OPTIMAL AND ADAPTIVE FEEDFORWARD REGULATORS

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ABSTRACT

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When disturbances can be measured, a large improvement of disturbance rejection becomes possible, compared with the use of output feedback only. Polynomial LQG design methods for feedforward and combined feedforward-feedback control of discrete time single input systems are presented. These regulators attain optimal disturbance rejection in cases where complete cancellation of disturbances is impossible, for example when systems are non-minimum phase. The optimal regulator structure is discussed. With the help of this structure, the feedforward control quality is independent of the choice of feedback. The methods handle deterministic as well as stochastic disturbances. Disturbance measurement signals affected by the input can be utilized. An application to load feedforward power frequency control of hydro power stations is studied. A close correspondence is shown to exist between feedforward control and input estimation problems. Two adaptive feedback control algorithms are extended with adaptive feedforward: An LQG self-tuner and explicit criterion minimization, which both converge to the optimal controller. Their performance is tested, and compared to minimum variance and extended minimum variance self-tuners with feedforward terms. Both the LQG self-tuner and explicit criterion minimization attain very good control behaviour. With the help of some simple safeguards, their performance is robust under a wide range of conditions.

Key-words: Feedforward control, Linear quadratic control, Disturbance rejection, Disturbance decoupling, Power system control, Deconvolution, Adaptive control, Optimal control.

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APPENDICES

1. INTRODUCTION

With the spread of James Watt's steam engine in the last two decades of the eighteenth century, the industrial revolution was born. Watt had not actually invented the steam engine. Instead, his contribution was the introduction of several technical improvements, and a very successful marketing effort. One of the most crucial improvements came in 1788. A device for controlling the engine speed automatically, the centrifugal governor, was introduced. It had been used earlier for controlling the speed of windmills. This regulator acted through *feedback control*, cf Figure 1.1: By measuring the engine speed, a control action was produced which counteracted deviations caused by load changes.

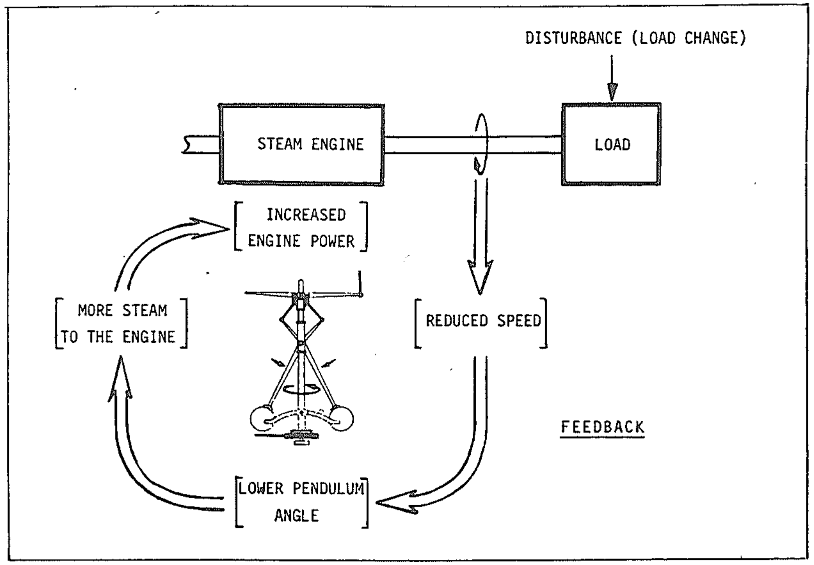

Figure 1.1 Engine speed feedback control with a centrifugal governor. When the load is increased, the engine speed is reduced. A lower speed reduces the angle of rotating pendulum. Through a mechanical connection, this opens the steam valve, and increases the power and torque of the engine. (The origin of centrifugal governors, and feedback control in general, is described in Mayr (1970).)

While the number of steam engines produced by Boulton & Watt increased rapidly, other inventors tried to get an edge in the competition by improving the speed regulation. The centrifugal governor reduced speed deviations, but it could not eliminate them. (Integrating control had not yet been invented.) Furthermore, a load change first had to produce a significant speed deviation before the regulator could begin to counteract it. In 1826, the French mathematician and engineer Jean-Victor Poncelet proposed a radically different control principle: Measure the disturbance instead of the controlled output, and derive a counteracting control action from the disturbance measurement. This control principle, which is based on *disturbance cancellation*, is now called *feedforward control*.

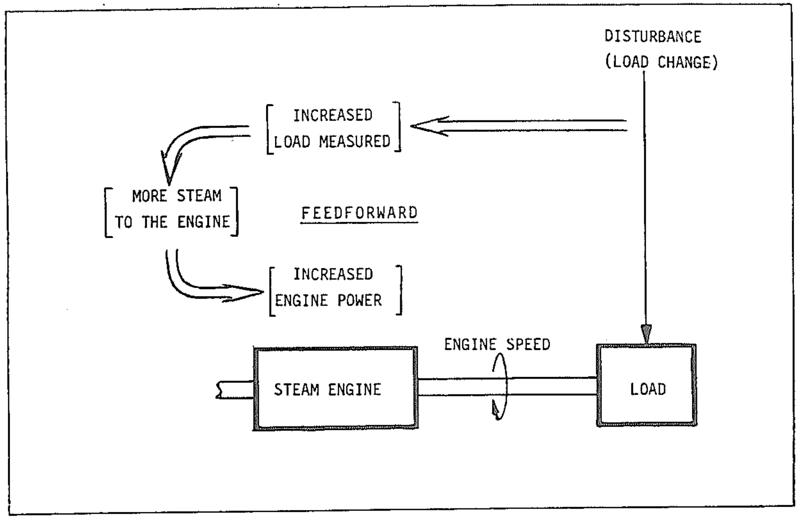

Figure 1.2 Engine speed regulation by load feedforward. (Poncelet's regulator is described in detail in Rörentrop (1971). The original reference is Poncelet (1826).)

It takes some time for a disturbance to affect the output. A feedforward controller can begin to act before this has happened. In certain situations, the control action can be made to meet the disturbance at some point inside the system, and *cancel it completely*. This can never be achieved with feedback from the controlled variable.

Poncelet was well aware of these advantages. He invented an ingenious device for measuring the torque on the engine axis, and tested his principle on several engines. It did not work. Poncelet and other designers of feedforward regulators since then have had to face several difficulties: A feedforward regulator continually balances the control signal against the disturbance. To achieve this delicate balance, the design must be based on an accurate model of the system. In addition, it requires accurate disturbance measurements. Finally, it can only compensate for measurable disturbances. Additional unmeasurable disturbances are often present, and their influence will remain uncompensated. For these reasons, regulator design has, until recently, been based almost exclusively on the feedback principle.

Still, the possibility of cancelling disturbances has continued to intrigue a number of investigators. For example, in the new generation of passenger airplanes with turbofan engines, too much of the engine sound may leak into the passenger compartment. The noise level could be reduced by introducing <u>more</u> engine sound into the cabin, with an array of about 50 loudspeakers. As Elliot and Nelson (1986) have suggested, the sound from the loudspeakers can be amplified, inverted and delayed with adaptive filters in just the right amount to cancel the sound coming directly from the engines. This use of "anti-sound" illustrates the somewhat paradoxical way in which feedforward works. Disturbances are utilized, and made to neutralize themselves, by introducing them into the system through a different signal path. The cancellation principle has numerous applications in signal processing, cf Widrow et al (1975).

For some systems, direct disturbance measurement (which may be impractical) is not needed for achieving disturbance cancellation. State feedback or feedback from auxiliary outputs (which are situated between the disturbance entry point and the controlled output) may accomplish cancellation. Such generalizations of the feedforward control problem, also called *disturbance decoupling* or *disturbance localization* problems, have recieved considerable interest in recent years, beginning with Wonham (1974).

