Tracking of Time-varying Mobile Radio Channels with WLMS Algorithms: A Case Study on D-AMPS 1900 Channels

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Abstract - Low-complexity WLMS adaptation algorithms, of use for channel estimation, have been presented in a companion paper. Their use and design is here evaluated on the fast fading radio channels encountered in TDMA systems based on IS-136. An exact analytical expression for the tracking MSE on two-tap FIR channels is presented and utilized. With this tool, the MSE performance and robustness of WLMS algorithms based on different statistical models can be investigated. A simulation study compares the uncoded bit error rate of detectors, where channel trackers are used in decision directed mode in conjunction with Viterbi algorithms.

A Viterbi detector combined with WLMS, based on second order autoregressive fading models possibly combined with integration, provides good performance and robustness at a reasonable complexity.

I. Introduction and Outline

In D-AMPS 900 and 1900 (or IS-136) digital mobile TDMA systems, a relatively low symbol rate and long data slots (6.67 ms) cause severe fading. In 1900 MHz systems, one or two fading dips can be expected within each data slot. Furthermore, large variations in fading rates and frequency selectivity are encountered, so well designed channel estimators are crucial for obtaining acceptable performance in the presence of intersymbol interference. Channel estimates obtained from training sequences cannot be used over the whole frames and interpolation of channel estimates between training sequences provides inadequate performance. The same is true for the decision-directed LMS and RLS algorithms.

In a related contribution to VTC2000 [1], we have presented the WLMS algorithm, which can efficiently utilize the fading statistics. It enables the design of high-performance adaptation laws with LMS computational complexity for white regressors. An early design related to this class of algorithms [6] has been successfully applied to tracking problems in both the D-AMPS 900 and 1900MHz systems [2, 4, 11].

We will here investigate opportunities, design choices and possible problems when applying the WLMS algorithm in a realistic scenario.

With a two-tap fading channel and a symbol alphabet with constant modulus, an exact performance analysis can be performed. Analytical expressions for the mean square parameter tracking error are presented. The performance is investigated with this tool for fading rates for which the adaptation laws are tuned, as well as for other fading rates. The bit error rate performance is then evaluated by simulation, for adaptive Viterbi detectors in decision-directed mode.

II. The Channel Model

A symbol-spaced baseband mobile radio channel is assumed described by the time-varying linear regression

$$y_t = (u_t \ldots u_{t-M+1}) \begin{pmatrix} h_{0,1} \\ \vdots \\ h_{M-1,1} \end{pmatrix} + v_t = \varphi_t^* h_t + v_t$$  \hspace{1cm} (1)

where $y_t$ is the received baseband signal, here assumed to be a scalar. The possibly multi-variable channel with $M$ taps is represented by $h_t$ and $\{u_t\}$ are transmitted symbols, with zero mean. The noise $v_t$ has zero mean and a variance $\sigma_v^2$. The autocorrelation matrix $E[\varphi_t \varphi_t^*] = R_\varphi$ of the regressor sequence $\{\varphi_t\}$ is known and nonsingular. When $u_t$ is white, $R = \sigma_v^2 I$.

In D-AMPS systems, $M = 1$ (flat fading) or $M = 2$.

The channel coefficients will be subject to fading characterized by the maximum Doppler frequency $\omega_D$, which may not be perfectly known. For the purpose of our investigation, we shall use Jakes’ fading model [3]. When the vehicle velocity is constant, the channel coefficients will then be stationary, circular Gaussian processes with zero means and covariance function

$$r_h(\ell) = E \{h_t h_{t-\ell}^*\} = R_h J_0(\Omega_D \ell) \quad \ell = 0, \pm 1, \ldots$$  \hspace{1cm} (2)

which yields the classical fading spectrum

$$\phi_h(\Omega) = \begin{cases} \frac{2}{\sqrt{\Omega_D^2 - \Omega^2}} R_h & |\Omega| < \Omega_D \\ 0 & |\Omega| > \Omega_D \end{cases}$$  \hspace{1cm} (3)

Here, $R_h = E\{h_t h_t^*\}$, $J_0()$ denotes the Bessel function of the first kind and zero order and $\Omega = \omega T$, $\Omega_D = \omega_D T$. The symbol time $T$ is 41.15μ in IS-136.
III. The Channel Estimator

A WLMS design begins with the selection of a hypermodel, describing the second order statistics of $h_t$.

