ROBUST WIENER DESIGN OF ADAPTATION LAWS WITH CONSTANT GAINS

Mikael Sternad*, Lars Lindbom†, and Anders Ahlén*

*Signals and Systems, Uppsala University, PO Box 528, SE-75120, Uppsala, Sweden.
†Ericsson Infotech, PO Box 1038, SE-65115 Karlstad

mikael.sternad@signal.uu.se (corresponding author), fax +46 18 555096

Abstract: Filters can be introduced into LMS-like adaptation algorithms to improve their tracking performance. This paper discusses the systematic model-based design of such filters. Parameter variations in coefficients of linear regression models are modeled as ARIMA-processes. The aim is to provide high performance filtering, prediction or fixed lag smoothing estimates for arbitrary lags. The properties of the time-varying parameters are in general not known exactly, so a robust design for a set of possible models will be of interest. We minimize the average tracking MSE, based on probabilistic descriptions of the model uncertainty. The method is based on a novel signal transformation that recasts the algorithm design into a robust Wiener filtering problem. The performance is illustrated on the tracking of mobile radio channels in IS-136 systems, based on a model of the time-variations affected by parametric uncertainty.

1. INTRODUCTION

Algorithms that estimate time-varying parameters of models and filters in linear regression form are important tools for signal processing, control and digital communication. Kalman estimators are the optimal linear algorithms when the statistics of the parameter variations are known. However, their complexity is often unacceptable in high-speed applications.

The required low complexity of channel estimators in mobile radio systems has motivated us to develop a class of adaptation laws that for channel tracking attain close to the optimal Kalman performance, at a computational complexity close to that of LMS algorithms (Lindbom 1995, Sternad et al. 2000, Lindbom et al. 2000a). The approach also opens up new ways of analyzing adaptation laws for fast variations (Ahlén et al. 2000), that provide accurate estimates of the parameter error variance. These and related algorithms can, for example, be applied effectively on the fading 1900MHz channels of North American IS-136 mobile radio systems (Lindbom et al. 2000b). Design of a related class of algorithms has been investigated by Benveniste et al. (1990) for slowly varying parameters. Section 4 and 5 outline an iterative Wiener design that is effective also for fast variations.

A model-based Wiener design may become sensitive to the assumed model. We here propose a method for decreasing this sensitivity. Using tools from Sternad and Ahlén (1993) and Öhrn et al. (1995), the design equations are in Section 6 modified to minimize the average tracking MSE, based on probabilistic descriptions of the uncertainty in the parameter models. In Section 7, time-varying mobile radio channels in IS-136 systems are estimated. The model of the time-variations is there affected by parametric uncertainty in the Doppler frequency and the robust design is performed by using an averaged covariance function.

Notation: Here, $R(q^{-1})$, $R(q^{-1})$ and $\mathcal{R}(q^{-1})$ denote polynomials, polynomial matrices and causal rational matrices, respectively. Conjugate matrices $P_q(q)$ or $\mathcal{R}_q(q)$ are obtained by conjugating complex coefficients, transposing and substituting the forward shift operator $q$ for the backward shift operator $q^{-1}$.

2. OUTLINE OF THE PROBLEM

A sequence of measurement vectors $\{y_t\}$ of dimension $n_y$ is assumed available at the discrete time instants $t = 0, 1, 2, ...$ and to be generated by a linear regression

$$y_t = \phi_t^* h_t + v_t \tag{1}$$

where $v_t$ is a noise vector that is uncorrelated with the $n_y/n_h$ regression matrix $\phi_t^*$. We assume the possibly complex-valued regressors to be known at time $t$ and to be persistently exciting, so that

$$R \overset{\Delta}{=} \mathbb{E} \{\phi_t \phi_t^*\} \tag{2}$$

is nonsingular. The covariance matrix $R$ will here be assumed time-invariant, but it can in practice be slowly varying. The time-varying parameter vector