Feedforward control is mostly used in combination with feedback. The feedback increases the robustness of the design and takes care of unmeasurable disturbances. The feedforward compensates the most important measurable disturbance. While perfect cancellation of disturbances normally is impossible, dramatic improvements in the control performance can be achieved, compared with output feedback only. Application of feedforward facilitates the design of stabilizing feedback regulators. With the feedforward link compensating for most of the disturbances, designers can concentrate on the robust stability, rather than the disturbance rejection, of the feedback system.

Despite of these advantages, only a small minority of all auxiliary measurements which could concievably be used for feedforward/disturbance decoupling control in industry are in fact used in this way. There are several reasons for this. The need for an accurate process model, which requires a time consuming and expensive modelling effort, has already been mentioned. For nonlinear systems, feedforward controllers based on a linear design will depend on the operating point. If perfect cancellation is not possible or desired, the feedforward design will depend on the type of disturbance, which is often time-varying. The use of *adaptive controllers* with feedback and feedforward, which attune themselves to system and disturbance properties, is a very promising solution to these problems. Such regulators exist, and some are now commercially available. See, for example, Bentsson and Egardt (1984). The adaptive feedforward regulators proposed so far, commercial or not, have, however, several unfortunate limitations. In particular, they are unable to provide

optimal feedforward control for non-minimum phase systems. Additional limitations are discussed in Section 5.1.

The possibilities inherent in the feedforward principle, and the problems described above, have motivated the present work. The goal has been to develop adaptive regulators which use auxiliary measurements in an optimal way. The control strategies should include non-minimum phase as well as minimum phase systems, and deterministic as well as stochastic disturbances. Control based on discrete time linear models with one input has been considered.

The example below indicates the results which can be obtained with adaptive feedforward control. A discrete time system (system 1 from Chapter 6) is affected by a square wave disturbance. Figure 1.3 shows the output of the uncontrolled system. In Figure 1.4, adaptive control (the LQG algorithm of Section 5.2) is applied. The regulator measures the output y(t) and an additional auxiliary output, which is affected by the input and the disturbance. At the beginning, the regulator filters are zero. The first 20 samples are used for open loop identification of a model of the system. White noise is used as an input signal. At time 20, the regulator is connected. From time 40, virtually complete cancellation of the disturbance is achieved.

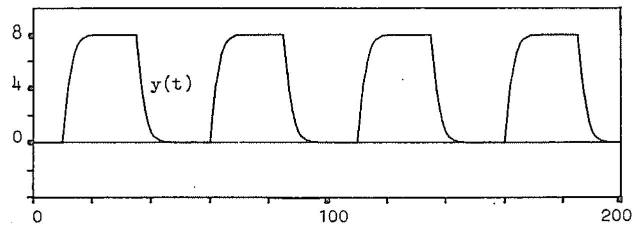

Figure 1.3 The output of the uncontrolled system. (Desired output value: y(t)=0.)

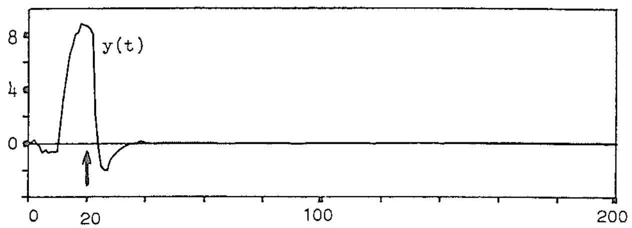

Figure 1.4 The result of adaptive disturbance decoupling control.

OUTLINE OF THE THESIS

(A more complete summary is obtained by reading the introductory pages of each chapter, the concluding discussion of Chapters 3 and 6, and the conclusions of Chapter 7.)

Chapter 2 describes different methods which have been suggested in the literature for designing feedforward and disturbance decoupling regulators. It is indicated which methods are, and which are not, reasonable as a basis for adaptive control.

In Chapter 3, a solution to the feedforward control problem, based on a linear quadratic gaussian approach in polynomial form is presented. Optimal feedforward regulators for scalar non-minimum phase systems can be computed in a simple way, using a spectral factorization and a polynomial equation. Auxiliary measurement signals affected by the input are included in the design. The optimal structure of regulators combining feedback and feedforward is clarified. When this regulator structure is used, the feedforward control quality is shown to be independent of the choice of stabilizing feedback. LQG feedback-feedforward regulators can be designed by a simple two step procedure: The feedback is first optimized with respect to unmeasurable disturbances. The feedforward is then optimized with respect to the measurable disturbance.

Feedforward filters can be optimized for deterministic disturbances (steps, sinusoids) and for nonstationary stochastic disturbances. Integrating regulators can be included in the design. Input estimation (deconvolution) problems are shown to be very closely related to feedforward problems. One problem can in fact be transformed into the other.

In Chapter 4, the application of the LQG method for off-line design is considered. The solution of a feedforward problem for a non-minimum phase system is discussed in some detail: Load feedforward power frequency control of hydro power stations.

In Chapters 5 and 6, two adaptive feedback control algorithms are extended with adaptive feedforward: An LQG self-tuner and explicit criterion minimization. Their performance is tested by simulation, and compared to minimum variance and extended minimum variance self-tuners with feedforward terms. Both the LQG self-tuner and explicit criterion minimization attain very good control behaviour. With the help of some simple safeguards, their performance seems to be robust in a wide range of situations. Some ways of reducing the computational load in LQG self-tuners are also discussed.

In the concluding Chapter 7, directions for further research are indicated. The development of adaptive algorithms for signal processing problems involving cancellation or deconvolution is of special interest. The hardware development of recent years has made adaptive control and signal processing based on polynomial LQG methods practical, despite of the relatively large computational requirements.

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2. FIVE APPROACHES TO FEED-FORWARD AND DISTURBANCE DECOUPLING CONTROL

This chapter reviews different feedforward/disturbance decoupling regulator design methods in the literature. In a critical evaluation of the approaches, it is discussed which ideas are of use for attaining our design goal: Adaptive algorithms that remove the difficulties now preventing widespread use of feedforward control. Let us review some of the complications encountered in designing a feedforward filter C in the problem described by Figure 2.1. For simplicity, all signals are scalars.

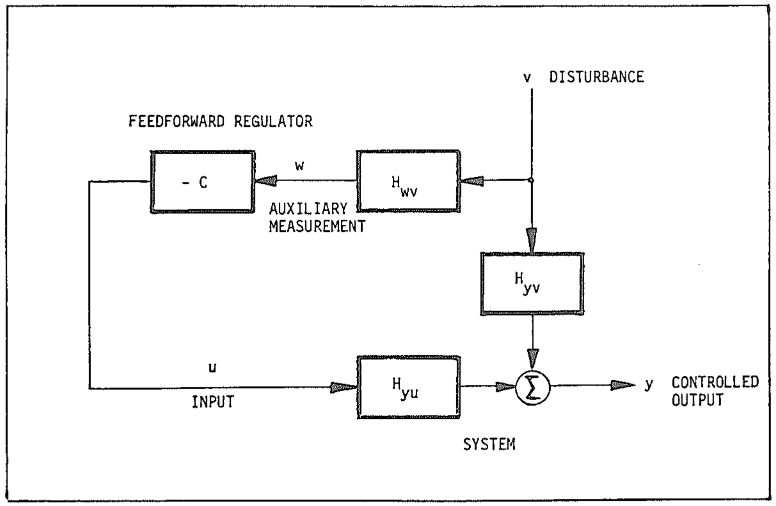


Figure 2.1 Feedforward control. H_{wv}, H_{yv} and H_{yu} are continuous or discrete time rational transfer operators. C is the controller to be designed.

