Tracking of time-varying Mobile Radio Channels with WLMS Algorithms:

A Case study on D-AMPS 1900 Channels

Mikael Sternad*, Lars Lindbom¹ and Anders Ahlén*

*Signals and Systems group, Uppsala University, Sweden.
www.signal.uu.se

¹Ericsson Infotech, Karlstad, Sweden.
www.ericsson.se/infotech
Outline

WLMS (Wiener LMS):
- novel design of adaptation laws with constant gains
- prediction, filtering and fixed-lag smoothing
- close to optimal Kalman performance, but
- much lower complexity

Example of MSE tracking performance and complexity.

Two-tap fast fading channel. (See paper 4 in session 1.01):

<table>
<thead>
<tr>
<th>Input (symbol) properties</th>
<th>Kalman</th>
<th>WLMS</th>
<th>LMS</th>
<th>RLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>White and constant modulus:</td>
<td>0.010</td>
<td>0.012</td>
<td>0.020</td>
<td>0.026</td>
</tr>
<tr>
<td>White and Gaussian:</td>
<td>0.011</td>
<td>0.015</td>
<td>0.032</td>
<td>0.038</td>
</tr>
<tr>
<td>Colored and Gaussian:</td>
<td>0.020</td>
<td>0.032</td>
<td>0.085</td>
<td>0.075</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># of add/mult</th>
<th>White inputs:</th>
<th>Colored inputs:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>214</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>72</td>
</tr>
</tbody>
</table>

LMS and RLS are here inadequate
- Assume channel taps to have (coupled) ARIMA statistics.

- Filters are introduced into LMS, and tuned via an iterative Wiener procedure.

- An exact MSE tracking expression is used in the tuning.

- The design is here evaluated on fast fading channels in IS-136.

- It is used in decision-directed mode together with Viterbi detectors, in which prediction estimates are required.
An illustration

LMS tracking

Channel estimator

dotted - LMS
dashed - WLMS
solid - correct channel

WLMS tracking

Channel estimator + Viterbi
The Channel Model

Time-varying linear regression:

- Measurement: \( y(n) = \frac{\begin{bmatrix} u(n) & u(n-1) & \ldots & u(n-M+1) \end{bmatrix}}{\phi^*(n)} \)
- Regressor (symbols)
- Channel coefficients
- Noise: \( + v(n) \)

Autocorrelation matrices:
\[
R = E\{\varphi(n)\varphi^*(n)\} \quad (R = \sigma_u^2 I) \\
R_h = E\{h(n)h^*(n)\}
\]

Classical fading spectrum:
\[
\phi_h(\Omega) = \frac{2}{\sqrt{\Omega^2 - \Omega_D^2}} R_h, \ |\Omega| < \Omega_D
\]

Noise variance \( \sigma_v^2 \)
The channel estimator

One possible implementation of the WLMS algorithm

The algorithm recursions:

\[ \varepsilon(n) = y(n) - \varphi^*(n)\hat{h}(n|n-1) \]
\[ \hat{h}(n|n) = \hat{h}(n|n-1) + \mu R^{-1}\varphi(n)\varepsilon(n) \]
\[ \hat{h}(n + k|n) = P_k(q^{-1})\hat{h}(n|n) \]

Minimize the steady-state tracking MSE

\[ \lim_{n \to \infty} E \| h(n + k) - \hat{h}(n + k|n) \|^2 \]

by tuning \( \mu \) and the filters \( P_k(q^{-1}) \)

Coefficient smoothing-prediction filters

In WLMS:

\[ P_k(q^{-1}) = \frac{Q_k(q^{-1})}{Q_0(q^{-1})} \]

\( k < 0: \) Smoothing
\( k > 0: \) Prediction

In LMS

\[ P_k(q^{-1}) = I \]
\[ R^{-1} = I \]
WLMS design

Set the parameter-drift-to-noise ratio

Select a model describing the dynamics of \( h(n) \)

Evaluate via simulations - analytical MSE expressions

Solve Diophantine equations

Design equations

\[ r^k \gamma C(z^{-1})C(z) = r \frac{Q_k(z^{-1})}{Q_0(z^{-1})} + \frac{zD(z^{-1})}{D(z^{-1})} \]

Iterate

Evaluate via simulations

Select a model describing the dynamics of \( h(n) \)