$$h_t = (h_{0,t} \ldots h_{n_h-1,t})^T \tag{3}$$

is to be estimated, with the order $n_h$ assumed known. Models describing the variation of $h_t$ are sometimes called hypermodels (Benveniste et al. 1990). We here use linear time-invariant stochastic models

$$h_t = \mathcal{H}(q^{-1}) e_t \tag{4}$$

where \( e_t \) is white noise with covariance matrix \( \mathbf{R}_e \) and where \( H(q^{-1}) \) is an \( n_h \times n_a \) matrix of stable or marginally stable transfer operators. The model \( (4) \) is for now assumed known, but will in Section 6 be assumed uncertain. Denote the tracking error by

\[
\hat{h}_{t+k|t} \triangleq h_{t+k} - \hat{h}_{t+k|t}
\]

(5)

where the estimate \( \hat{h}_{t+k|t} \) may be obtained by filtering \( (k = 0) \), prediction \( (k > 0) \) or fixed lag smoothing \( (k < 0) \). Kalman estimators, based on \( (1) \) and on state-space realizations of \( (4) \), are the linear estimators that minimize the tracking covariance matrix

\[
\mathbf{P}_{k,t} \triangleq \mathbb{E} \{ \hat{h}_{t+k|t} \hat{h}_{t+k|t}^* \} .
\]

(6)

Since \( \varphi_t^* \) in \( (1) \) is time-varying, the Kalman gains will not converge as \( t \to \infty \), so on-line Riccati updates are required.

We here consider a class of adaptation laws that avoid on-line Riccati updates. Instead, pre-designed linear time-invariant filters \( \mathbf{M}_k(q^{-1}) \) operate on the negative instantaneous gradient of \( |\varepsilon_t|^2 \) with respect to \( \hat{h}_{t|t-1} \),

\[
\varepsilon_t = y_t - \varphi_t^* \hat{h}_{t|t-1}
\]

(7)

\[
\hat{h}_{t+k|t} = \mathbf{M}_k(q^{-1}) \varphi_t \varepsilon_t .
\]

(8)

The LMS algorithm

\[
\hat{h}_{t+1|t} = \frac{\mu}{1-q^{-1}} \mathbf{I} \varphi_t \varepsilon_t ,
\]

(9)

where \( \mu > 0 \) is a scalar gain, constitutes a simple special case of the structure \( (7),(8) \). The rational matrix \( \mathbf{M}_k \) can be selected to asymptotically minimize \( (6) \) under various constraints and assumptions.

3. THE LOOP TRANSFORMATION

The algorithm \( (7),(8) \) can be expressed as a stable and causal filter, denoted the learning filter \( \mathcal{L}_k(q^{-1}) \), that operates on a signal vector

\[
f_t \triangleq \varphi_t \varepsilon_t + \mathbf{R} \hat{h}_{t|t-1}
\]

(10)

Since \( (7),(8) \) give

\[
\hat{h}_{t|t-1} = q^{-1} \mathbf{M}_t(q^{-1}) \varphi_t \varepsilon_t ,
\]

\[
\varphi_t \varepsilon_t = (\mathbf{I} + q^{-1} \mathbf{R} \mathbf{M}_t(q^{-1}))^{-1} f_t .
\]

Thus, we obtain

\[
\hat{h}_{t+k|t} = \mathbf{M}_k(q^{-1})(\mathbf{I} + q^{-1} \mathbf{R} \mathbf{M}_t(q^{-1}))^{-1} f_t \triangleq \mathcal{L}_k(q^{-1}) f_t .
\]

(11)

By \( (7) \) and \( (1) \),

\[
\varphi_t \varepsilon_t = \varphi_t \varphi_t^* \hat{h}_{t|t-1} + \varphi_t v_t .
\]

(12)

Adding and subtracting \( \mathbf{R} \hat{h}_{t|t-1} \) on the right-hand side of \( (12) \) gives

\[
\varphi_t \varepsilon_t = \mathbf{R} \hat{h}_t - \mathbf{R} \hat{h}_{t|t-1} + (\varphi_t \varphi_t^* - \mathbf{R}) \hat{h}_{t|t-1} + \varphi_t v_t .
\]