With the disturbance influence on the output being described by

$$y = (H_{yv}-H_{yu}CH_{wv})v \qquad (2.1)$$

perfect feedforward, i.e. complete disturbance cancellation, would require a control law given by

$$u = -Cw = -\frac{H_{yv}}{H_{yu}H_{wv}} w \qquad (2.2)$$

for scalar systems. Unfortunately, even when accurate noise-free measurements of w can be obtained, and an accurate model of the system is available, perfect feedforward control may well be unobtainable or impractical, for a number of reasons:

- o Stability must be considered. In (2.1) and (2.2), the uncontrolled system and C must be stable. A good design method should not just ignore the question about what to do when perfect feedforward would lead to instability. It should provide a stabilizing regulator with the best (in some sense) disturbance rejection properties.

- o The controller C must be proper. The filter (2.2) will often require multiple differentiation of w in continuous time, or measurement of future values of w in discrete time. In such cases, a sensible design method should provide a proper (causal) filter with good disturbance rejection properties. (Continuous time systems with delays will not be considered. In such problems, noncausal controllers are, of course, also out of the question.)

- o Even if stable and causal, the filtering (2.2) might result in unrealistically large input signals. A design method should provide provisions for tradeoffs between input energy and disturbance rejection.

Additional aspects have to be kept in mind:

- o In some feedforward control problems, disturbances are best described as stochastic. In others, they are deterministic. (Loosely speaking, they occur infrequently, and their shape is known.) A design strategy handling both types of disturbances would be valueable.

- o It must be taken into account that the auxiliary measurement w may very well be affected by the input u. It is these kinds of feedforward control problems that will be called disturbance decoupling problems.

The common feedforward design methods in the process industry are discussed briefly in Section 2.1. Sections 2.2 and 2.3 review state space and input-output approaches to disturbance decoupling. The disturbance prediction and inferential control viewpoints are mentioned in Section 2.4. Optimization methods are introduced in Section 2.5.

2.1 STATIC FEEDFORWARD AND SIMPLE LEAD/LAG FILTERS

In the process industry, the use of auxiliary measurements can improve control of numerous processes. In product lines with multiple steps, measurements of product quality or flow from earlier steps can, for example, be used for feedforward. Descriptions of application of feedforward can be found, for example, in Shinskey (1967) and Smith and Corripio (1985). (It should be mentioned that auxiliary measurements are of use not only for feedforward: Cascade control and gain scheduling are other alternatives.) Feedforward control is used almost exclusively in combination with feedback. (The best way to combine feedback with feedforward signals in linear regulators will be discussed in Chapter 3.) Feedforward can increase the bandwidth where good disturbance rejection is achieved. It improves the disturbance transient performance within the feedback regulator settling time. This is frequently achieved with C being a *static gain* calculated from a simple static model (2.1):

$$C = H_{yvstat}/H_{yustat}H_{wvstat}$$

In some applications, nonlinear static feedforward is of use. The signal u=-Cw is then multiplied or divided with the feedback control signal.

When static feedforward is insufficient, *dynamic compensation* with a stable proper approximation of (2.2) can be used. If (2.2) is a stable but nonproper continuous time filter, the approximation

$$C(s) = \frac{H_{yv}(s)}{H_{yu}(s)H_{wv}(s)} \frac{1}{(1+\tau s)^n} \tag{2.3}$$

with n chosen such that the filter is strictly proper, could be implemented. Thus, additional poles are introduced to avoid differentiation of the feedforward signal w. A tradeoff between noise sensitivity and regulator bandwidth determines τ. There are complications with this design method. A filter $1/(1+\tau s)^n$ not only reduces the high-frequency gain; it also introduces a phase lag. The filter (2.3) may very well have a large gain at frequencies where the phase of $1/(1+\tau s)^n$ is $\approx -180^0$. In this frequency range, (2.3) will *amplify* the disturbances

2.2 THE GEOMETRIC APPROACH: MAKE THE DISTURBANCE UNOBSERVABLE

Disturbance decoupling problems can be seen in a state space context. Let us assume that all states of the system are measurable, but that the disturbance v itself cannot be measured. Disturbance decoupling is attained when v has no effect on an output vector y. Consider a controllable continuous time system

$$\dot{x} = Ax+Bu+Dv$$
$$y = Cx \tag{2.4}$$

where $x \in R^n$, $u \in R^m$, $v \in R^q$ and $y \in R^\ell$.

Wonham (1974) derived the following condition for *disturbance decoupling by state feedback* to be possible: let $\mathcal{V}^*$ be the maximal (A,B) invariant subspace of kernel C.

Disturbance decoupling by state feedback is possible if and only if

$$\text{Im } D \subset \mathcal{V}^* \tag{2.5}$$

A necessary condition for (2.5) is

$$CD = 0$$

In other words: A state feedback u=-Fx should be constructed which makes as much as possible of the state space unobservable from y. This is the space $\mathcal{V}^*$. The disturbance v will be unobservable from the output y if and only if D projects it into $\mathcal{V}^*$. The disturbance will then never be able to affect the complement of $\mathcal{V}^*$, which is observable from y. The subspace controllable from v will thus be unobservable from y.

A state feedback designed exclusively to make a part of the state space unobservable could have unintended side-effects, such as making the system unstable. A more realistic version of the condition (2.5) is obtained by substituting V_g^* for V^*. V_g^* stands for the largest subspace that can be made unobservable from y by a *stabilizing* state feedback u=-Fx.

Algorithms for computing the space V^*, and the corresponding state feedback F, can be found in Wonham (1974). Kümmel et al (1984) have developed a simplified design method. Kümmel et al (1984) and Takamatsu et al (1979) have tested the geometric design by simulations on a linear binary distillation column model, and have found it to work well.

In distillation columns, the whole state vector is measurable. In most problems this is not the case. The problem of *disturbance decoupling by measurement feedback*, using the measurement vector

$$w = Hx \quad ; \quad w \in R^p \tag{2.6}$$

and a proper dynamic compensator, has been solved by Akashi and Imai (1979) and Schumacher (1980). They combined the state feedback with an observer for systems with unknown inputs. These solutions may, however, be unstable. The problem is generically solvable (if we ignore the stability problem) if and only if the following mild conditions are satisfied:

$$CD = 0, \; m \geq \ell \quad \text{and} \quad p \geq q \tag{2.7}$$

Thus, the number of inputs should be no less than the number of controlled outputs. The number of measurements should be no less than the number of disturbances. The conditions (2.7) were derived by Willems and Commault (1981). They also presented a solvability condition and a solution method which take stability into account.

Bhattacharyya et al (1983) have derived an illuminating interpretation of the condition (2.5). Disturbance decoupling by state feedback for continuous time systems will be possible if and only if the system can be decomposed into two subsystems interconnected in the way described by Figure 2.2.

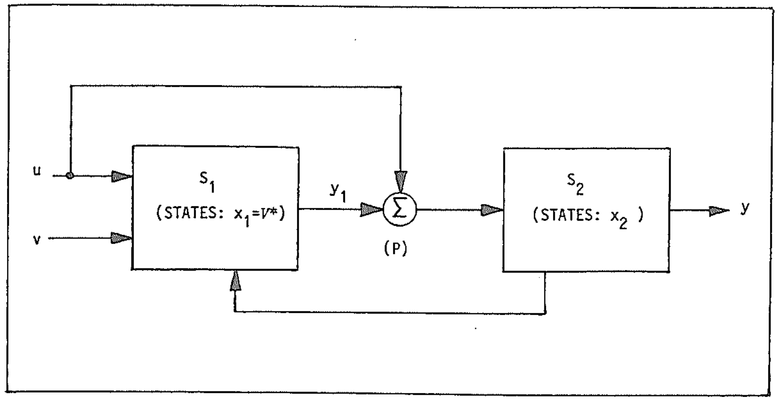


Figure 2.2 The structure of continuous time systems (2.4) for which disturbance decoupling by state feedback is possible. The state feedback law $u=-y_1-Lx_2$ will prevent the disturbance vector v from affecting the output y. The states of the subsystems S_1, which are made unobservable from y, constitute the subspace V^*.

The important property shared by all continuous time systems where disturbance decoupling is possible is evident in Figure 2.2: The disturbance influence passes through a point (P) which is directly influenced by the input.

