the channel impulse response. The power can then be predicted by adding the squared magnitudes of the predicted taps as in [3]. That will, however, give a biased power predictor, which underestimates the power. We propose a compensation for the bias that improves the power prediction performance.

Estimated snapshots of the channel impulse response are used as input to a set of channel tap predictors. The SNR of these samples is improved by using Wiener smoothers, which improves the total performance. Our approach to prediction of the received power will be evaluated on measured broadband data.

II. The Measured Channels

A. The Estimation Method

Measurements obtained by Ericsson Research in urban and suburban areas, at the carrier frequency 1880 MHz and with a 6.4 MHz baseband sampling rate, have been utilized. The sampling rate of the channel is 9.14 kHz whereas the highest Doppler frequency is about 160 Hz. Thus we have a highly oversampled channel [6].

The channel can be modeled as a time varying FIR-filter, with taps denoted by $h_k(t)$. Let n denote a discrete time index at the symbol rate. For each measurement location, the time-varying complex-valued channel taps $h_k(n)$ in

$$s(n) = \sum_{k=1}^{n_k} h_k(n)u(n-k) + w(n) \qquad (1)$$

were estimated. Here, $s(n)$ is the received baseband signal sampled at 6.4 MHz, $u(n)$ is a known filtered transmitted sequence, obtained from back-to-back measurements of the receiver connected to the transmitter, and $w(n)$ is noise.

A channel length $n_k = 120$, corresponding to a time span of 18.75 μs, was found to be sufficient at all measurement locations. The channel was assumed time-invariant over blocks of 700 samples (channel sampling period $T = 109.4$ μs ≈ 0.01 wavelength), an assumption that introduces negligible errors at these fading rates [5]. Block least squares estimates of the channel taps were calculated, resulting in 1430 channel samples, indicated by the time index t,

$$\bar{h}_k(t) = h_k(t) + v_k(t)\ ;\ k = 1, \ldots, 120\ ;\ t = 1, \ldots, 1430$$

covering 156 ms at each location. Here, $h_k(t)$ denote the noise-free tap samples, while the estimation error $v_k(t)$ is zero mean noise that is uncorrelated with $h_k(t)$.

B. Noise Reduction

We exploit the oversampling of the channel to reduce the noise with an FIR Wiener-smoother. The covariances of the channel taps $h_k(t)$ are in the filter design represented by a zero order Bessel function of the first kind (Jakes model) and

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the noise $v_k(t)$ is assumed to be white. The Jakes model is a good description of the average short term fading of the channel taps, although the different realizations will deviate significantly from it. For channel taps $h_k(t)$ estimated by a least squares procedure, here based on blocks of 700 samples, the estimation error $v_k(t)$ will be close to white when considering consecutive snapshots.

For a given smoothing lag and filter order, a FIR Wiener smoother is parameterized by the SNR and the Doppler frequency. These entities can be estimated from the power delay profile and the average Doppler spectrum [6].

In the measurements at hand, the Doppler frequency normalized by the channel sampling frequency is on the order of 10 mHz. This corresponds to 100 samples per traveled distance measured in wavelengths. Smoothing (use of future channel samples) provides much better noise reduction performance than filtering only. The performance improves with an increasing smoothing lag. When using a smoother with 128 coefficients on a Jakes model with 10 mHz maximal Doppler shift, almost full performance is obtained already at a smoothing lag of 10 samples and the performance saturates at a smoothing lag of about 20 samples.[1]

The use of a smoothing lag in the noise reduction introduces a corresponding delay in the data used by the predictor. This will affect the attainable predictor performance. To circumvent this problem, a bank of noise reduction smoothers are used, that have smoothing lags from zero up to a lag that is sufficient to obtain close to optimal noise reduction. Here, five smoothers with lags 0, 2, 5, 10 and 20 are used. The channel predictor, described in Section 3.3 below, can then always use regressor variables with appropriate time lags. These regressors have been given the best, at that time, available noise reduction.

1) The Wiener smoother: The correlation function for the channel in the Jakes model is given by the zero order Bessel-function of the first kind,

$$r_J(\tau) = \mathrm{J}_0(2\pi f_D \tau) \,, \tag{2}$$

where f_D is the Doppler frequency.