(13)

We now define

\[
Z_t \triangleq \varphi_t \varphi_t^* - \mathbf{R}
\]

(14)

\[
\eta_t \triangleq Z_t \hat{h}_{t|t-1} + \varphi_t v_t
\]

(15)

which we call the autocorrelation matrix noise and the gradient noise, respectively. The signal \( f_t \) which can be regarded as a fictitious measurement, can then by using \( (10),(13),(14) \) and \( (15) \), be expressed as

\[
f_t = \mathbf{R} \hat{h}_t + Z_t \hat{h}_{t|t-1} + \varphi_t v_t = \mathbf{R} \hat{h}_t + \eta_t ,
\]

(16)

see Fig. 1. The design of our adaptation law \( (7),(8) \) has now been transformed into a Wiener filter design for \( \mathcal{L}_k(q^{-1}) \), where \( \eta_t \) plays the role of noise, see Fig. 2.

![Figure 1: The adaptation algorithm (8) operates in closed loop. This loop can be decomposed into an inner time-invariant feedback of \( \mathbf{R} \hat{h}_{t|t-1} \), and an outer time-varying loop via the feedback noise \( Z_t \hat{h}_{t|t-1} \).](image)

**Figure 2:** The filter design problem. The vector \( \hat{h}_{t+k} \) is to be estimated from \( f_t \), such that the steady state tracking error covariance matrix of the parameter error \( \hat{h}_{t+k|t} \) is minimized.

The gradient noise \( \eta_t \) is affected by the term \( Z_t \hat{h}_{t|t-1} \), here called the feedback noise. It is shown in Ahlen et al. (2000) that the feedback noise is negligible either when \( \hat{h}_t \) has small increments or when the noise \( v_t \) has high variance. Such situations are denoted “slow variations” (Ahlen et al., 2000; Macchi, 1995). The optimal learning filter will then operate in open loop, with \( \eta_t \approx \varphi_t v_t \). Stability and convergence in MSE is then guaranteed by stability of the learning filter, which follows directly from a Wiener design. (While the learning filter \( \mathcal{L}_k(q^{-1}) \) must be stable, the filter \( \mathbf{M}_t(q^{-1}) \) need not be stable, since it works within the inner feedback loop of Fig. 1.) An iterative design must be performed when \( Z_t \hat{h}_{t|t-1} \) cannot be neglected, see Section 5.

4. LEARNING FILTER OPTIMIZATION

The criterion \( (6) \) for \( t \to \infty \) could be minimized directly by adjusting \( \mathcal{L}_k(q^{-1}) \), if \( H(q^{-1}) \) in \( (4) \) and the
properties of $\eta_t$ were known exactly. Using a polynomial approach to Wiener filtering, the learning filter is here designed under the constraint of stability, and under the following assumptions.

**Assumption A1:** The sequence $\{\varphi_t^*\}$ is stationary and known, with $R$ known and nonsingular.

**Assumption A2:** The gradient noise $\eta_t$ is white and stationary with zero mean and known covariance matrix $R_\eta$. The correlation of $\eta_t$ with $h_{t-i}$ and with $\hat{h}_{t+1}\mid t-1$, $i \geq 0$ is negligible.

**Assumption A3:** The time-varying parameters are described by a known vector-ARIMA process

$$D(q^{-1})h_t = C(q^{-1})e_t,$$  \hspace{1cm} (17)

with $R_e = E e_t e_t^*$ nonsingular, where $D(q^{-1}) = D_v(q^{-1})D_s(q^{-1})$. Moreover, $C(q^{-1})$ and $D_s(q^{-1})$ are monic and stably invertible, while the polynomial $D_u(q^{-1})$ has zeros on the unit circle.