By measuring y_1 (which is measurable since all states are measurable) the path from v to y can be effectively blocked. If a feedback law $u=-y_1-Lx_2$ can be found which stabilizes the system, the disturbance decoupling problem is solved. The state subspace V^* will simply be the states x_1 of the subsystem S_1, which are made unobservable from y.

As long as stability and the topology of Figure 2.2 is preserved, parameter variations of the system will not affect the disturbance decoupling. In this sense, disturbance decoupling by state feedback is robust, contrary to what may be suspected from an inspection of the condition (2.5) for the system (2.4).

Recently, Linnemann (1986) has in a similar way characterized continuous time systems (2.4),(2.6) where disturbance decoupling by measurement feedback is possible: In the signal flow path, the disturbances must first pass through a point which is directly observed via w. Then, they must pass through a point which is directly acted upon via the control u. (Further conditions are necessary to assure stability.) The nice robustness properties of the state feedback case do unfortunately not carry over to output feedback design.

There is an important problem which the methods mentioned above do not address: What are we to do if perfect decoupling is impossible because of the structure of the system,the stability requirement, or, in the measurement feedback case, because no proper compensator can be found?

The answers to these problems which have come up within the geometric state space approach are not very helpful. Willems (1981),(1982a) has studied the problem of "*almost disturbance decoupling*" by high gain feedback. In theory, a regulator can often achieve disturbance rejection arbitrarily close to perfect cancellation if some state feedback gain magnitudes are made sufficiently large. This approach can, however, lead to disasters if even a slight amount of measurement noise is present. Feedback laws which allow multiple differentiation of the states or of the measurement vector have been suggested. See, for example, Imai et al (1981), Willems (1982b), Seraji (1986), Wang and Tsuchiya (1985) or Armentano (1985). In continuous time, such control laws could concievably be implemented if they were complemented with low-pass filters to limit the high-frequency gain. As was noted at the end of Section 2.1, this approach is, however, not without problems.

ASYMPTOTIC DISTURBANCE REJECTION

While a proper stabilizing regulator often cannot achieve perfect cancellation, asymptotic rejection of certain classes of disturbances will in general be possible through either main output feedback, auxiliary output feedforward or state feedback. After an initial

transient, deterministic disturbances such as steps, ramps, exponential functions and sinusoids can be cancelled asymptotically, as long as the input does not attain any bound. The use of integrating feedback to cancel static control errors asymptotically is a well known example. Design methods for asymptotic cancellation of deterministic disturbances have been developed by Davison (1973), (1976) and Johnson (1986). These techniques require a disturbance model in the form of a difference or differential equation.

Deterministic disturbances are often reasonable descriptions of disturbances occuring in practice. However, design methods concentrating only on asymptotic disturbance rejection might lead to a bad transient behaviour.

2.3 ZERO THE TRANSFER FUNCTION FROM DISTURBANCE TO OUTPUT

The disturbance decoupling problem has been studied for many years from a transfer function viewpoint. In the Soviet Union, theoretical investigations have been proceeding since the 1930's. Disturbance cancellation has there been called the *invariance principle*. (See references in Kučera (1983b) and Preminger and Rootenberg (1964).)

Let the continuous or discrete time system be described by

$$\begin{pmatrix} y(t) \\ w(t) \end{pmatrix} = \begin{pmatrix} H_{yu} & H_{yv} \\ H_{wu} & H_{wv} \end{pmatrix} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \qquad (2.8)$$

where H_{yu} etc are rational matrices in the differential operator p or the forward shift operator q. $y \in R^{\ell}$, $w \in R^p$, $u \in R^m$ and $v \in R^q$ are the controlled output, measured output, input and disturbance, respectively. (If some controlled signals are also measured, they are here included in w. In Chapter 3, the notation w is reserved for auxiliary measurements which differ from y.)

The goal is to find a proper (causal) regulator, which stabilizes the controlled system

$$u(t) = -Cw(t) \qquad (2.9)$$

In addition, the transfer function from v to y should be zero, i.e.

$$0 = H_{yv} - H_{yu}C(I+H_{wu}C)^{-1}H_{wv} \qquad (2.10)$$

This equation is to be solved with respect to C. For scalar systems, Kučera (1983) has presented a solution to the problem, which provides several insights. It is discussed below, in a somewhat simplified form.

DISTURBANCE DECOUPLING IN SCALAR SYSTEMS

Write the system (2.8), with ℓ=p=m=q=1 in polynomial form with a common denominator

$$\begin{pmatrix} y(t) \\ w(t) \end{pmatrix} = \frac{1}{A} \begin{pmatrix} G & H \\ B & C \end{pmatrix} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \qquad (2.11)$$

(In Chapter 3, a different polynomial notation is used.) To simplify the notation, assume A and C to be stable. Assume G/A and B/A to be strictly proper. As can be easily verified, the regulator of minimal degree zeroing the transfer function from v(t) to y(t) is then given by

$$u(t) = -\frac{HA}{GC-BH} w(t) \qquad (2.12)$$

with common factors cancelled.

For scalar systems, this way of solving the problem is much simpler than the use of algorithms from the geometric state-space approach.

The regulator (2.12) will be *proper* if and only if

$$\deg H + \deg A \le \max\{\deg B + \deg H, \deg C + \deg G\}$$

since degA>degB this is equivalent to

$$\deg H + \text{dag} A \le \deg C + \deg G$$

or

$$\deg A - \deg H \ge (\deg A - \deg C) + (\deg A - \deg G) \qquad (2.13)$$

In discrete time, (2.13) means: The time delay for the disturbance in the control signal path (through C/A, the regulator and G/A) should be no larger than the delay from the disturbance to output directly, through H/A. Otherwise, at least parts of a disturbance would have time to pass through the system before the regulator could do anything about it. Note that the presence or absence of an input influence on the measurement w (the polynomial B) does not affect the possibilities to achieve disturbance decoupling. Also note that (2.13) implies that disturbance decoupling through main output feedback ($w(t)=y(t)$, $H=C$) is impossible since G/A is strictly proper (strictly causal) i.e. $\deg A - \deg G > 0$. A generalization of the degree condition (2.13) to multivariable systems has been given by Bhattacharyya (1982).

Let D_{GH} contain all common factors of G and H

$$G = D_{GH}G_o \quad ; \quad H = D_{GH}H_o$$

The regulator (2.12) will result in a *stable* controlled system if and only if

$$G_o \text{ is stable} \qquad (2.14)$$

When $D_{GH}=1$, this implies that the transfer function from u(t) to main output y(t) must have a stable inverse, i.e. be *minimum phase*. This stable inverse condition carries over to multivariable systems. It is an important reason why exact disturbance decoupling often cannot be attained, especially for sampled systems in which G is frequently unstable. When $D_{GH}\neq 1$, the condition (2.14) does, however, imply that exact disturbance decoupling will be possible for certain non-minimum phase systems: Cases when the unstable zeros are common factors of G and H, i.e. are included in D_{GH} while G_o is stable. Such cases will be given an interpretation in Section 3.3.

The inverse of the system (2.11) is given by

$$\begin{pmatrix} \frac{AC}{GC-BH} & \frac{-HA}{GC-BH} \\ \frac{-AB}{GC-BH} & \frac{AG}{GC-BH} \end{pmatrix}$$

We recognize the disturbance decoupling regulator (2.12) as the (1,2)-element of the inverse. This is no coincidence. It has been proven by Sternad (1984) that if a disturbance decoupling regulator (2.9) exists, it will, under mild conditions on the system, be a (1,2)-block of an inverse to (2.8). (A left inverse of $\ell+p \geq m+q$. A right inverse if $\ell+p<m+q$.)