A FIR Wiener smoother with N coefficients and smoothing lag ℓ has the coefficient vector

$$\mathbf{w}^{[\ell]} = (\mathbf{R}_J + \mathbf{I}\sigma_e^2)^{-1}\mathbf{r}_J^{[\ell]} \,, \tag{3}$$

where $\mathbf{R}_J$ is the covariance matrix for a Jakes model channel, $\mathbf{r}_J^{[\ell]} = [r_J(-\ell) \dots r_J(N-1-\ell)]^T$ while σ_e^2 is the variance of the channel estimation error normalized by the channel tap variance. The FIR-Wiener smoothing estimate with lag ℓ is thus

$$\tilde{h}_k(t-\ell|t) = \bar{\boldsymbol{\varphi}}_k^H(t)\mathbf{w}^{[\ell]} \,, \tag{4}$$

[1]The performance of the FIR-smoother is similar to the IIR-smoother proposed in [4]. This FIR-smoother needs on the order of hundred of coefficients whereas the IIR-smoother needs only on the order of ten coefficients. Still, the FIR-smoother is here preferred due to numerical sensitivity of the coefficients in the IIR-smoother.

where $\bar{\boldsymbol{\varphi}}_k^H(t)$ is a vector with current and delayed samples of the estimated channel

$$\bar{\boldsymbol{\varphi}}_k(t) = \left[\bar{h}_k(t) \;\; \bar{h}_k(t-1) \dots \bar{h}_k(t-N+1))\right]^H \,. \tag{5}$$

III. Channel Predictors

We first discuss prediction of complex channel taps $\bar{h}_k(t)$. In the considered measurements, different taps are only weakly correlated, so they will be predicted individually. We then consider tap power predictors that are quadratic functions of the noise reduced taps.

A. Linear Prediction of Complex Taps

Even though the mobile radio channel changes dramatically over short traveled distances, the parameterization of the amplitude and phase is slowly time varying. We can thus assume that the optimal linear predictor coefficients change slowly and without abrupt changes. In this section, we study the performance of a linear FIR-predictor for prediction of a complex time series with stationary statistics. This is a valid approximation for a block based method using channel statistics collected over a few meters.

Consider a flat Rayleigh fading channel. It can be modeled as a correlated complex circular Gaussian stochastic variable with zero mean, that is here denoted $x(t)$ instead of $h_k(t)$. Furthermore, assume the channel estimation error, here denoted $v(t)$, to be circular complex Gaussian distributed with zero mean.

In a vector formulation of the FIR-predictor, the signal (tap) can be predicted from past noisy observations as

$$\hat{x}(t+L|t) = \boldsymbol{\varphi}^H(t)\boldsymbol{\theta} \,, \tag{6}$$

where $\boldsymbol{\theta}$ is a vector with prediction coefficients and the components of the regressor $\boldsymbol{\varphi}(t)$ consist of smoothed observations of $x(t)+v(t)$ with different delays. The components of $\boldsymbol{\varphi}(t)$ will thus also be complex circular Gaussian stochastic variables. From the Wiener-Hopf equations, the MSE of a predictor like (6) is minimized by $\boldsymbol{\theta} = \boldsymbol{\theta}_c$ with

$$\boldsymbol{\theta}_c = \mathbf{R}_\varphi^{-1}\mathbf{r}_{x\varphi} \,, \tag{7}$$

where $\mathbf{r}_{x\varphi} = \mathrm{E}\{x(t)\boldsymbol{\varphi}(t-L)\}$ is the cross-correlation between the signal and the regressor, and $\mathbf{R}_\varphi = \mathrm{E}\{\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^H(t)\}$ is the correlation matrix for the regressors. With $r_x = E|x(t)|^2$ the minimal prediction MSE is

$$\mathrm{E}|x(t+L) - \hat{x}(t+L|t)|^2 = r_x - \boldsymbol{\theta}_c^H\mathbf{R}_\varphi\boldsymbol{\theta}_c \,. \tag{8}$$

B. Quadratic Power Prediction

1) Biased power prediction: From past and present noisy observations, the power of $x(t)$ at time $t+L$, that is $|x(t+L)|^2$, is to be predicted.