Assumption A3 implies that e.g. random walks, integrated random walks and filtered random walk models can be considered, but that the unstable dynamics $D_u(q^{-1})$ must then affect all the elements of $h_t$. We can now present the optimal learning filter.

**Theorem 1:** Under Assumptions A1-A3, the stable and causal learning filter minimizing the asymptotic parameter covariance matrix (6) is

$$\hat{h}_{t+1}\mid t = L^p_k f_t = D_s^{-1} Q_k \beta^{-1} D_s R^{-1} f_t,$$  \hspace{1cm} (18)

where the polynomial matrix $\beta(q^{-1})$ of dimension $n_h \times n_h$ and degree $n_\beta = \max(n_c, n_d)$ is the stable left spectral factor obtained from

$$\beta \beta^* = C R_e C^* + D R^{-1} R_\eta R^{-1} D^*.$$  \hspace{1cm} (19)

The unique solution to the Diophantine equation

$$q^k C R_e C^* = Q_k \beta^* + q D L_{k*}$$  \hspace{1cm} (20)

provides polynomial matrices $Q_k(q^{-1})$ and $L_{k*}(q)$ of dimension $n_h \times n_h$ with generic degrees.

$$n_q = \max(n_c - k, n_d - 1), \hspace{0.5cm} n_L = \max(n_c + k, n_\beta - 1)$$  \hspace{1cm} (21)

respectively. The estimation error $\hat{h}_{t+1}\mid t$ will be stationary with finite covariance matrix and zero mean.

**Proof:** See Sternad et al. (2000), where a generalization to colored gradient noise is also presented.

Under Assumptions A1-A3, the (generalized) innovations model of $f_t = R h_t + \eta_t$ can be expressed as

$$f_t = RD^{-1}(q^{-1})\beta(q^{-1})\epsilon_t$$  \hspace{1cm} (22)

where $\epsilon_t$ is the white zero mean innovation sequence with unit covariance matrix. By defining the signal

$$\bar{\epsilon}_t \triangleq \frac{1}{D_u(q^{-1})} \epsilon_t = \beta^{-1}(q^{-1}) D_s(q^{-1}) R^{-1} f_t,$$  \hspace{1cm} (23)

the adaptation law (18) can be realized as in Fig. 3. The polynomial matrix $Q_k(q^{-1})$ can be obtained from closed-form expressions, see Sternad et al. (2000). In particular, $Q_1(q^{-1}) = q(\beta(q^{-1}) - D(q^{-1})\beta_0)$, with $\beta_0$ being the leading coefficient matrix of $\beta(q^{-1})$.

With this expression and (11), (18), the Wiener optimized filter matrix $M_k(q^{-1})$ in (8) can for white gradient noise be shown to be given by

$$M_k^{opt}(q^{-1}) = D^{-1}(q^{-1}) Q_k(q^{-1}) \beta_0^{-1} R^{-1}.$$  \hspace{1cm} (24)

The filter has the inverse regressor covariance matrix $R^{-1}$ as a right factor. The optimized tracker can thus be seen as a generalization of the LMS-Newton adaptation law ( Widrow and Stearns 1985).

5. **ITERATIVE WIENER DESIGN**

For slow time-variations, the feedback noise $Z_t \hat{h}_{t+1}\mid t-1$ is negligible, so we may perform a one-shot design by assuming $\eta_t = \varphi_t e_t$. Otherwise, the properties of $\eta_t$ depend on $L_1(q^{-1})$ via (15). The multiplication by $Z_t$ in (15), see also Fig. 1, acts as a scrambler. For FIR models (1) with white regressors, it will reduce the correlation between the feedback noise and $\hat{h}_{t+1}\mid t-1$. Assumption A2 will then hold for white regressor elements and a design for fast parameter variations can be obtained iteratively. (See Sternad et al. (2000) for a design example.)

1. **Design a one-step predictor for slow variations, i.e.** use $R_\eta = E \{ \varphi_t v_t^* \varphi_t^* \}$ to design $L_1(q^{-1})$. Verify that the closed loop around $L_1(q^{-1})$ of Fig. 1 is stable. If not, scale up $R_\eta$ to decrease the gain of $L_1(q^{-1})$.

