Considering Downlink Intermodulation Distortion in Switched Multibeam Antennas for Cellular Radio Systems

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Abstract

The performance and feasibility of switched multibeam array antennas in cellular systems is investigated. We assume that multicarrier power amplifiers (MCPA:s) are used in the base-station, to co-amplify the transmitted signals. The MCPA generates intermodulation distortion and it is shown how the multibeam antenna makes the radiated intermodulation distortion dependent on the azimuthal lookdirection. Given the carrier to intermodulation distortion ratio of the MCPA, semi-analytical expression for the outage probability for the mobiles is derived assuming a lognormal fading radio channel. Furthermore, the impact of the frequency reuse factor, trunkpool scheme and number of antennas is investigated. The analysis show that when the number of antennas are increased, it is possible to reduce the requirements on the MCPA linearity.

1. Introduction

Using switched multibeam antennas at base stations (BS) in cellular FDMA systems has shown to either improve the capacity or to extend the radio coverage by increasing the carrier to interference ratio (CIR)[1]. The reduced interference levels allows for a reduction in frequency reuse distance, thereby increasing the spectrum efficiency of the cellular system. One drawback with the use of array antennas at the BS sites is the increased amount of hardware. Each antenna and frequency channel requires an single carrier power amplifier (SCPA) for the downlink transmission. Thus, to reduce the size and complexity of the BS, multicarrier power amplifiers (MCPA) have been suggested [2] where the signals are combined at low power and then coamplified to avoid filter combining at high power. The coamplification of several carriers on different frequencies in an MCPA generates intermodulation products (IMP). Often the third order IMP are considered (of type $2f_i$ - f_k and $f_i + f_k - f_l$), as they fall onto frequency channels that also are used in the system and thereby, cannot be removed by

filtering. Hence, linearisation of MCPA:s are commonly adopted to reduce the IMP to a level acceptable for the system.

The problem addressed in this paper is, how is the linearity requirements on the MCPA related to the frequency reuse distance in the cellular system and the number of antennas used? We assume that the IMP output power from the MCPA is limited by the linearisation techniques to a carrier to intermodulation level of λ dBc. Different frequency planning techniques is studied as well as the effect of other influencing parameters as the beam-width and the side lobe level (SLL) of the multibeam antenna. As a performance measure, the outage probability, or the area-averaged probability that the received power at the MS is below a target value is used. Furthermore, the radio channel is assumed to be a fading channel modeled with a log-normal distribution.

One property of MCPA:s in conjunction with multibeam antennas is that the radiated IMP will be spatially filtered by the beam-pattern of the antenna array [3]. Assume that the transmitted power in one of the beams from the BS is P_T [dBW]. In some directions the IMP level $P_T - \lambda$ [dBW] will be further suppressed by the SLL of the multibeam radiation pattern and in other directions, the IMP level will be increased by the gain of the antenna array by coherent addition of the IMP from all antennas. Hence, the amount of IMP interference power that a MS experiences from BS in it's "own" cell and from BS:s in adjacent cells depends on the number of antennas, i.e. the beam-width, SLL and also on the trunkpool type adopted for the current system.

The analysis in this paper considers the worst case scenario with a fully loaded FDMA system and flat-top beamformers are assumed to simplify the analysis. Furthermore, power control is not addressed, each BS transmits with the same power P_T .

2. System model

This section describes the multibeam antenna, the characteristics of intermodulation distortion in multibeam antennas and also the assumptions made about the cellular system frequency plan.

2.1. Switched multibeam antennas

It is assumed that the BS in each each sector is equipped with an antenna array capable of transmitting the downlink signal in N beams covering a 120° horizontal angle. Hence, the beam-width (BW) of each beam is approximately $120^{\circ}/N$ degrees where N is the number of antenna elements of the array. The beams are assumed fixed and are often generated using a Butler matrix [4] which can be implemented in hardware. This gives a low cost and robust solution to the beamforming problem, although there is no possibility to steer nulls in the radiation pattern in directions where co-channel mobiles exists [5]. Hence, to reduce the co-channel interference it is desirable to have a narrow BW, which is determined by the number of antennas, and low SLL of the radiation pattern, which can be controlled by the aperture tapering. Uniform aperture taper give SLL of -13.2 dB compared to the direction of maximum intensity (main beam), by using a Hamming taper window, the SLL can be reduced to -42.8 dB below maximum intensity at the expense of 25% loss in gain relative to the uniform aperture [6].