We first examine the use of $|\hat{x}(t+L|t)|^2$, with $\hat{x}(t+L|t)$ given by (6), as a power predictor,

$$\hat{p}_b(t+L|t) = |\hat{x}(t+L|t)|^2 = \boldsymbol{\theta}^H \boldsymbol{\varphi}(t)\boldsymbol{\varphi}^H(t)\boldsymbol{\theta} \ . \quad (9)$$

This predictor utilizes quadratic functions of the regressor variables and is thus a nonlinear predictor. This predictor for the power is used for channel prediction in e.g. [3] and [4]. For any $\boldsymbol{\theta}$, the average power prediction error will be

$$\mathrm{E}\{\varepsilon_{p_b}(t)\} = \mathrm{E}\{|x(t)|^2 - |\hat{x}(t|t-L)|^2\} = r_x - \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta}, \quad (10)$$

where $r_x = \mathrm{E}\{|x(t)|^2\}$ is the variance (power) of the channel. The absolute square of the complex prediction is thus a *biased power predictor*. With $\boldsymbol{\theta} = \boldsymbol{\theta}_c$, this bias becomes equal to the variance (8) of the optimized complex prediction error. The power of $x(t)$ is thus underestimated.

2) Unbiased power prediction: To avoid underestimation of the power, we propose an unbiased power predictor, that modifies (9) with a compensation for the bias, as

$$\hat{p}(t+L|t) = |\hat{x}(t+L|t)|^2 + r_x - \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta} \ . \quad (11)$$

The prediction error $p(t) - \hat{p}(t|t-L)$ will then have zero mean. The modification is especially noticeable when the complex channel is hard to predict, i.e. when the optimal $\boldsymbol{\theta} \to 0$. The unbiased prediction then becomes the average power r_x, whereas the biased predictor will predict zero power. The proposed unbiased power predictor thus offers a graceful transition from perfectly predicted channel states to situations where only the average power is known.

C. Unbiased Power Predictors for Frequency Selective Channels

Tools from the preceding sections can now be combined. The linear regression

$$\hat{h}_k(t+L|t) = \boldsymbol{\varphi}_k^H(t)\boldsymbol{\theta}_k \ , \quad (12)$$

represents an L step ahead FIR-prediction of a channel tap $h_k(t)$. Here, $\boldsymbol{\theta}_k$ is a vector of predictor coefficients. The regression vector $\boldsymbol{\varphi}_k(t)$ consists of smoothed samples (4) of tap k up to time t, with a sub-sampling factor of m, where the largest available smoothing lag is used:

$$\boldsymbol{\varphi}_k(t) = \left[\tilde{h}_k(t|t) \ \ \tilde{h}_k(t-m|t) \ldots \tilde{h}_k(t-(M-1)m|t)\right]^H . \quad (13)$$

Here M denotes the order of the predictor (12). If $(M-1)m$ is larger than the largest available smoothing lag provided by the bank of smoothers, then the corresponding smoothed signal with the largest smoothing lag is used instead.

The unbiased power predictor (11) is then given by

$$\hat{p}_k(t+L|t) = |\hat{h}_k(t+L|t)|^2 + r_{h_k} - \boldsymbol{\theta}_k^H \mathbf{R}_{\varphi_k} \boldsymbol{\theta}_k \ , \quad (14)$$

where $r_{h_k} = \mathrm{E}\{|h_k(t)|^2\}$ and $\hat{h}_k(t+L|t)$ is given by (12).

Each tap of a frequency selective channel can be regarded as a flat fading channel. For a Rayleigh fading tap, that is $h_k(t) \sim \mathbf{CN}(0, r_{h_k})$, an optimal choice of $\boldsymbol{\theta}_k$ is then given in Appendix A as

$$\boldsymbol{\theta}_k = \mathbf{R}_{\varphi_k}^{-1} \mathbf{r}_{h_k \varphi_k} \ , \quad (15)$$

where $\mathbf{R}_{\varphi_k}$ is the covariance matrix for the regressor to tap k and $\mathbf{r}_{h_k\varphi_k} = \mathrm{E}\{h_k(t)\boldsymbol{\varphi}_k(t)\}$. Somewhat remarkably, this is the same adjustment as (7), which optimizes the L-step prediction of the complex tap.[2]

An unbiased predictor for the total power is finally obtained by summing contributions from each significant tap

$$\hat{P}(t+L|t) = \sum_{k=1}^{n_k} \left(|\hat{h}_k(t+L|t)|^2 + r_{h_k} - \boldsymbol{\theta}_k^H \mathbf{R}_{\varphi_k} \boldsymbol{\theta}_k \right) . \quad (16)$$

If all channel taps are unpredictable, then the channel power will be predicted by its average $\sum_k r_{h_k}$. In receivers that use only a subset of the taps, the available power is the sum over this subset. The predicted available power is then the sum over the predictions for this subset, whereas the other taps act as interference.