2. **Based on a simulation of $\varphi_t$, $v_t$, $h_t$ and of $\hat{h}_{t+1}\mid t-1$, estimate $R_\eta$ from $\tilde{\eta}_t = \varphi_t e_t - R(\hat{h}_t - \hat{h}_{t+1}\mid t-1)$ (see (10),(16)), by using sample averages over $\tilde{\eta}_t$.

3. **Design a new estimator $L_1(q^{-1})$.

Repeat 2. and 3. until the difference in consecutive $\hat{h}_{t+1}\mid t$ becomes small. Then, design $L_1(q^{-1})$ for lag $k$.

It will be possible to find an initial stable solution under mild conditions. If $H(q^{-1})$ is stable, then $L_1(\omega) \to 0 \forall \omega$ when the assumed noise power is increased. If $Z_t$ has bounded elements, then the small gain theorem implies that the closed loop of Fig. 1...
can be stabilized by assuming a sufficiently high noise power in the design of \( \mathcal{L}_1(q^{-1}) \).

6. ROBUST WIENER DESIGN

The statistics of time-varying parameters will rarely be exactly known. Small uncertainties can be disregarded, but large uncertainties should be taken into account in a model-based design.

One can then use a gain scheduling approach or a minimax robust design, as investigated in Lindbom et al. (2000b). Another alternative is to optimize the parameter error covariance on average over a set of possible dynamics for \( h_t \). Such an approach to the design of robust Wiener filters was originally suggested by Speyer and Gustafsson (1975) The minimization of averaged quadratic criteria has been developed into a systematic design methodology in Sternad and Ahlen (1993), Öhrn et al. (1993) and Öhrn (1996) and it can be applied directly here. Assume that a set of hypermodels is described by the probabilistic extended design model

\[
h_t = (\mathcal{H}^o(q^{-1}) + \Delta \mathcal{H}(q^{-1})) e_t .
\]

(25)

Here, \( \mathcal{H}^o(q^{-1}) \) is the nominal model, while the error model \( \Delta \mathcal{H}(q^{-1}) \) represents the set of possible deviations, described by a set of time-invariant transfer functions, parametrized by random coefficients.

Define the operation \( \mathbb{E}(\cdot) \) as an average over all random coefficients parameterizing the error model. We assume that

\[
\mathbb{E}(\Delta \mathcal{H}(q^{-1})) = 0
\]

so the nominal model is defined as being the average model of the set. For a set of models (25), it will be possible to minimize the average of the asymptotic tracking error covariance matrix (6)

\[
\tilde{\mathbf{P}}_k = \mathbb{E}\left(\lim_{t\to\infty} \mathbf{P}_{k,t}\right)
\]

(26)

if A3 is substituted by the following assumption:

**Assumption A4:** The set of hypermodels can be represented by

\[
h_t = \frac{1}{\tilde{D}_u(q^{-1})}(\mathcal{H}^o(q^{-1}) + \Delta \mathcal{H}_u(q^{-1})) e_t ,
\]

(27)

where the known polynomial \( \tilde{D}_u(z^{-1}) \) has zeros on \( |z| = 1 \), \( \mathcal{H}^o_u(q^{-1}) \) is a known, stable and stably invertible rational matrix and \( \Delta \mathcal{H}_u(q^{-1}) \) is a set of stable random rational matrices that are independent of \( e_t \), with zero mean and known second order statistics. The noise \( e_t \) is white, with nonsingular covariance matrix \( \mathbf{R}_e = \mathbb{E}\{e_t e_t^*\} \).

The known polynomial \( \tilde{D}_u(z^{-1}) \) must thus include all marginally stable factors of transfer function denominators appearing in the set (25). If the marginally stable modes are unknown, there exists no single learning filter which can provide a finite average covariance matrix \( \tilde{\mathbf{P}}_k \).