THE MULTIVARIABLE PROBLEM

For multivariable systems, a stable proper rational matrix solution X to the equation

$$H_{yv} = H_{yu}XH_{wv} \qquad (2.15)$$

is sought, where X represents, cf (2.10),

$$X = C(I+H_{wu}C)^{-1}$$

It can then be shown that the corresponding regulator,

$$C = (I-XH_{wu})^{-1}X \qquad (2.16)$$

will be proper and stabilizing. Different methods for solving this problem have been developed by Pernebo (1981), Özgüler and Eldem (1985) and Antoulas (1985). Using the Youla parametrization, Antoulas (1985) provides a method for parametrizing all stabilizing proper rational matrices C which attain disturbance decoupling for a given problem.

This short review of polynomial and transfer function methods has provided some important insights into the problem, and a simple solvability condition and solution for the scalar case. We do, however, still not have any answer to the question posed at the end of Section 2.2: What should the designer do in the numerous cases when perfect feedforward/disturbance decoupling cannot be attained?

2.4 PREDICT THE DISTURBANCE INFLUENCE, AND CANCEL IT

If the disturbance can be measured through an auxiliary output w, its influence on the main output y can be predicted, if a good model of the system is available. A control action can then be computed which counteracts the predicted disturbance influence. This approach to disturbance decoupling, *inferential control*, has been proposed by Brosilow and Tong (1978). Figure 2.3 explains the structure of their controller. $\hat{H}_{wu}$ etc are estimates of system transfer functions.

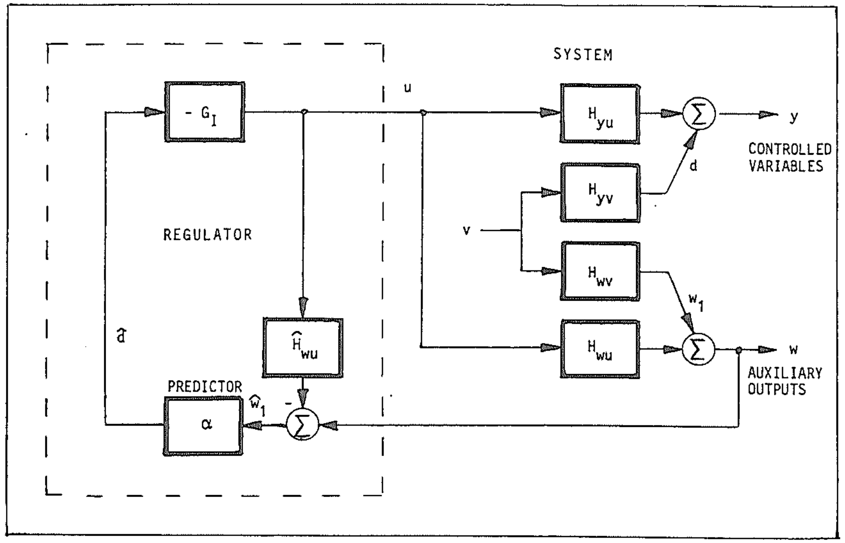


Figure 2.3 Inferential control. The vector y is to be controlled through feedback from the auxiliary output vector w. From a measurement of w, with an estimate $\hat{H}_{wu}u$ subtracted away, a prediction $\hat{d}$ of the disturbance influence d on the main output is computed. The prediction is filtered through an inverse of H_{yu} to get a control action that will cancel d.

In the scalar case, the predictor α would ideally be given by

$$\hat{d} = \hat{H}_{yv}\hat{v} = \frac{\hat{H}_{yv}}{\hat{H}_{wv}}\hat{w}_1$$

The filter G_I should, ideally, equal $1/\hat{H}_{yu}$. These expressions frequently have to be modified, since $G_I\alpha$ must be stable. The regulator transfer function

$$C = (I-G_I\alpha\hat{H}_{wu})^{-1}G_I\alpha \quad , \quad u = -Cw \qquad (2.17)$$

must be causal. For multivariable systems for which perfect disturbance decoupling is possible, the transfer function and prediction approaches lead to the same problem. Compare (2.17) to (2.16) with $X=G_I\alpha$. For scalar systems, the transfer function (2.17) will be the same as (2.12), with A as an additional stable common factor. ($G_I=A/G$, $\alpha=H/C$, $\hat{H}_{wu}=B/A$.)

If perfect disturbance decoupling is unattainable because of the properness condition (2.13), or its multivariable equivalent, disturbance prediction still works. For stochastic disturbances v, the approach provides a minimum variance feedforward regulator. (It will be defined in Section 5.1, equation (5.12).)

If the system is non-minimum phase (H_{yu}^{-1} unstable) there are, however, problems. It is then not obvious what kind of stable filter αG_I gives the best disturbance rejection for a given class of disturbances v. This question will be clarified in Chapter 3.

Adaptive combined feedback-feedforward regulators based on prediction have been developed previously. We will discuss them in Section 5.1. Adaptive signal processing algorithms, used in applications involving cancellation, are also mostly based on the prediction approach. See, for example, Widrow et al (1975).

The regulator in Figure 2.3 contains a useful element, which will be used later on: By subtracting away the input influence on the signal w, the problem is reduced to the simpler feedforward control problem of Figure 2.1.

2.5 MINIMIZE THE DISTURBANCE INFLUENCE

If perfect disturbance cancellation is unobtainable, or requires unrealistic input signals, optimization of the regulator is a natural approach. It is also the most general: If perfect feedforward is possible and desired, a good optimization algorithm should provide a regulator attaining it as a limiting case. If not, it should provide the best (in some sense) compromise.

PARAMETERIC OPTIMIZATION OF FEEDFORWARD FILTERS

The parameters of a filter with a given structure can be optimized numerically, if a model of the system and of the disturbance known. For a given parameter vector, a criterion value is computed from the response of the controlled model, for example the disturbance step response. A numerical minimization routine, for example a gradient method, can be used to optimize the parameters. Isermann (1977) and Takegaki and Matsui (1985) have suggested such approaches. In Section 5.3, an adaptive algorithm is described which is also very useful for off-line optimization of regulators with predetermined (not necessarily optimal) structure.

WORST CASE DESIGN IN THE FREQUENCY DOMAIN

The H_∞ approach (Francis 1985, Francis 1987) could be used for optimizing feedforward filters. If the properties of the disturbance v were considered unknown, the result would be a design which minimizes the largest gain of the disturbance transfer function. The worst case (the case when the disturbance happens to be concentrated in the frequency range where the disturbance rejection is worst) is optimized, with the added constraint of stability. This would be a sensible approach in many situations. Self-tuning regulators based H_∞-control theory, if they were developed, could become interesting competitors to the regulators to be discussed in the following.

LINEAR QUADRATIC GAUSSIAN DESIGN

Linear quadratic optimization fulfills all requirements stated at the beginning of this chapter. In contrast to prediction approaches, LQ methods provide stable regulators for non-minimum phase systems, under the mild conditions of stabilizability and detectability. The input energy can be limited with an input penalty.

The linear quadratic approach will be used, and compared to prediction-based methods, in the following. LQ state feedback design with measurable stochastic disturbances is described in Kwakernaak and Sivan (1972). Since the whole state vector is rarely measurable, we will concentrate on (auxiliary) output feedback problems. Infinite horizon criteria will be used. State-space LQ-design leads to an unfortunate problem. If exact cancellation of disturbances were desired, it would require the use of a cost function without input penalty. This case, cheap control, leads to problems with the solution of the Riccati equation. Input-output model based methods avoid these difficulties. They also lead to simpler calculations, at least for scalar systems. Therefore, a polynomial-based LQG design will be the main technique used.

3. FEEDFORWARD LQG REGULATOR DESIGN

A method for designing feedforward regulators, described previously in Sternad (1985), (1986a), will be presented. An infinite horizon quadratic criterion is minimized with respect to parameters in the controller transfer functions. A *polynomial LQG* technique is used. The method leads to simpler design calculations (polynomial equations) than the corresponding state-space method (Riccati difference equations). It is suitable both for off-line design (see Chapter 4) and as a basis for adaptive control (Chapters 5 and 6).