Solve the spectral factorization

Design equations

Select a model describing the dynamics of \( h(n) \)

Evaluate via simulations - analytical MSE expressions

Solve Diophantine equations

Design equations

\[ r^k \gamma C(z^{-1})C(z) = r \frac{Q_k(z^{-1})}{Q_0(z^{-1})} + \frac{zD(z^{-1})}{D(z^{-1})} \]
Selection and adjustment of fading models

In this case study, we consider AutoRegressive (AR) modelling, possibly with integration (ARI), of $h(n)$

$$ h(n) = \frac{1}{D(q^{-1})} e(n) \quad \text{Re} = E\{e(n)e^*(n)\} $$

Some design examples:

Second order AR (AR2)

$$ D_2(q^{-1}) = 1 - 2\rho \cos \frac{\Omega_D}{\sqrt{2}} q^{-1} + \rho^2 q^{-2} $$

$$ \rho = 0.999 - 0.1 \Omega_D $$

AR2I

$$ D(q^{-1}) = (1 - q^{-1})D_2(q^{-1}) $$

Random walk (RW)

$$ D(q^{-1}) = 1 - q^{-1} $$

$$ P_k(q^{-1}) = 1 $$
Low complexity fading models

First and second order fading models lead to simple design equations.

Prediction:

\[
D(q^{-1}) = 1 + d_1 q^{-1} + d_2 q^{-2} \quad C(q^{-1}) = 1
\]

\[
p = \frac{d_1 d_2 (1 - \mu)}{1 + d_2 (1 - \mu)} \quad 0 < \mu \leq 1
\]

\[
Q_k(q^{-1}) = \mu \begin{bmatrix} 1 & q^{-1} \\ -d_1 & 1 \\ -d_2 & 0 \end{bmatrix}^k \begin{bmatrix} 1 \\ p \end{bmatrix} \quad k \geq 0
\]

Example: Integrated RW (IRW)

\[
D(q^{-1}) = (1 - q^{-1})^2 = 1 - 2q^{-1} + q^{-2}
\]

\[
p = \frac{-2(1 - \mu)}{2 - \mu} \quad (\text{"The pole"})
\]

\[
Q_1(q^{-1}) = \mu((2 + p) - q^{-1})
\]

\[
Q_0(q^{-1}) = \mu(1 + pq^{-1})
\]

\[
P_1(q^{-1}) = \frac{(2 + p) - q^{-1}}{1 + pq^{-1}}
\]

\[
Q_0(z^{-1}) \text{ is always minimum phase!}
\]

The step-size \( \mu \) is obtained via \( r \) or directly used as a design variable

Smoothing: see conference proceeding
Tools for Performance Analysis

The steady-state mean square one-step parameter prediction error

\[ \text{trP} = E \left\| h(n) - \hat{h}(n|n-1) \right\|_2^2 \]

**ASSUMPTIONS:**
- Signals and noise in the channel model are mutually independent and have zero means.
- The symbols and the noise are white sequences.
- The symbols have constant modulus (PSK-modulation).
- The number of channel coefficients \( M \) are < 3 and \( (M-1)\Sigma < 1 \)

**EXACT TRACKING MSE:**

\[ \text{trP} = \frac{\Gamma + M10^{-\text{SNR}/10} \Sigma}{1 - (M-1)\Sigma} \text{trR}_h \]

\[ \Gamma = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{Q_1(e^{i\Omega})}{\beta(e^{i\Omega})} \right|^2 d\Omega \]

\[ \Sigma = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{D(e^{i\Omega})}{\beta(e^{i\Omega})} \right|^2 \frac{\phi_h(\Omega)}{\text{trR}_h} d\Omega \]

**Use the MSE expression for analysis and for the tuning of the WLMS**

**For a more general MSE expression, see conference proceeding**
MSE Performances

Performance indicator:

\[ V = 10 \log \left( \frac{\sigma_{u}^{2} trP + \sigma_{v}^{2}}{\sigma_{v}^{2}} \right) \] (dB)

WLMS based on fading model:

- RW - dashed-dotted (LMS)
- IRW - dashed
- AR2 - circle
- AR4 - solid

AR fading models adjusted to classical Rayleigh fading

\[ \Omega_D - \text{Normalized Doppler frequency} \]
Simulation study

MSE optimized WLMS trackers are used together with Viterbi detectors for adaptive equalization of D-AMPS 1900 channels.