Under Assumptions A1, A2 and A4, a robust design can be obtained by a modification of Theorem 1. The key modification of the derivation in Sternad et al. (2000), is the use of an averaged measurement spectrum

\[
\tilde{\mathbb{E}}(\phi_f) \triangleq \mathbb{E}(\mathbf{R}(\mathcal{H}^o + \Delta \mathcal{H}) \mathbf{R}_e (\mathcal{H}^o + \Delta \mathcal{H})^* \mathbf{R}) + \mathbf{R}_\eta
\]

(28)

Introduce the averaged hypermodel \( \tilde{D}^{-1}(q^{-1}) \tilde{C}(q^{-1}) \), with \( \tilde{D}(q^{-1}) = D_u(q^{-1}) \tilde{D}_u(q^{-1}) \) where \( D_u(q^{-1}) \) and \( \tilde{C}(q^{-1}) \) are assumed stable. It is a (generalized) innovation description, having spectral density equal to that of the average of the set (25),

\[
\tilde{D}^{-1} \tilde{C} \tilde{C}^* \tilde{D}^{-1} = \mathbb{E}(\mathcal{H}^o + \Delta \mathcal{H}) \mathbf{R}_e (\mathcal{H}^o + \Delta \mathcal{H})^* \frac{1}{\tilde{D}_u} .
\]

(29)

By defining an averaged spectral factorization

\[
\tilde{\beta} \tilde{\beta}^* = \tilde{C} \tilde{C}^* + \tilde{D} \mathbf{R}^{-1} \mathbf{R}_\eta \mathbf{R}^{-1} \tilde{D}^* ,
\]

(30)

the averaged measurement spectrum becomes

\[
\tilde{\mathbb{E}}(\phi_f) = \mathbf{R} \tilde{D}^{-1} \tilde{\beta} \tilde{\beta}^* \tilde{D}^{-1} \mathbf{R} .
\]

The square polynomial matrix \( \tilde{\beta}(q^{-1}) \) is monic and \( \text{det} \tilde{\beta}(z^{-1}) \) is stable since the right hand sides of (28)-(30) are nonsingular on the unit circle, due to the assumed non-singularity of \( \mathbf{R}_e \) and stability of \( \tilde{C}(q^{-1}) \). The Diophantine equation (20) is in a similar way modified by substituting \( \tilde{C}(q^{-1}) \tilde{C}^*(q) \) for \( C(q^{-1}) \mathbf{R}_e C^*(q) \) and \( \tilde{D}(q^{-1}) \) for \( D(q^{-1}) \):

\[
q^k \tilde{C} \tilde{C}^* = \tilde{Q}_k \tilde{\beta}^* + q \tilde{D} \tilde{L} \tilde{K}^* .
\]

(31)

The averaged robust design is summarized as follows:

**Theorem 2:** For the model set (25), with specified second order moments, a learning filter can be designed under Assumptions A1, A2 and A4 which minimizes the average covariance matrix (26), by obtaining a polynomial spectral factor \( \tilde{\beta}(q^{-1}) \) from (30) and a polynomial matrix \( \tilde{Q}_k(q^{-1}) \), together with \( \tilde{L}_k(q) \), as the unique solution of the Diophantine equation (31). The robust learning filter is then given by

\[
\mathcal{L}_k^{rob} = \tilde{D}^{-1}_s \tilde{Q}_k \tilde{\beta}^{-1} \tilde{D} \mathbf{R}^{-1}
\]

(32)

At frequencies where \( \mathbb{E}(\Delta \mathcal{H}(e^{-j\omega}) \mathbf{R}_e \Delta \mathcal{H}_e(e^{j\omega})) \) is