Since a large majority of practical regulator design problems can be decomposed into single input problems, we will discuss scalar systems. Because computer control now dominates in process control applications, discrete time regulators will be discussed. Even for continuous time systems with stable inverses, sampling often leads to sampled systems with unstable inverses. Cf Åström, Hagander and Sternby (1984). Therefore, a feedforward design method handling non-minimum phase systems is needed.

Let us emphasize some features of the method, to be explained and discussed in the following:

o Optimal feedforward regulators may be calculated for non-minimum phase systems, and for systems where large time delays make perfect cancellations impossible.

o Problems where the auxiliary measurement signal is affected by the input can be handled.

- o Frequency dependent tradeoffs between input energy and disturbance rejection can be included in the design.

- o The achievable feedforward control quality is not affected by the choice of feedback, if the optimal regulator structure is used.

- o A main output feedback may first be optimized with respect to unmeasurable disturbances. The auxiliary output feedforward is then optimized with respect to the measurable disturbance. The combined regulator resulting from this step-by-step procedure is shown to be optimal.

- o Feedforward filters can be optimized for deterministic disturbances (steps, sinusoids) and for nonstationary stochastic disturbances. Integrating regulators can be included in the design.

- o Input estimation (deconvolution) problems are very closely related to feedforward problems. One can in fact be transformed into the other. Thus, equations for solving feedforward control problems can be used to compute deconvolution filters. They may also be used to solve noise cancelling signal processing problems.

- o As with sampling deadbeat feedback regulators, inter-sample oscillations may occur. Such problems are easily handled by minimizing a criterion with a small nonzero input penalty. For a given input energy, LQG regulators achieve the best rejection of stochastic disturbances (measured in continuous time), among all discrete time control strategies.

Optimal filtering and feedback control problems were originally solved by the Wiener-Hopf technique (Wiener 1949, Newton et al 1957, Youla et al 1976). In the 1960's, state space methods were

refined. With the minimum variance regulators of Åström (1970) and Peterka (1972), it became popular to approach such problems via polynomial equations. See, for example, Shaked (1976), Kučera (1979a,b,1983a) and Grimble (1984, 1985). In Appendix A2, a method for finding optimal controller polynomials is presented. It is a simple alternative to other methods discussed in the literature. Criterion derivatives with respect to the parameters are expressed as path integrals around the unit circle, using Parseval theorem. By selecting the parameters such that the integrands are analytic inside the integration path, all integrals are made to vanish. The conditions for this to happen define polynomial equations, which give the optimal filter.

A solution to the combined feedback-feedforward LQG control problem was presented by Peterka (1984). It applies to cases when the measurable disturbance is described by an autoregressive process. This solution does, however, not exhibit the simplifying decoupling of the feedback design from the feedforward regulator design. Recently, Grimble (1986) has derived a regulator equivalent to the one to be presented in Theorem 3.6, using another proof technique.

3.1 PROBLEM FORMULATION

Consider a linear time invariant system

$$\begin{pmatrix} y(t) \\ w(t) \end{pmatrix} = \begin{pmatrix} H_{yu} & H_{yv} \\ H_{wu} & H_{wv} \end{pmatrix} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} + \begin{pmatrix} \ell(t) \\ m(t) \end{pmatrix} \qquad (3.1)$$

where H_{yu} etc are discrete time rational transfer functions. The *main output* $y(t)$, the *auxiliary measurement* $w(t)$ and the *input* $u(t)$ are all scalar signals. The system is affected by a scalar *main disturbance* $v(t)$ and possibly by additional disturbances $\ell(t)$ and $m(t)$.

Our goal is to control $y(t)$ in an optimal way with a linear regulator. Feedforward from $w(t)$, possibly combined with feedback of $y(t)$ will be used:

$$u(t) = - G_w w(t) - G_y y(t) \qquad (3.2)$$

In particular, we may wish to achieve disturbance decoupling, i.e. to zero the transfer function from $v(t)$ to $y(t)$ with a stabilizing causal regulator. Let the problem of designing an optimal regulator (3.2) for the system (3.1) be called the *scalar disturbance decoupling problem*, although perfect disturbance decoupling may not be possible. Special cases where $H_{wu}=0$ are called *feedforward control problems*, cf Figure 2.1.

The following assumptions about the system will be made in this chapter:

- o The true system is described exactly by the linear model (3.1).

- o The system is detectable from (y,w) and stabilizable from u.

- o Since the purpose is to control $y(t)$ (and not $w(t)$), stability of H_{wu} and H_{wv} is assumed. (In Section 3.5, we do, however, consider systems where H_{wv} has poles on the stability limit.) In addition, H_{wv}^{-1} is assumed to be stable.

- o H_{yu} does not have zeros on the unit circle. (If it does, a stable optimal solution may not exist.)

- o For simplicity, $m(t)$ is assumed to be zero. (Problems where $m(t) \neq 0$ are investigated in the adaptive control experiments of Chapter 6.)

- o The time delay of H_{wv} is not larger than the time delay of H_{yv}. (Otherwise, the use of the auxiliary output would be of limited interest, since changes in $v(t)$ could be noticed faster in the main output $y(t)$.)

From the last assumption, $H_{yv}H_{wv}^{-1}$ is causal. The system (3.1) may then be rewritten as a causal model with $w(t)$ as an input:

$$\begin{aligned} y(t) &= (H_{yu} - H_{yv}H_{wv}^{-1}H_{wu})u(t) + H_{yv}H_{wv}^{-1}w(t) + \ell(t) \\ w(t) &= H_{wu}u(t) + H_{wv}v(t) \end{aligned} \qquad (3.3)$$

The following polynomial parametrization will be used:

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + q^{-d}D(q^{-1})w(t) + C(q^{-1})e(t) \qquad (3.4)$$

$$H(q^{-1})w(t) = q^{-n}N(q^{-1})u(t) + G(q^{-1})v(t) \qquad (3.5)$$

where A, B,... are polynomials in the backward shift operator q^{-1} with degrees na,nb,... Nonzero time delays k and n are assumed while $d \geq 0$. The polynomials A, C, H and G are monic (their leading coefficient is 1). Stability of C, G and H is assumed. All unmeasurable disturbances affecting y(t) are represented by (C/A)e(t). The disturbances are represented by equivalent stochastic models, with e(t) and v(t) being uncorrelated white stationary random sequences. They have zero means and variances Λ_e and Λ_v, respectively. Nonzero means and nonstationary disturbances are discussed in Section 3.5.

The infinite horizon criterion

$$J = \lim_{N\to\infty} \frac{1}{2N} \sum_{t=0}^{N} Ey(t)^2 + \rho E(\tilde{\Delta}(q^{-1})u(t-k))^2 \qquad (3.6)$$

is to be minimized.

The input penalty ρ and the polynomial $\tilde{\Delta}(q^{-1})$ are chosen by the user. The special case $\rho=0$ (minimum variance control) is discussed in Section 3.3. An example where *differential input penalty* ($\tilde{\Delta}(q^{-1})=1-q^{-1}$) has to be used is discussed in Section 4.2.

(The k step delay of u(t) in (3.6) is unimportant for the infinite horizon criterion value. It is included because it is crucial in one of the adaptive algorithms, explicit criterion minimization.)

With the parametrization (3.4),(3.5), five common feedforward control/disturbance decoupling problems can be represented:

o Situations when an *unmeasurable process variable* y(t) is to be controlled by feedback from a measured signal w(t). A regulator (3.2) with $G_y=0$ is then used, and the system is assumed to be stable. This is also called *inferential control* (Brosilow and Tong, 1978).

o *Scalar disturbance decoupling* problems where w(t) may be affected by the input. Control problems for mass transport systems often have this structure. (u(t) is then the transport speed while y(t) and w(t) are two measurement points along the transport path.)

o Measurable *load disturbances*. Set $N(q^{-1})=0$ and let $w(t)=(G/H)v(t)$ represent a stochastic model of the disturbance. Often, w(t) is added directly to the output. Set d=0 and $D(q^{-1})=A(q^{-1})$ in such cases.

o Measurable *feed disturbances* (entering at the input). $Ay(t)=B(u(t)+w(t))$. Set $N(q^{-1})=0$, d=k and $D(q^{-1})=B(q^{-1})$.

o *Ratio control*. See Figure 3.1. In our terminology, $q^{-k}B(q^{-1})/A(q^{-1})$ represents the servo system and u(t) is its reference input. Set N=0 and $q^{-d}D(q^{-1})=-cA(q^{-1})$, where c is the desired ratio f(t)/w(t).