In this case study we consider autoregressive and possibly integrating models of order $n_p$, with equal dynamics for all channel taps, described by

$$h_t = \frac{1}{D(q^{-1})} \mathbf{I} e_t + \frac{1}{1 + d_1 q^{-1} + \cdots + d_{n_p} q^{-n_p}} \mathbf{I} e_t$$

or

$$h_t = d_1 h_{t-1} + \cdots + d_{n_p} h_{t-n_p} + e_t,$$

with real-valued scalar coefficients $\{d_i\}$. Here, $q^{-1}$ denotes the backward shift operator and $e_t$ is a white zero mean random vector sequence with covariance matrix $\mathbf{R}_e$. For example, when $n_p = 2$ the model is denoted AR(2), while for an AR(2) model, $n_p = 3$ and one pole is at $q = 1$.

When the Doppler speed is known, the model should be adjusted to the autocorrelation function (2). Perfect adjustment would require models of infinite degree, but good performance can be obtained with simple models. For AR(2) models, we use

$$D(q^{-1}) = 1 - 2\rho \cos \frac{\Omega_D}{\sqrt{2}} q^{-1} + \rho^2 q^{-2}$$

where $\Omega_D$ is the nominal maximal (normalized) Doppler frequency and $\rho = 0.999 - 0.1 \Omega_T$, which works well for $\Omega_D \lesssim 0.1$.

For higher order AR models, we adjust $D(q^{-1})$ by considering row $j$ of (5). Introduce the set of covariances

$$\{r_{t,j} \triangleq \mathbb{E} h_{j,t} h_{j,t-1}^\top \}_{j=1}^N,$$

where $h_{j,t}$ denotes element (tap) $j$ of $h_t$ and where $\ell_j$ are integers such that $0 < \ell_j < \ldots < \ell_N$. Multiplying row $j$ of (5) by $h_{j,t-1}^\top$, and taking the expectation gives the set of equations

$$r_{t,j} + d_1 r_{t-1,j} + \cdots + d_{n_p} r_{t-n_p,j} = 0; \quad i = 1, 2, \ldots, N.$$  

(6)

Assuming Jakes model, we replace $r_{t,j}$ by $J_0(\Omega_0(\ell_j - \ell))$ for a known or estimated $\Omega_0$, and solve the possibly over-determined system of equations (6) by the least squares method [10].

When (4), $\mathbf{R}_e$, $\mathbf{R}$ and $\sigma_e$ are given, we can optimize a WLMS algorithm for tracking $h_t$ in (1):

$$\bar{e}_t = y_t - \phi_t^\top \hat{h}_{t|t-1}$$

$$\hat{h}_{t|t} = \hat{h}_{t|t-1} + \mu \mathbf{R}^{-1} \varphi_t e_t$$

$$\hat{h}_{t+k|t} = \frac{Q_k(q^{-1})}{Q_0(q^{-1})} \mathbf{R}^{-1} h_t$$  

(9)

Here, $\hat{h}_{t+k|t}$ is an estimate of $h_{t+k}$ at time $t$, $\mu$ is a scalar gain and $Q_k(q^{-1})/Q_0(q^{-1})\mathbf{R}$ in (9) is the coefficient smoothing-prediction filter. An equivalent implementation can be expressed in terms of the learning filter $L_k(q^{-1})$ [1, 9]:

$$f_t = \mathbf{R} \hat{h}_{t|t-1} + \varphi_t e_t$$

$$\hat{h}_{t+k|t} = L_k(q^{-1}) f_t = \frac{Q_k(q^{-1})}{\beta(q^{-1})} \mathbf{R}^{-1} f_t$$  

(10)

where

$$\beta(q^{-1}) = D(q^{-1}) + q^{-1} Q_1(q^{-1}).$$  

(11)

The polynomials $Q_k(q^{-1})$ depend on the selected hyper-model and on the SNR, and are calculated via Theorem 1 in [1] or [9] to minimize

$$E[\hat{h}_{t+k|t}]^2 = \text{tr} \mathbf{E} \hat{h}_{t+k|t} \hat{h}_{t+k|t}^\top \triangleq \text{tr} \mathbf{P}_k$$  

(12)

where $\hat{h}_{t+k|t} = h_{t+k} - \hat{h}_{t+k|t}$.