\footnote{More general cases are covered by Theorem 4.1 in Öhrn (1999) which take into account also uncertainty in the transducer, here corresponding to \( \mathbf{R}_e \) and in the noise model.}
significant as compared to the nominal spectral density $\mathcal{H}(e^{-j\omega}) R_{\mathcal{H}(e^{-j\omega})}$, the averaged hypermodel (29) will have higher gain than the nominal hypermodel. At such frequencies, the principal gains of the robust learning filter (32) will be larger than the principal gains of the nominal filter (18), since the average signal-to-noise ratio is higher than the nominal SNR. Note that only second order moments

$$E(\Delta \mathcal{H}_s(q^{-1}) R_q \Delta \mathcal{H}_s(q))$$

need to be specified, since the type of distribution, and higher order moments, will not affect the design. The required function on the right-hand side of (29) can be obtained by averaging over hypermodels. Draw $n$ samples from the set of their stable parts $\{\mathcal{H}_{s_i} = \mathcal{H}_{s_i} + \Delta \mathcal{H}_{s_i}, i = 1, \ldots, n\}$. The corresponding spectra (or covariance functions) can then be averaged to obtain (28),(29).

7. EXAMPLE: TRACKING OF IS-136 CHANNELS WITH UNCERTAIN DOPPLER ESTIMATES

In IS-136 mobile radio systems, the symbol-spaced sampled baseband channels can be described by FIR filters with one or two time-varying taps

$$y_t = h_{0,t} y_t + h_{1,t} y_{t-1} + v_t = \varphi_t h_t + v_t \,,$$

(33)

where $y_t$ is the received scalar complex-valued signal while $v_t$ is noise and co-channel interference. The transmitted symbols $u_t$ are here assumed known, although they would in reality partly be estimated by the receiver. They have variance $\sigma_u^2$ and are mutually uncorrelated, so $R_u = \sigma_u^2 I_2$ is known exactly. For a mobile terminal, the channel coefficients $h_{0,t}$ and $h_{1,t}$ will be subject to fading characterized by the maximum Doppler frequency $\omega_D = 2\pi v_c / \lambda$, where $v_c$ denotes the speed of the mobile and $\lambda$ is the carrier wavelength, which in the following is assumed to be 16 cm (~1900 MHz). For the purpose of our investigation, we shall use Jakes’ (1974) fading model, which assumes an infinite number of nearby scatterers and is parametrized by $\omega_D$. When the vehicle velocity is constant, the channel coefficient vector $h_t = (h_{0,t} h_{1,t})^T$ will then be a stationary, complex circular Gaussian process with zero mean and covariance function

$$r_h(\ell) = E\{h_t h_{t-\ell}^*\} = R_h J_0(\omega_D \ell) \ell = 0, \pm 1, \ldots \,.$$ 

(34)

Here, $R_h \triangleq E\{h_t h_{t}^*\}$, $J_0(\cdot)$ denotes the Bessel function of the first kind and zero order and $\Omega = \omega T$, $\Omega_D = \omega_D T$. The symbol time $T$ is 41.15 $\mu$s in IS-136. This yields the classical Rayleigh fading spectrum

$$\phi_h(\Omega) = \begin{cases} \displaystyle \frac{2}{\Omega_D^2 - \Delta^2} R_h & |\Omega| < \Omega_D \\ 0 & |\Omega| > \Omega_D \end{cases}.$$ 

(35)

When $\Omega_D$ is known, the model (4), (17) can be adjusted to the autocorrelation function (34). Perfect adjustment would require models of infinite degree, but good performance can be obtained with simple models. In the following, third order autoregressive models (AR3 models) $(1/D(q^{-1})) I_2$ will be fitted to the relevant covariance function.