Other representations of feedforward problems can easily be transformed to the form (3.4),(3.5). See, for example, Figure 3.2.

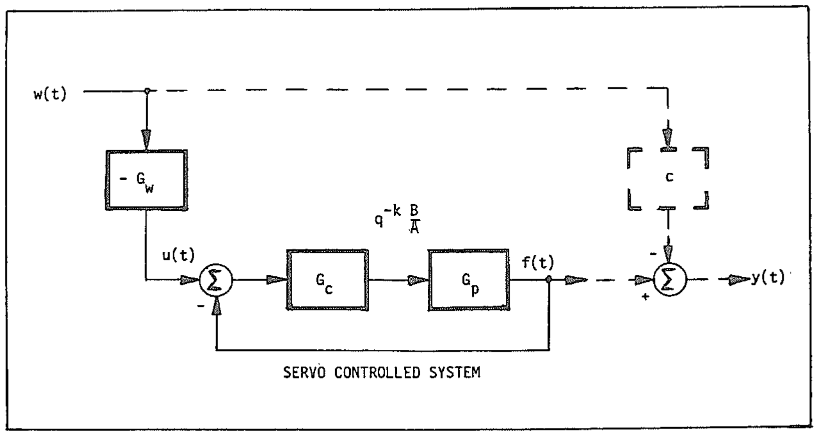


Figure 3.1 Ratio control seen as a feedforward problem. The reference input u(t) to a servo controlled system is to be calculated such that there is a constant ratio c between its output f(t) and a signal w(t). In other words, we wish to minimize the deviation

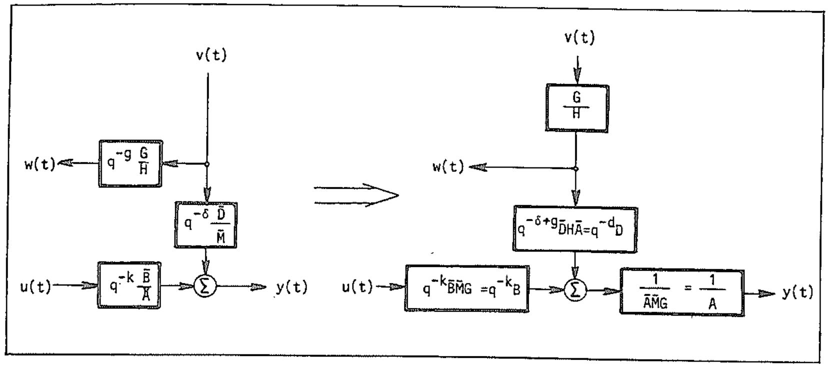


Figure 3.2 Transformation of a system with measurement dynamics $q^{-g}G(q^{-1})/H(q^{-1})$ and unequal transfer function denominators to the form (3.4),(3.5).

REMARKS ON THE NOTATION

The following different types of polynomials will be of use:

o $D \triangleq D(z) = d_o+d_1z+\ldots+d_{nd}z^{nd}$, where z replaces q^{-1}.

o *Conjugate polynomials*: $D_* \triangleq D(z^{-1}) = d_o+d_1z^{-1}+\ldots+d_{nd}z^{-nd}$

o *Reciprocal polynomials*: $\bar{D} \triangleq z^{nd}D_* = d_oz^{nd}+d_1z^{nd-1}+\ldots+d_{nd}$

The polynomial arguments (q^{-1}, z and z^{-1}) are often omitted to simplify the notation. The zeros of $\bar{D}$ are the zeros of D, reflected in the unit circle. If D is stable, $\bar{D}$ will be unstable. D_s and D_u denote the stable and unstable parts of a polynomial D.

3.2 OPTIMAL FEEDFORWARD AND FEEDBACK REGULATORS

SOME PRELIMINARY RESULTS

Problems where $N(q^{-1})\neq0$ are considered in Section 3.4. We now specialize on systems where w(t) is unaffected by the input:

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t)+q^{-d}D(q^{-1})w(t)+C(q^{-1})e(t) \tag{3.7}$$

$$w(t) = \frac{G(q^{-1})}{H(q^{-1})}v(t)$$

where H, C and G are assumed stable and (A,B) have no unstable common factors. $B(q^{-1})$ has no zero on the unit circle.

The regulator (3.2) is parametrized in the following way:

$$R(q^{-1})u(t) = -\frac{Q(q^{-1})}{P(q^{-1})}w(t)-S(q^{-1})y(t) \tag{3.8}$$

R and P are monic and P is required to be stable. The choice of regulator structure will be motivated later in this section. The corresponding parameter vector is

$$\theta = (p_1,\ldots,p_{np},\ Q_o,\ldots,Q_{nQ},\ r_1,\ldots,r_{nr},\ s_o,\ldots,s_{ns})^T \tag{3.9}$$

Simple calculations give the following input and output signals when (3.8) is used to control (3.7):

$$y(t) = \frac{MG}{P\alpha H}v(t)+\frac{RC}{\alpha}e(t) \tag{3.10}$$

$$u(t) = -\frac{FG}{P\alpha H}v(t)-\frac{SC}{\alpha}e(t) \tag{3.11}$$

where

$$\alpha \triangleq AR+q^{-k}BS \qquad (3.12)$$

$$M \triangleq q^{-d}DRP-q^{-k}BQ \qquad (3.13)$$

$$F \triangleq AQ+q^{-d}DSP \qquad (3.14)$$

The characteristic polynomial of the closed loop system is $P\alpha H$. The polynomial α is required to be stable.

The problem of minimizing (3.6) will be solved by seeking a stationary point where $dJ/d\theta=0$. It will be shown that for sufficiently high regulator polynomial degrees, stationary points are global minima. In Appendix A1, it is shown that the partial derivatives of the criterion (3.6) with respect to the regulator coefficients (the sensitivity functions) are given by:

$$\frac{\partial J}{\partial p_i} = \frac{\Lambda_v}{2\pi i} \oint z^i \frac{QGW(z,z^{-1})}{P^2\alpha H} dz \quad ; i=1,\dots,np \qquad (3.15)$$

$$\frac{\partial J}{\partial Q_j} = -\frac{\Lambda_v}{2\pi i} \oint z^j \frac{PGW(z,z^{-1})}{P^2\alpha H} dz \quad ; j=0,\dots,nQ \qquad (3.16)$$

$$\frac{\partial J}{\partial r_\ell} = \frac{\Lambda_v}{2\pi i} \oint z^\ell \frac{FGW(z,z^{-1})}{P\alpha^2 H} dz + \frac{\Lambda_e}{2\pi i} \oint z^\ell \frac{SC(z^k BR_* - \rho A\tilde{\Delta}\tilde{\Delta}_* S_*)C_*}{\alpha^2 \alpha_*} \frac{dz}{z}$$

$$; \ell=1,\dots,nr \qquad (3.17)$$

$$\frac{\partial J}{\partial s_m} = -\frac{\Lambda_v}{2\pi i} \oint z^m \frac{MGW(z,z^{-1})}{P\alpha^2 H} dz - \frac{\Lambda_e}{2\pi i} \oint z^m \frac{RC(z^k BR_* - \rho A\tilde{\Delta}\tilde{\Delta}_* S_*)C_*}{\alpha^2 \alpha_*} \frac{dz}{z}$$

$$; m=0,\dots,ns \qquad (3.18)$$

where

$$W(z,z^{-1}) = \frac{(z^k BM_* - \rho A\tilde{\Delta}\tilde{\Delta}_* F_*)G_*}{P_*\alpha_* H_* z} \qquad (3.19)$$