IV. Tools for Performance Analysis

For one- or two-tap fading channels and symbol alphabets $\{u_k\}$ with constant modulus, there exists an exact expression for the MSE performance (12), for a given algorithm in a given fading environment [7, 8]. This expression is valid for arbitrary fast fading rates.

**Lemma 1.** Consider the channel model (1), with $M < 3$. Assume $h_t$, $\phi_t$ and $u_t$ to be mutually independent and stationary. The spectrum $\phi_h(\Omega)$ of $h_t$ is described by (3), and the zero mean noise $v_t$ has variance $\sigma_v^2$. Let the zero mean symbols $u_t$ be white, with constant modulus and variance $\sigma_u^2$ ($\mathbf{R} = \sigma_u^2 \mathbf{I}$).

Assume $(M-1)\Sigma_1 < 1$, where $\Sigma_1$ is defined in (14) below. If an estimator for $h_{t+k}$ with the structure (10) or (7)–(9) is used, then the steady-state mean square estimation error (12) is given by

$$\text{tr} \mathbf{P}_k = \frac{\Gamma_k + M10^{-\frac{SNR}{10}} \Sigma_k + (M-1)G_k}{1 - (M-1)\Sigma_1} \text{tr} \mathbf{R}_e$$

(13)

where

$$\Sigma_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{Q_k(e^{j\Omega})^2}{\beta(e^{j\Omega})} d\Omega$$

(14)

$$\Gamma_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\beta(e^{j\Omega}) - e^{j\Omega} Q_k(e^{j\Omega})^2}{\beta(e^{j\Omega})} \text{tr} \phi_h d\Omega$$

(15)

and

$$\text{SNR} \triangleq 10 \log \frac{\sigma_u^2}{\sigma_v^2} \mathbb{E} |h_t|^2$$

(17)

Proof: Given in [7], [8].

Above, $(M-1)\Sigma_1 < 1$ is a condition for convergence in MSE. This condition will always be fulfilled for flat fading channels. Note also that the term $G_k$ vanishes for $k = 1$. All preconditions for Lemma 1 are fulfilled in the IS-136 TDMA system: The symbol sequence is white and circular with constant modulus. The delay spread is not larger than one symbol.
interval $T$, so channel models with $M \leq 2$ are appropriate. The noise $v_t$ represents mainly co-channel interference and can be assumed independent of both $u_t$ and $h_t$.

Lemma 1 can be generalized to fading statistics other than the Jakes' Rayleigh fading model, by modifying the fading spectrum $\phi_\delta(\Omega)$ used in (15).

It is of interest to know to what extent improved linear regression modelling can improve the end result for which it is intended. Filtering or detection performance is essentially determined by the ambient SNR. With Lemma 1, the variance of the "tracking noise" $\sum_{i=0}^{M-1} h_{t+i} u_{t-i}$ at the channel model output, caused by non-perfect tracking, can be calculated and compared to the variance of the noise $v_t$. As a rough but useful performance indicator, we define the relative noise level

$$ V \triangleq 10 \log \left( \frac{\sigma_v^2 \text{tr } P_k + \sigma_n^2}{\sigma_n^2} \right) \text{ (dB)} \quad (18) $$

where the numerator describes the total tracking plus noise variance, if $h$, $u$ and $v$ are mutually uncorrelated.

When $V$ is above 3dB, the tracking noise dominates over the output noise $v_t$. It is then worthwhile to consider a superior adaptation law based on, for example, a higher order hypermodel. If $V$ is below 1dB, then the noise $v_t$ dominates, so even the total elimination of any remaining tracking error would result in marginal improvements of the performance of a filter or detector based on the estimated model.