IV. Results

A. Simulations

The expected performance for prediction of a flat Rayleigh fading channel with Jakes spectrum will indicate the performance on true measured channels. We thus first examine the Jakes model to obtain reasonable choices for the delay spacing (the sub-sampling factor m measured in wavelengths) in predictors for one tap. The channel correlation is then given by, $r_x(\tau) = r_x \mathrm{J}_0(2\pi f_D \tau)$. The delay spacing and the prediction range are measured in wavelengths, as fading is a spatial phenomenon. The time-frequency product $f_D\tau = v\tau/\lambda$, where v is the speed of the mobile, is the traveled distance measured in wavelengths.

For a channel corrupted by white noise, the theoretically obtainable prediction NMSE can be calculated, given the prediction range, the order of the predictor and the delay spacing. This can also be done for a predictor using smoothed regressors. Our finding is that when using the smoothed regressors the performance is rather insensitive to the choice of delay spacing, as long as it is in the range of 10 to 20 samples per traveled wavelength.

Figure 1 shows how predictable a Jakes channel is with and without smoothed regressors. The predictor has eight coefficients and a robust choice of delay spacing, that gives close to optimal performance over SNRs $E|h_k(t)|^2/Ev_k(t)^2$ ranging from 10 to 50 dB.

An increase of the number of coefficients in the predictor results in only a minor decrease in the NMSE, see Figure 2. The major gain is attained by the use of noise reduction.

[2] All $\boldsymbol{\theta}_k$ that satisfy $\boldsymbol{\theta}_k = e^{i\omega}\boldsymbol{\theta}_c$, with $\omega \in [0\ 2\pi]$, are solutions, since the phase becomes irrelevant when the absolute square is taken.

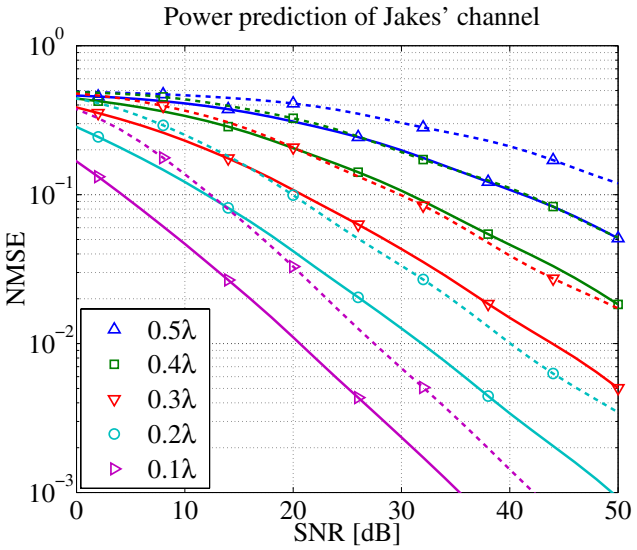

Fig. 1. The prediction NMSE of an unbiased quadratic power predictor with 8 coefficients, for a Jakes channel as a function of SNR. Solid lines use smoothed regressors, while dashed lines use noisy regressors. The uppermost curves corresponds to a prediction range L of 0.5 wavelengths. The prediction range decreases to 0.1 for the two consecutive lowest curves.

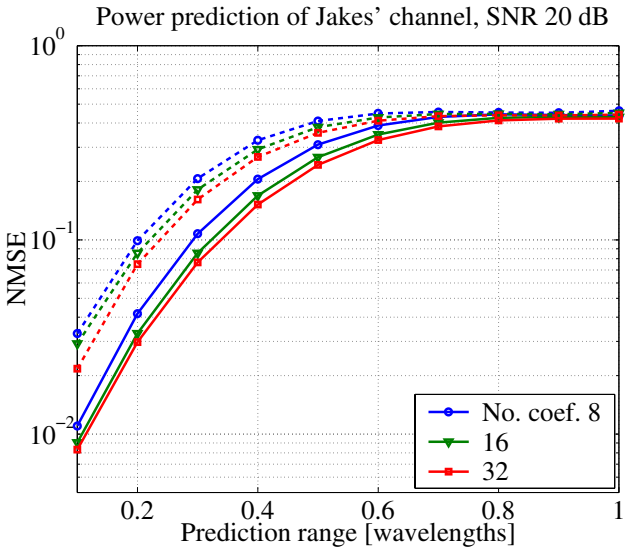

Fig. 2. The prediction NMSE at 20dB SNR of an unbiased quadratic power predictor for a Jakes channel as a function of the prediction range. Solid and dashed lines are using smoothed and noisy regressors respectively. The uppermost curve corresponds to a predictor with 8 coefficients, the next to 16 and the lowest to 32 coefficients.