The beam selection for the downlink transmission is based on information derived from the uplink such as the direction of arrival (DOA) in azimuthal angle of the uplink information. The DOA estimate is used to select one beam from the set of downlink beams. The DOA can be an actual estimate of direction to the mobile station or if the radio channel is a multi-path channel, then the best uplink beam is identified as the beam with the largest received power from the desired user. The chosen beam might not coincide with the true DOA beam of the mobile due to the reflections in the multi-path channel [1].

2.2. Intermodulation in multibeam antennas

Assume that an MCPA precedes each antenna in the array. See Figure 1. The MCPA amplifies all the downlink signals in use in the particular 120° sector. An FDMA system is assumed, hence the downlink signals are placed on separate non-overlapping frequency channels. Due to the non-ideal MCPA, the signals will generate intermodulation distortion (IMD) and the frequency channel and amplitude of the IMP is dependent on the characteristics of the MCPA, often defined as the AM/AM and AM/PM characteristics and the amplitude and frequency of the involved input signals. The degree of linearity can be measured as the carrier to intermodulation power ratio (CIMR).

Using an array antenna, the IMD will be direction dependent, so the IMD will be weighted by the array antenna



Figure 1. Switched beam antenna architecture using multicarrier power amplifiers

radiation pattern with sidelobes and nulls, where less or no IMD is radiated [7][3]. If we assume that the frequency channels are equally spaced, frequency channel k have center frequency f_k , where

$$f_k = f_0 + k\Delta f \tag{1}$$

and Δf is the frequency channel spacing and f_0 is the frequency reference channel. Due to the equally spaced frequency channels, it is possible drop the actual frequency and proceed in terms of frequency channels. Hence, the frequency channel of the third order in-band IMP generated by mixing channel a channel j and channel l is given as

$$k_{IMP1} = 2j - l \tag{2}$$

$$k_{IMP2} = 2l - j \tag{3}$$

which then corresponds to the frequencies $f_{k_{IMP1}}$ and $f_{k_{IMP2}}$. Note that these frequencies might fall outside the frequency band of interest, if for example $f_{k_{IMP1}} < f_0$ or $k_{IMP1} < 0$ and they can then easily be removed by the transmit filters.

Furthermore, a FFT based beamforming network (BFN) often implemented as a Butler matrix [4] is assumed. It has the property of a constant phase gradient $\Delta \varphi_k$ over the antenna array aperture. If φ_{nk} is the phase weight of the signal on frequency channel k at antenna element n, then

$$\varphi_{nk} = (n-1)\Delta\varphi_k \tag{4}$$

where n = 1, ..., N is the antenna element number and k is the frequency channel number. We assume, from here on, that the amplitude weights or aperture tapering is constant and equal for all N antennas, hence an uniform aperture.

In the same way as the frequencies of the IMP was calculated above, it is possible to calculate the phase gradient of the IMP beams. Hence, the two signals on frequency channel j and l are assumed to be connected to the BFN ports to give output phase gradients $\Delta \varphi_j$ and $\Delta \varphi_l$ respectively and the third order IMP has the phase gradients [8]

$$\Delta \varphi_{IMP1} = 2\Delta \varphi_j - \Delta \varphi_l \tag{5}$$

$$\Delta \varphi_{IMP2} = 2\Delta \varphi_l - \Delta \varphi_j \tag{6}$$

The phase gradient, $\Delta \varphi_k$ belongs to a set Ω_S of N phase gradients, unique for each beam the BFN can generate,

$$\Omega_S = \left\{ \frac{2\pi p}{N} \right\}_{p=0}^{N-1} \tag{7}$$

The finite set Ω_S is a closed group under multiplication and addition, following *modulo-N* algebra. These properties implies that the main-lobe direction of the desired signals are also main-lobe directions for the IMD. The direction of maximum IMD can then be calculated as, for example [3]:

$$\theta_{max,IMD} = \arcsin\left(\frac{2\Delta\varphi_l - \Delta\varphi_j}{\pi}\right) \quad . \tag{8}$$