V. MSE Performance

The theoretical MSE according to Lemma 1 can be used, for example, to compare the MSE performance of WLMS algorithms based on hypermodels (4) of different complexity. This is done in Table 1 and the lower part of Figure 1 for two-tap Rayleigh fading channels with Jakes' statistics. We evaluate the use of a random walk (RW) model $h_t = h_{t-1} + e_t$ for which WLMS reduces to an LMS adaptation law. It is compared to the use of second and fourth order autoregressive models (AR2 and AR4), adjusted to the fading statistics. The transmitted QPSK symbols $u_t$ are here assumed to be known. Results are given for maximal normalized Doppler frequencies $\Omega_D$ between 0.02 and 0.06, corresponding to 45 km/h and 137 km/h, respectively, at 1900 MHz.

It can be seen that the use of a higher order model improves the performance. At 15dB for example, a WLMS tracker based on AR4 modeling provides a lower MSE at 137 km/h than LMS tracking at 45 km/h (Table 1). In terms of the effect of the noise level on the tracking MSE, more than 10 dB can be gained at both $\Omega_D = 0.02$ and $\Omega_D = 0.06$ by using an AR4 model instead of a random walk model (Figure 1).

The top diagrams in Figure 1 display the relative tracking noise level (18), under the assumption $\sigma_n^2 = 1$, $E|h_{0,t}|^2 = E|h_{1,t}|^2 = 1$ and SNR = $10 \log(2/\sigma_v^2)$.

![Figure 1: Optimized tracking error $E\|h_{t+1}\|_2^2 = \text{tr } P_1$ (lower part) and relative tracking noise level $V$ (dB) (upper part) in Section V for WLMS algorithms based on RW modeling (dashed-dotted), integrated random walks (dashed) AR2 (circle) and AR4 (solid). All AR models are matched to the true normalized Doppler frequency $\Omega_D$.](image)

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<td>AR2</td>
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<tr>
<td>AR4</td>
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Table 1: The attainable tracking error $\text{tr } P_1$ for WLMS algorithms based on different hypermodels.

For LMS tracking (WLMS based on random walks), the tracking error is in many of the considered cases so large that it dominates the total noise ($V > 3$dB).

The performance and robustness of incorrectly tuned algorithms, computed by Lemma 1, is investigated in Figure 2. The algorithms were matched to a maximum Doppler frequency of 140Hz ($\Omega_D = 0.035$) and SNR 15 dB. The results indicate that the AR2-I hypermodel provides superior robustness if we underestimate the SNR and overestimate the Doppler frequency.

The use of predicted channel estimates has also been investigated and was shown to improve the tracking performance significantly [10].

VI. Simulation Study

We investigate the bit error rate performance of adaptive decision-directed Viterbi receivers in combination with LMS, AR2 and AR2-I-based trackers. The best performance is obtained with a decision delay 3 in the detector. Due to an additional feedback delay, $k = 4$ step prediction of the channel is then required. (For LMS, $k = 3$ gives the best performance.)
VI.A Specifications

We focus on a set-up suitable for the D-AMPS 1900 standard IS-136 with the following conditions.

- **Slot structure:** As in the forward link of IS-136 with $N = 162$ differential QPSK-modulated symbols, including 14 leading training symbols.$^1$

- **Channel properties:** A two tap Rayleigh fading symbol-spaced baseband channel model with independently fading taps$^2$ is simulated:

  $$ y_t = h_{0,t}u_t + h_{1,t}u_{t-1} + v_t ; \quad R = I $$

  with $R_e$ diagonal and $E[h_{0,t}]^2 = E[h_{1,t}]^2 = 1$. The taps $h_{i,t}$ are generated according to [3], using 12 offset oscillators with uniformly distributed ($0, 2\pi$) phases. Hence, the level crossing statistics are close to classical Rayleigh fading. All estimators are initialized from least squares estimates of the channel taps in the form of robustified linear trends, based on the initial training sequence. We also study the flat fading case.

- **Disturbances:** The scenario is interference-limited with burst-synchronized interferers propagating via the same type of fading channel as the signal. The color of the interference is not estimated. (In a noise-limited scenario with Gaussian noise, the BER performance improves.)