B. Power Prediction on Measured Channels

As only noisy observations of the channel taps are available, the true tap is taken as the smoothed tap obtained with smoothing lag 20. This is a good approximation of the true taps for SNRs above 0 dB. A total of 37 measurement locations are used in the evaluation. Only measurements where the average channel-to-estimation-error power ratio is above 10 dB are used.

The algorithms are designed for channels with stationary statistics. If abrupt changes are detected new prediction parameters would have to be estimated. To avoid such situations in the evaluation, a few channels where the average power changed by more than 3 dB from the first quarter of the data to the last quarter, were ruled out.

1) Quadratic power prediction: For each tap the estimated sample covariances $\hat{\mathbf{R}}_{\varphi_k}$ and $\hat{\mathbf{r}}_{h_k\varphi_k}$ are used to obtain the prediction coefficients (15) of the unbiased power predictor (14). The prediction of the total power is obtained as in (16). The same holds for the biased power predictor (9) for comparison.

Independent of the prediction range, 900 samples are used in the target vector for the training. The validation interval was on the order of 400 down to 100 samples, depending on prediction range and Doppler frequency.

The delay spacing was taken as the robust choice, as obtained from simulations using the Jakes model, depending on the Doppler frequency but not on the SNR. A predictor using eight coefficients was used, as simulations show that little can be gained by increasing the number of coefficient above this value if noise reduction is performed in a separate step, cf Figure 2.

2) Average and last sample prediction: The two simplest power predictors, the average power and the last sample predictor, are also included in the evaluation. The average power is estimated using the same training interval as for the more advanced predictors and it is evaluated on the same validation interval.

3) Performance: The average NMSE of the power prediction algorithms in Figure 3 shows that the unbiased power predictor provides the best performance for all prediction horizons measured over the validation set. As the horizon L increases, its NMSE slowly approaches that of the average predictor from below. The biased power predictions and the last sample predictions become slightly worse than using the average power at large prediction ranges.

V. Conclusions

Using smoothed noise reduced regressors result in a significant increase of the prediction performance on channels described by the Jakes model. The combined effect of improved noise reduction that eliminates the lag in the regressors and unbiased power estimation has resulted in significant performance improvements, as compared to our previous results reported in [4]. The proposed unbiased power predictor offers a graceful transition from perfect channel state information to just knowledge of the average power. This is demonstrated in evaluations of the algorithm on measured channels.

Appendix

For convenience, the time index is dropped for $\hat{x}(t|t-L)$, $x(t)$ and $\boldsymbol{\varphi}(t)$. The covariance of $x(t)$ is $\mathrm{E}\{|x|^2\} = r_x$ and the

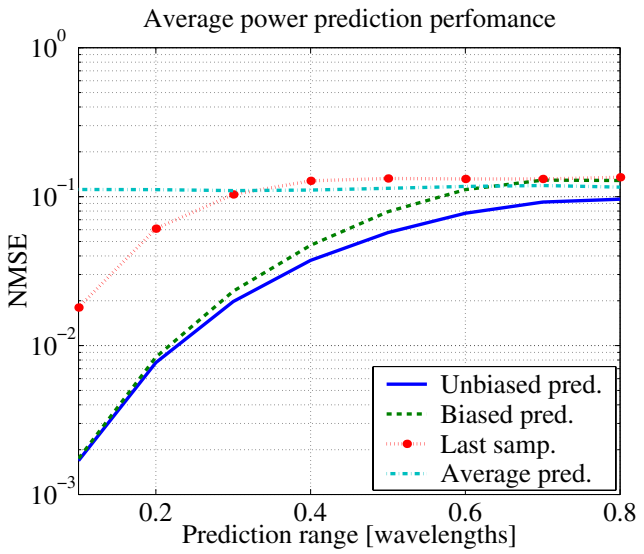

Fig. 3. The average power prediction normalized MSE evaluated at 37 measurement locations.

covariance for the complex prediction is

$$r_{\hat{x}} = \mathrm{E}\{|\hat{x}|^2\} = \mathrm{E}\{\boldsymbol{\theta}^H \boldsymbol{\varphi}\boldsymbol{\varphi}^H \boldsymbol{\theta}\} = \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta} \,. \quad (17)$$