Hence, two signals, say $s_1(t)$ and $s_2(t)$, separated according to (1), with equal phase gradients ($\Delta \varphi_1 = \Delta \varphi_2$, i.e. same input port of the beamforming network), will generate third order IMP with equal phase gradients $\Delta \varphi_{eq} = \Delta \varphi_1 = \Delta \varphi_2$, i.e. IMP at the frequencies $f_3 = 2f_2 - f_1$ and $f_0 = 2f_1 - f_2$ are radiated in the same beam as the amplified original signals $s_1(t)$ and $s_2(t)$. If, however the two signals are to be transmitted in distinct beams, $\Delta \varphi_1 \neq \Delta \varphi_2$ (different beamforming network input ports) and the third order IMP are radiated with phase gradients $2\Delta \varphi_2 - \Delta \varphi_1$ and $2\Delta \varphi_1 - \Delta \varphi_2$ at the frequencies $f_3 = 2f_2 - f_1$ and $f_0 = 2f_1 - f_2$ respectively. Note that f_0 is outside the defined transmitter band.

The above discussion shows that the IMP is radiated with the same radiation pattern as the linear (desired) amplified signals.

2.3. Cell sectoring and frequency reuse

Depending on how the cellular system is sectorized and on the frequency reuse distance, the system sensitivity to the IMD will vary. In this section, the cell sectoring and frequency planning used in the study in this paper is defined. We assume that the cells are circular with radius Rand that K cells in a group make up a cluster of cells. Assume that a total of C frequency channels are used by the operator for the downlink transmission. Then each cell use a fraction C/K of these frequency channels, so the frequency reuse distance is $D = \sqrt{3KR}$ [9], see Figure 2. Because of the tiling properties of hexagonal shapes, the clusters will accommodate only a certain number of cells, such as $K = 1, 3, 4, 7, 9, 12, \ldots$ When K is decreased, with a constant cell radius R, the traffic capacity is increased due to a more dense reuse of the C available frequency channels. However the transmission quality is also decreasing with decreasing frequency reuse distance D due to the increased interference levels. The interference reducing properties of the multibeam antenna allows a smaller D and thereby an improved capacity. It is assumed that each cell has three switched multibeam antennas covering 120° each, hence three linear arrays with N antenna elements each is situated at the BS site. In the omni-trunkpool frequency plan [10] all the C/K channels assigned to a cell can be used in any beam in any sector, see Figure 2. In this case, no hand-offs are needed unless the mobile crosses a cell boundary. In the sector-trunkpool case, the C/K channels assigned to each cell are further divided into three groups, see Figure 3. The sector can thus be considered as a new cell with a substantial reduction in the co-channel interference [9], however the omni-trunkpool technique have a lower channel usage efficiency due to the loss in trunking efficiency. In both



Figure 2. Frequency plan with omni-trunkpool cells and K=4, C=12. The numbers represents the FDMA frequency channels.

of these frequency plans, it is easy to see that the generated IMP are falling on frequency channels used in the same cell they are generated. For example, frequency channel 1,5 and 9 generate third order IMP of type $2f_j - f_k$ on frequency channel $2 \cdot 5 - 1 = 9$ and of type $f_j + f_k - f_l$ on frequency channel 9 + 1 - 5 = 5 (where the third order IMP:s falling



Figure 3. Frequency plan with sectorizedtrunkpool cells and K=4, C=12. The numbers represents the FDMA frequency channels.

outside frequency channels 1,5,9 have been omitted). As a result, the IMP will also have a reuse distance of D.

3. Outage probability

In this section, the outage probability of a mobile in a cellular system using switched multibeam antennas at the BS is calculated. The radio channel shadowing is assumed to have a log-normal distribution and the mobiles are assumed to be uniformly area distributed in each sector.