$^1$A known CDVCC sequence of six differential symbols is placed after 85 symbols of the slot. They are here not used to improve the tracking performance, since this would complicate the performance evaluation.

$^2$The more realistic case of correlated taps would result in higher bit error rates due to partial loss of diversity, but will otherwise not provide any new fundamental problems for the tracking.

- **Idealized simulation conditions:** We have compared decision directed adaptation to the use of correct symbols $u_t$ as regressors. To quantify the loss of performance due to imperfect initialization, we also compare to initialization with known channel taps.

VI.B BER Performance for Two-tap Channels

Channels with two symbol-spaced taps of equal magnitude are simulated. The taps are independently Rayleigh fading, as described by Jakes’ model, with normalized Doppler frequency $\Omega_D = 0.04$ (90 km/h at 1900 MHzz). WLMS tracking algorithms based on random walk (LMS), AR$_2$ and AR$_2$I fading models are evaluated in combination with a Viterbi algorithm.

Figure 3: The Bit error rate as a function of the signal-to-interference ratio for the adaptive Viterbi equalizer with $k = 4$ ($k = 3$ for LMS). The BER with correct channel (lower solid) is compared to WLMS tracking with AR$_2$I modelling with true $u_t$ as regressors (lower dash-dotted) and estimated regressors (upper dash-dotted) and to WLMS tracking with AR$_2$ modelling with true $u_t$ as regressors (lower dashed) and estimated regressors (upper dashed). Compare to LMS with optimized step length and true $u_t$ as regressors (middle solid) and with estimated regressors (upper solid). Also shown is AR$_2$I tracking using true $u_t$ and correct initialization (dotted). 10000 slots are considered for each simulation case.

Figure 3 presents the uncoded bit error rate when the correct $\Omega_D$ and signal-to-interference ratios (SIRs) are used in the design.

Comparing the dotted to the lower dash-dotted curve in Figure 3 we see that not much performance is lost due to imperfect initialization. (If the algorithms were initialized with levels instead of linear trends, the performance would deteriorate further by 1-2dB.)

Decision-directed adaptation results in a performance loss due to nonlinear feedback effects. It is approximately 3dB for WLMS based on AR$_2$ and AR$_2$I models in Figure 3.

In Figure 3, WLMS based on AR$_2$I models show the best performance, but the performance of AR$_2$-based trackers is rather close. LMS tracking will in
this case be completely inadequate, partly due to its inapplicable structure and not least due to its inability to predict the channels; With a random walk model, $h_{t+1|t} = h_{t|t}$. This results in a significant lag error, which will not vanish at low disturbance levels. Hence, the error floor at 1.7% BER.

To test our conclusions from Section V, we have designed AR2 and AR3I-based WLMS algorithms for $f_D = 160$Hz and $SIR = 15$dB and evaluated their performance at other operating points. The results, presented in Figure 4, confirm that one single fixed adaptive filter, designed at the high end of the uncertainty interval of the Doppler frequency and the low end of the SIR range can indeed be used over the whole parameter range. If the operating area is bounded by $SIR = [15,25]$dBi and $f_D = [0,160]$Hz, then this filter does in fact constitute a minimax robust design, since the so-called saddle-point condition [5] is fulfilled: The resulting performance attains its worst value at the nominal (worst-case) design point. In the most critical regions, with low SIR and/or high Doppler frequency, the performance for an AR3I-based design is about the same as for an AR2-based design.

**VI.C BER Performance for Flat fading**

In the flat fading case, with $h_{1,t} = 0$, not much can be gained by improving the tracking, see Table 2. An exception is at high SNR, where for true regressors a significantly lower BER is attained for AR2 or AR3I-based designs as compared to LMS. This can be predicted by the values of V from (18) in the right-hand part of Table 2. For flat fading channels, all the algorithms provide about the same performance. The detector becomes trivially simple, so no channel prediction beyond $k = 1$ is required.

Table 2: Flat fading at $\Omega_0 = 0.04$, $E|\mathbf{h}_0|^2 = 1$, $E|\mathbf{h}_1|^2 = 0$. In row 4 to 6, a true symbol is used as regressor. The relative noise level (V) is obtained with $k = 1$.

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**References**