The cross-correlation between x and the prediction $\hat{x}$ is

$$r_{x\hat{x}} = \mathrm{E}\{x\hat{x}^*\} = \mathrm{E}\{\boldsymbol{\theta}^H \boldsymbol{\varphi} x\} = \boldsymbol{\theta}^H \mathbf{r}_{x\varphi} \,. \quad (18)$$

When using the biased power predictor (9) the MSE for the power prediction error in (10) becomes

$$\begin{aligned} \mathrm{E}\{|\varepsilon_{p_b}|^2\} &= \mathrm{E}\{(|x|^2 - |\hat{x}|^2)^2\} \\ &= \mathrm{E}\{|x|^4 - 2|x|^2|\hat{x}|^2 + |\hat{x}|^4\} \,. \end{aligned} \quad (19)$$

Examine the three terms of (19) separately. As both x and $\hat{x}$ are circular complex Gaussian the first and third terms are $2r_x^2(0)$ and $2r_{\hat{x}}^2(0)$ respectively. For the same reason we can express the second term as

$$\begin{aligned} \mathrm{E}\{|x|^2|\hat{x}|^2\} &= \mathrm{E}\{|x|^2\}\mathrm{E}\{|\hat{x}|^2\} + \mathrm{E}\{x\hat{x}^*\}\mathrm{E}\{x^*\hat{x}\} \\ &= r_x r_{\hat{x}} + |r_{x\hat{x}}|^2 \,. \end{aligned} \quad (20)$$

The MSE (19) for the biased power predictor is thus

$$\begin{aligned} &\mathrm{E}\{|\varepsilon_{p_b}|^2\} = \\ &\quad 2r_x^2 - 2\boldsymbol{\theta}^H \mathbf{r}_{x\varphi}\mathbf{r}_{x\varphi}^H \boldsymbol{\theta} - 2r_x \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta} + 2|\boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta}|^2 . \end{aligned} \quad (21)$$

This is the criterion that should be minimized to obtain the coefficient vector $\boldsymbol{\theta}$ that provides the best power predictor of the type (9).

The MSE for the unbiased power predictor is obtained from (21) by subtracting the square of the bias,

$$\begin{aligned} \sigma_{\varepsilon_p}^2 = \mathrm{E}\{|\varepsilon_p|^2\} &= \mathrm{E}\{(|x|^2 - |\hat{x}|^2)^2\} - |r_x - \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta}|^2 \\ &= r_x^2 - 2\boldsymbol{\theta}^H \mathbf{r}_{x\varphi}\mathbf{r}_{x\varphi}^H \boldsymbol{\theta} + |\boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta}|^2 \end{aligned} \quad (22)$$

A. Optimal coefficients

To find the $\boldsymbol{\theta}$ that minimizes $\sigma_{\varepsilon_p}^2$ in (22), we set the partial derivative of the MSE with respect to $\boldsymbol{\theta}$ equal to zero

$$\begin{aligned} 0 &= \frac{\partial \sigma_{\varepsilon_p}^2}{\partial \boldsymbol{\theta}} \\ &= -2\boldsymbol{\theta}^H \mathbf{r}_{x\varphi}\mathbf{r}_{x\varphi} + \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta}\boldsymbol{\theta}^H \mathbf{R}_\varphi + \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta}^H \mathbf{R}_\varphi \boldsymbol{\theta} \,, \end{aligned} \quad (23)$$

which result in the equation

$$\boldsymbol{\theta}^H (\mathbf{r}_{x\varphi}\mathbf{r}_{x\varphi}^H - \mathbf{R}_\varphi \boldsymbol{\theta}\boldsymbol{\theta}^H \mathbf{R}_\varphi) = 0 \,. \quad (24)$$

Here, $\boldsymbol{\theta} = 0$ is the trivial solution. The other solutions are given by

$$\boldsymbol{\theta}\boldsymbol{\theta}^H = \mathbf{R}_\varphi^{-1} \mathbf{r}_{x\varphi}\mathbf{r}_{x\varphi}^H \mathbf{R}_\varphi^{-1} \,, \quad (25)$$

with one obvious solution as $\boldsymbol{\theta} = \boldsymbol{\theta}_p$, where

$$\boldsymbol{\theta}_p = \mathbf{R}_\varphi^{-1} \mathbf{r}_{x\varphi} \,. \quad (26)$$

These are the same coefficients that are optimal for the complex prediction (6).

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