3.1. Interference in cellular systems

We consider an interference limited system in which the thermal noise in the receivers is negligible compared to the co-channel interference power. Hence the carrier to interference ratio (CIR) is used. The CIR at the mobile is a random variable, affected by phenomena as mobile location, base station location, fading characteristics and the directional antenna properties given by the multibeam antenna. Design specifications often require that a minimum CIR, called the power protection ratio γ , must be achieved over a large percentage of the coverage area (usually 90%) [9].

We neglect co-channel interference from cells outside the first tier of co-channel cells and we assume that the MCPA:s are linearized to give an CIMR of λ [dBc]. Therefore, the CIR (γ_d) (actually the carrier to interference plus intermodulation interference) of the desired user can be written as

$$\gamma_d = \frac{S_d}{S_I} = \frac{S_d(r)}{\sum_{i=1}^{N_I} S_i(r_i) + S_{IMD}(r)}$$
(9)

where $S_d(r)$ [W] is the received power at the desired mobile at a distance r from its BS and S_I [W] is the total interfering power, which is from BS in the first tier and intermodulation distortion $S_{IMD}(r)$ [W] from the BS the mobile is connected to. A total of N_I number of first tier interfering sources exists and the distance between the interfering BS *i* and the mobile under study is r_i . If a sectorized trunkpool is used, with 120° sectors, $N_I = 2$, otherwise the number of interferers is a random variable ranging from zero to a maximum of $N_I = 6$ [10]. It is further assumed that the interfering signals are non-correlated.

An area uniform distribution of the mobiles in a 120° sector relative to their BS is given in polar co-ordinates by [11]

$$p_r(r) = \frac{2(r-R_0)}{(R-R_0)^2}$$
 $R_0 \le r \le R$ (10)

$$p_{\theta}(\theta) = \frac{3}{2\pi} \qquad -\pi/3 \le \theta \le \pi/3 \qquad (11)$$

where R_0 is the minimum allowed BS to mobile distance, set to 20 meters and R is the cell radius.

3.2. The outage probability in a fading channel

A fully loaded system is assumed, that is, all N_I interferers in the first tier are active and a flat-top beamformer is used in the calculations, see Figure 4. We assume that the beam-width (BW) is $BW = 2\pi/3N$, hence, the 120° sector is covered by N beams.

For each BS in the first tier that interfers with the mobile in the center cell under study, three cases exists:

- 1. BS is interfering the mobile and the main beam is pointing towards the mobile, hence the power P_T dB is transmitted in the direction of the mobile.
- 2. BS is interfering the mobile, but main beam is not pointing in the direction of the mobile, hence the power $P_T q$ dB is pointing in the direction of the mobile.
- 3. BS is not transmitting in the sector that cause interference to the mobile, this is only possible in the omnitrunkpool scheme.

As the mobiles are distributed according to an uniform area distribution, and the N_I interfering cells in the first tier are independent, the probability that n_i interfering first tier



Figure 4. Radiation pattern of a flat-top beamformer covering one sector with beam-width BW and side-love level of $P_T - q$ dB where P_T is the transmitted power.

beams are pointing towards the mobile is binomially distributed [12]:

$$P_I(n_i) = p_a \cdot \binom{N_I}{n_i} \left(\frac{1}{N}\right)^{n_i} \left(1 - \frac{1}{N}\right)^{N_I - n_i}$$
(12)

for $n_i = 0, ..., N_I$. In (12), N is the number of antennas, equal to the number of beams. The factor p_a is 1/3 for omnitrunkpool schemes and 1 for sectorized-trunkpool schemes, to take the fact that in omni schemes, the interferer might be transmitting in another sector facing in another direction and in that case it does not interfer at all with the mobile.