Simulation conditions:
- Forward link of IS-136
- Two tap channel
  \[ y(n) = h_0(n)u(n) + h_1(n)u(n - 1) + v(n) \]
- Classical Rayleigh fading, with equal tap powers
- Independently fading taps (ideal synch)
- Differential QPSK
- Burst synchronised interferers (colored)

Adaptive equalization:
- LMS best with decision delay k=3, otherwise k=4 is used.
- Channel tracking
  - \[ q^{-k} \]
  - \[ \hat{h}(n | n - k) \]
  - \[ \hat{u}(n - k) \]
Uncoded Bit Error Rates

TWO TAPS OF EQUAL MAGNITUDE, 160Hz FADING

Detector performance:
The BER with correct channel is compared to WLMS based on different fading models.

WLMS based on fading model:
- RW - solid (LMS)
- AR2 - dashed
- AR2I - dashed-dotted, dotted

Correct channel

Upper curves: decision-directed mode
Lower curves: WLMS fed with true symbols.
Dotted curve: true symbols and correct initialization.
Summary

- WLMS channel trackers based on AR2 and AR2I fading models provide good performance at low computational complexity in D-AMPS 1900.

- We obtain superior BER, tracking MSE and prediction performance compared to LMS or RLS for two-tap channels. (For flat fading channels and single branch, not much can be gained by improved channel tracking.)

- Fading models need not to be very accurate: Channel estimators designed for one Doppler frequency and SNR (SIR) provide close to “tuned” performance also when used at lower speeds and higher SNR’s. (See paper.)

- WLMS is a special case of a class of adaptation algorithms with time-invariant gains, based on vector-ARIMA fading models

\[ D(q^{-1})h(n) = C(q^{-1})e(n) \]

where \( C \) and \( D \) are not necessarily diagonal. More general fading models and trackers provide higher performance e.g. in multi-user detectors with mobiles at differing speeds, see

www.signal.uu.se/Publications/abstracts/r001.html
The fictitious signal model

Measurement

\[ y(n) \xrightarrow{\Sigma} \varepsilon(n) \]

\[ \phi(n) \]

\[ \phi^*(n) \]

Fictitious measurement

\[ f(n) \xrightarrow{\Sigma} \]

\[ \hat{h}(n|n-1) \]

\[ q^{-1} \]

\[ \hat{h}(n+1|n) \]

Estimate

\[ f(n) = \phi(n)\varepsilon(n) + R\hat{h}(n|n-1) \]

\[ = \phi(n)y(n) - \left( \frac{\phi(n)\phi^*(n) - R}{Z(n)} \right) \hat{h}(n|n-1) \]

\[ = \phi(n)(\phi^*(n)\hat{h}(n) + v(n)) - Z(n)\hat{h}(n|n-1) \]

\[ = \underbrace{Rh(n)}_{\text{“Signal”}} + \underbrace{Z(n)(h(n) - \hat{h}(n|n-1)) + \phi(n)v(n)}_{\text{“Noise”}} \]

\[ \eta(n) = Z(n)(h(n) - \hat{h}(n|n-1)) + \phi(n)v(n) \]

\[ \text{“Feedback noise”} \]
**Wiener filtering**

**Assumption:**
The “noise” $\eta(n)$ is white and uncorrelated with the “signal” $R h(n)$
The dynamics of the channel taps

$$h(n) = \frac{C(q^{-1})}{D(q^{-1})} e(n)$$

---

**Diagram:**

- **“Whitening filter”**
  - $f(n)$
  - $\hat{h}(n+1|n)$

- **“Innovations”**
  - $\hat{h}(n+1|n)$
  - $\xi(n)$
  - $Q_k(q^{-1})$
  - $D(q^{-1})$

- **“Causal factor”**
  - $\hat{h}(n+k|n)$
  - $Q_1(q^{-1})$
  - $D(q^{-1})$

**The parameter-drift-to-noise ratio:**

$$\gamma = \frac{\text{tr} R_e}{\text{tr} R^{-1} R \eta R^{-1}}$$

$$R_e = E\{\eta(n)\eta^*(n)\}$$