The maximum normalized Doppler frequency $\Omega_D$ can be estimated from data, but such estimates will be imperfect. We therefore here investigate the design of an algorithm for tracking $h_t$ that is robustified against uncertainties in the Doppler estimate. The Doppler spectrum averaged with respect to the uncertain $\Omega_D$ constitutes our averaged model (29):

$$\tilde{\phi}_h(\Omega) = \frac{1}{\pi} E_{\Omega_D}(\phi_h(\Omega)) = \int_{-\pi}^{\pi} \phi_h(\Omega) p(\Omega_D) d\Omega_D \,.$$ 

(36)

Here, $p(\Omega_D)$ denotes the assumed probability density function of the normalized maximum Doppler frequency $\Omega_D$. When assuming Jakes model giving (34), the covariance function corresponding to (36) is

$$\tilde{r}_h(\ell) = \int_{-\pi}^{\pi} R_h J_0(\Omega_D \ell) p(\Omega_D) d\Omega_D \,.$$ 

(37)

In Figure 4, an element of the averaged covariance function (37) is displayed for a uniformly distributed probability density function, with different uncertainty regions. A wider uncertainty region will increase the damping of the averaged covariance function, yielding a spectrum with a less pronounced peak.

![Figure 4: Auto-correlation function $r_h(\ell) = J_0(\Omega_D \ell)$ with $\Omega_D = 0.02$ (solid) and the averaged covariance function (37), with $\Omega_D \in [0.01, 0.03]$ (dotted) and $\Omega_D \in U[0.015, 0.025]$ (dashed).](image)

A (stable) averaged AR model can now be adjusted to $\tilde{r}_h(\ell)$ (Lindbom et al. 2000b) and a robust tracking algorithm is then designed, as outlined in Section 6, using this model. Assume the signal-to-noise ratio to be 15 dB and let $R_y = I_2$. Use a nominal $\Omega_D = 0.02$, while the true Doppler frequency is varied. A nominal Wiener design is now performed by adjusting an AR3 model to (34) and then using Theorem 1 once, to design an algorithm that minimizes the one-step prediction error. (Iterations are needed in this case.) Then, robust designs are performed based on

2In the work Lin et al. (1995), a windowed LS algorithm was designed to take uncertainties about the Doppler frequency into account. The averaged covariance function (37) was then utilized in the choice of the adaptation window.
AR₃ models adjusted to (37).

The effect of using the robust designs is presented in Figure 5, where it is compared to the nominal design. We also compare to a perfectly matched AR₃ model-based design, based on the correct Doppler frequency. To compute the one-step prediction tracking MSE tr P₁, we use a novel analytical expression which is exactly valid for two-tap FIR channels with white inputs, and which gives very good approximations for higher order FIR models. It is derived in Ahlén et al. (2000), and utilized extensively also in Lindbom et al. (2000b). The averaged AR₃ model is matched to the variance functions described by the dotted and the dashed lines in Figure 4, for lags < 200. The left hand diagram of Figure 5 reflects the performance when the uncertainty level of Ω₀ is assumed moderate: Ω₀ ∈ U[0.015 0.025], whereas the right hand diagram displays the performance for a larger uncertainty interval, Ω₀ ∈ U[0.01 0.03].

When the Doppler frequency is known, the Wiener design (dash-dotted) is much superior to LMS tracking by (9) (dotted). Its performance is almost equal to that of a time-varying Kalman predictor designed for the same AR₃ model. For uncertain Ω₀, the averaged robust design improves both the worst-case performance and the average performance significantly. The average MSE performance (area under solid line) is 33% higher with an averaged robust design, than for a known Ω₀ (area under dash-dotted line). It would be 80% higher for the nominal design (dashed). The effect is significant also for the more moderate uncertainties (left-hand diagram).

As a generalization of the design above, deviations from the idealized Jakes' model can be introduced. They can be regarded as unstructured uncertainty, which could also be incorporated in an averaged robust design, using methods described in Sternad and Ahlén (1993) and Öhrn (1996).

Figure 5: One-step prediction MSE performance of Wiener designs based on AR₃ fading models: Averaged robust (solid), nominal design for Ω₀ = 0.02 (dashed) and optimally matched to known model (dash-dotted). The assumed uncertainty Ω₀ ∈ U[0.015 0.025] (left) and Ω₀ ∈ U[0.01 0.03] (right). Also shown is a LMS design tuned for a known Ω₀ (upper dotted in right figure).

REFERENCES