Furthermore, we assume that the main beam of the radiated IMP is in one of the N beams with equal probability. Hence, the probability $P_{IMD}(n_{IMD})$ that the mobile is in the main beam of the IMD is

$$P_{IMD}(1) = 1/N$$
 In main beam (13)

$$P_{IMD}(0) = 1 - 1/N \qquad \text{In side lobe} \qquad (14)$$

and the power of the IMD is $P_T - \lambda$ dB and $P_T - q - \lambda$ dB in the mainbeam and in the side-lobe respectively. Using conditional probability theory, the outage probability can now be expressed as

$$P(outage) = P(\gamma_d < \gamma) = \sum_{n_i=0}^{N_I} \sum_{n_{IMD}=0}^{1} P(\gamma_d < \gamma | n_i, n_{IMD}) P_I(n_i) P_{IMD}(n_{IMD})$$
(15)

since all possible values of the number of co-channel interferers pointing with the main beam towards the mobile must be considered. The local mean power, μ , of a signal in a log-normal fading channel is a stochastic variable with the pdf:

$$f(\mu, m, \sigma) = \frac{1}{\sqrt{2\pi\sigma\mu}} \exp\left(-(\ln(\mu) - m)^2/2\sigma^2\right) \quad (16)$$

where σ^2 and *m* are the logarithmic variance and the area mean power of the signal, respectively [13]. The area mean power is assumed to follow a *p*-th power law propagation model:

$$m = K/r_i^p \tag{17}$$

where R is the BS to mobile distance and K is a constant that depends on the transmitted power. We assume in the following that all BS transmit the same power P_T and that all radio channels have the same variance σ .

The interfering signals will add non-coherently at the mobile, and the sum of stochastic independent log-normal variables can be approximated by another log-normal variable [14][15]. If we denote the desired signal area mean power as m_d we get

$$m_d = \tilde{P}_T / r^p \tag{18}$$

and for the interference

$$\tilde{m}_{I}^{(i)} = \frac{\tilde{P}_{T}}{r_{i}^{p}} (\eta_{i} + \tilde{q}(1 - \eta_{i})) \qquad \text{for } i = 1, \dots, N_{I} \quad (19)$$

and

$$\tilde{n}_{I}^{(IMD)} = \frac{\tilde{P}_{T}\tilde{\lambda}}{r^{p}} (\eta_{IMD} + \tilde{q}(1 - \eta_{IMD})) \qquad (20)$$

for the intermodulation interference, where, $\eta_i = 1$ if the mobile is in the main beam of interfering BS *i*, otherwise $\eta_i = 0$ and similar for the IMD. The notation $\tilde{\cdot}$ is used to denote the linear scale, e.g. $\lambda = 10 \cdot \log_{10}(\tilde{\lambda})$.

The conditional outage probability $P(\gamma_d < \gamma | n_i, n_{IMD}, r, \phi)$ can now be calculated by using Wilkinsons:s method of approximating the sum of log-normal distributed variables [14]. Denote m_I as the sum of all the interferers (in nats) obtained by Wilkinsons method to be used as the mean of the log-normal pdf (16). Now, to find the conditional outage probability, we integrate the pdf:s (16):

$$P(\gamma_d < \gamma | n_i, n_{IMD}, r, \phi) =$$

= $\int_0^\infty d\mu_I \int_0^{\gamma \mu_I} f(\mu_d, m_d, \sigma) f(\mu_I, m_I, \sigma) d\mu_d$ (21)

and the result is [13]:

$$P(\gamma_d < \gamma | n_i, n_{IMD}, r, \phi) = Q\left(\frac{m_d - m_I - \ln(\gamma)}{\sqrt{2\sigma^2}}\right).$$
(22)

To find the unconditional outage probability, (22) must be averaged over all possible mobile locations (r, ϕ) weighted by the pdf of the mobile location (10),(11). The difficulties in performing this step is circumvented by performing an Monte Carlo simulation, to avoid lengthy calculations in direct analysis or to avoid roundoff and stability problems in numerical integration methods.

3.3. Monte Carlo simulation

The Monte Carlo simulation is composed by the following steps:

- 1. The position of the desired mobile (r, θ) is randomly picked according to (11).
- 2. Loop through all the N_I first tier interferers and calculate the distances from BS i to the desired mobile.
- 3. Loop through all 2^{N_I+1} possible combinations of $\eta_1, \ldots, \eta_{N_I}, \eta_{IMD}$ and for each combination, use Wilkinsons method to calculate the sum of the lognormal faded interfering sources (\tilde{m}_I) [15].
- 4. Calculate m_d and m_I and insert these into (22) to get the conditioned outage probability.
- 5. Calculate the probability that the specific combination of $\eta_1, \ldots, \eta_{N_I}, \eta_{IMD}$ occurred.
- 6. Multiply the probability with the conditional outage probability (22) and sum up according to (15).
- 7. Store the calculated outage probability and repeat the procedure from step 1 until a sufficient accuracy has been obtained.

4. Results and discussion

The system was analysed with the following parameters. The path loss exponent p was set to 4, the log-normal variance σ =4 dB and the cell radius R=1000 m and R_0 =20 m. A power protection ratio γ =17 dB was used, as common in AMPS cellular systems. The side-lobe level q=13 dB, as the uniform aperture weighted array.

Figure 5 shows results from using the sector-trunkpool scheme. The figure shows the outage probability as a function of $-\lambda$, the carrier to intermodulation level. When $\lambda < 40$ dBc, the curves reaches a threshold, and in the region less than -40 dBc the system is interference limited by cochannel interference from first tier BS. Hence, improving the MCPA linearity further does not improve the outage probability. Given a service grade of 90% (outage probability 0.1), we see that a system with frequency reuse in every cell (*K*=1) cannot be implemented with either *N*=1,4 or 8 antennas. The K=4 system achieves an outage probability of 10% at an intermodulation level of 29.1 dB and 23.3 dB below the carrier respectively. Hence, a larger array with more antenna elements makes the system less sensitive to IMD, due to the narrower beam-widths.

For the K=7 system, the reduction in λ at outage probability 0.1 is 12.6 dB when increasing the number of antennas from N=1 to 4, but to further increase N gives only an additional 3.6 dB reduction in λ . Hence, the first additional antenna elements (when N > 1) gives the largest interference reduction. Note that this is valid for the switched multibeam antenna, an adaptive antenna algorithm can more efficiently use the increased number of antenna elements to place more and/or broader nulls towards interferers. We propose that aperture tapering should be used in switched multibeam antennas when the number of antenna elements are large, because at some point, when a large N gives a sufficiently narrow BW, lower sidelobes becomes a more important issue for reducing the co-channel interference than narrowing the BW.



Figure 5. Calculated outage probability for the sector-trunkpool scheme.

Figure 6 shows the outage probability for the omnitrunkpool scheme where a maximum of N_I =6 simultaneous interfering BS in the first tier is present as opposed to the sectorized system with only N_I =2 interfering BS:s. Due to the interference, we cannot achieve an outage probability < 0.1 for the K=1 case even with a perfectly linear (ideal) MCPA. However, using the multibeam antenna with N=4 and 8 antenna elements, this is achieved for the K=4 and 7 cluster sizes. For the K=4 case, the MCPA must have a carrier to intermodulation level of 33.8 dBc to achieve P(outage) < 0.1. In practice, to increase the capacity



Figure 6. Calculated outage probability for the omni-trunkpool scheme.

or spectrum efficiency of the cellular system, the frequency reuse factor K is reduced, hence, we continue by study the K=4 system in more detail, see Figure 7. Here the omni and sectorized trunkpool schemes are compared. The sectorized system has a lower outage probability for any given CIMR, as the number of possible interferers are less than in the omni case. Also, increasing the number of antenna elements N from 1 to 4 gives the largest improvement in outage probability, the improvement by increasing to 8 antennas is less. Given an outage probability, we see that it is possible to reduce the amplifier linearity when the number of antennas (and MCPA:s) are increased.

5. Concluding remarks

The performance of switched multibeam antenna systems have been analysed for cellular radio systems when a non-ideal MCPA is used in the transmit chain. We showed how the omni-trunkpool scheme has a larger outage probability than the sectorized scheme and how it is possible to reduce the linearity specification of the MCPA when the number of beams in the multibeam antenna is increased. There are certain limitations to the analysis presented in this paper. We have assumed that the system is fully loaded, that is , the worst case of interference. Furthermore, the interfering BS:s outside the first tier have been neglected. Also, the fading was modeled with a log-normal pdf, hence we have assumed that the carrier to interference ratio is averaged over the fast, often Rayleigh distributed fading.



Figure 7. Calculated outage probability for a frequency reuse factor K=4.

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