We now introduce a *spectral factorization*

$$r\beta\beta_* = BB_* + \rho A\tilde{\Delta}\tilde{\Delta}_* A_* \qquad (3.24)$$

where r is a positive scalar factor and β is a stable monic polynomial in z with degree

$$n\beta = \begin{cases} nb \text{ if } \rho=0 \\ \max(nb, na+\deg\tilde{\Delta}) \text{ if } \rho>0 \end{cases} \qquad (3.25)$$

Since B is assumed to have no zeros on the unit circle, β will be strictly stable for all $0\leq\rho<\infty$. Let $B=cB_sB_u$ where c is a constant, B_s is stable and monic and B_u is unstable with $\bar{B}_u$ being stable and monic. When minimum variance control ($\rho=0$) is used, the spectral factor will then be given by

$$\beta = B_s\bar{B}_u \qquad (3.26)$$

FEEDFORWARD FILTER OPTIMIZATION

The following theorem presents a method for optimizing the feedforward part (Q,P) of the regulator (3.8). The feedback (R,S), i.e. the parameters ($r_1,\dots,r_{nr}$, $s_o,\dots,s_{ns}$) in (3.9), are held fixed. A stable causal optimal feedforward filter Q/P can be computed for any stabilizing feedback (R,S).

Theorem 3.1: Optimal feedforward control

Consider the system (3.7) controlled by a stabilizing regulator (3.8) with fixed R,S. Feedforward filter parameters attaining the global minimum value of the criterion (3.6) with respect to ($p_1,\dots,p_{np}$, $Q_o,\dots,Q_{nQ}$) are given by

$$P = G\beta \qquad (3.27)$$

where β is the stable spectral factor of (3.24). The regulator polynomial Q and a polynomial L (giving the minimal criterion value) are the solution of

$$(BR_*-\rho z^{-k}A\Delta\tilde{\Delta}_*S_*)z^{-d+k}D_*G_* = r\beta Q_*+\alpha_*H_*zL \qquad (3.28)$$

with minimal degree with respect to L

$$nQ = \max\{n\alpha+nh-1, nd+ng+d-k+nr, (\text{if } \rho \text{ and } S\neq 0)\, nd+ng+d+ns+\deg\tilde{\Delta}\} \qquad (3.29a)$$

$$nL = \max\{n\beta, nb-d+k\}-1 \qquad (3.29b)$$

Consider any stable feedforward filter with polynomial degrees no less than np=ng+nβ and (3.29a). If it attains the minimal criterion value, it will have the same coprime factors as the filter defined by (3.27),(3.28).

If $\Lambda_e=0$ (only measurable disturbances), the minimal criterion value $2J_{FF}$ is

$$2J_{FF} = \frac{\Lambda_v}{2\pi i}\oint \frac{LL_*}{r\beta\beta_*}\frac{dz}{z} + \frac{\rho\Lambda_v}{2\pi i}\oint \frac{GG_*DD_*\tilde{\Delta}\tilde{\Delta}_*}{r\beta\beta_*HH_*}\frac{dz}{z} \qquad (3.30)$$

■

Proof: See Appendix A2.

Remarks and interpretations

o Equation (3.28) can be transformed into an equation with positive powers of z only, by multiplying both sides with z^{nQ}. A linear system of simultaneous equations is obtained by considering terms with equal power of z. It is easy to verify that the number of equations equals the number of unknowns if (3.29) is used. β and α_*H_* cannot have common factors, since $\beta(z)$ is stable while $z^{n\alpha}\alpha_*z^{nh}H_*=\bar{\alpha}(z)\bar{H}(z)$ will be unstable. Thus, the system has full rank, and a unique solution exists.

o If (3.29) assigns the degree -1, the polynomials should be set to zero.

o It is simple to generalize the design to systems with one input and several measurable disturbances $w_i(t)=(G_i/H_i)v_i(t)$. Use a regulator with feedforward links $(Q_i/P_i)w_i(t)$ calculated from

$$P_i = \beta G_i$$

and

$$(BR_*-\rho z^{k}A\Delta\tilde{\Delta}_*S_*)z^{-d_i+k}D_{i*}G_{i*} = r\beta Q_{i*}+\alpha_*H_{i*}zL_i$$

o The optimal regulator polynomials P,Q given by (3.27),(3.28) may contain (stable) common factors. They can be cancelled before implementation.

- Problems where the regulator degrees are restricted to $np<ng+n\beta$ or $nQ<$(3.29 a) are <u>not</u> covered by Theorem 3.1. Then, several local minima may exist. Such *restricted complexity optimal control* problems are nonlinear and often difficult to solve. The explicit criterion minimization algorithm described in Section 5.3 may be used to find local minima.

- In the minimum variance control case ($\rho=0$), $P=\beta G=B_s\bar{B}_uG$ has a straightforward interpretation: If the system has minimum phase zeros, they could safely be cancelled. (One exception to this rule will be discussed in Section 3.7) Non-minimum phase zeros should of course remain uncancelled. The optimal feedforward filter has poles in their *inverse points with respect to the unit circle*. In addition, regulator poles should cancel the (stable) zeros of the disturbance model G/H.

- When perfect feedforward is impossible, the optimal regulator will depend on our disturbance model: G and H enter into (3.27) and (3.28). (When $\rho=0$, $d\geq k$ and B is stable, (3.27),(3.28) reduce to the perfect feedforward regulator $Q/P=q^{-d+k}DR/B$, $L=0$. The disturbance spectrum then becomes irrelevant.) A disturbance model $G/H\neq 1$ gives information about the most likely future values of w(t). It is not surprising that this information should influence the control law.

With the procedure in Theorem 3.1, feedforward filters may be optimized for any given system controlled with a prespecified stabilizing feedback. We may ask how the choice of feedback (R,S) affects the achievable feedforward control performance.

<u>How feedback affects the feedforward performance</u>

<u>Theorem 3.2</u>

The minimal criterion value (3.30) for e(t)=0 is independent of the choice of feedback (R,S) in (3.8) if $\alpha=AR+q^{-k}BS$ is stable.

<u>Proof:</u> Suppose that the system

$$Ay(t) = q^{-k}Bu(t)+q^{-d}Dw(t) \qquad (e(t)=0) \qquad (3.31)$$

is controlled by

$$R_ou(t) = -\frac{Q_o}{P_o}w(t)-S_oy(t) \qquad (3.32)$$

for some stable P_o and stabilizing (R_o,S_o).

Then the same stationary output and input, and consequently the same criterion value, may be achieved by the regulator

$$Ru(t) = -\frac{Q}{P}w(t)-Sy(t) \qquad (3.33)$$

where (R,S) stabilize the system and P,Q are calculated from

$$\frac{Q}{P} = \frac{Q_o\alpha-q^{-d}DP_o(R_oS-S_oR)}{P_o\alpha_o} \qquad (3.34)$$

where $\alpha=AR+q^{-k}BS$ and $\alpha_o=AR_o+q^{-k}BS_o$ are stable. This can be verified by using the control law (3.33) on the system (3.31), with Q/P chosen according to (3.34).

This results in the closed loop system

$$\alpha P_o\alpha_oy(t) = (q^{-d}DR_oP_o-q^{-k}BQ_o)\alpha w(t)$$

$$\alpha P_o\alpha_ou(t) = -(AQ_o+q^{-d}DS_oP_o)\alpha w(t)$$

The modes corresponding to α are hidden. They are neither visible from the input nor from the output. Thus, the transfer functions are the same as if the regulator (3.32) had been used for control. Compare with (3.10)-(3.14).

We conclude that there cannot exist a feedback R_o, S_o which makes it possible to achieve a lower criterion value than with other feedbacks R,S. If (R_o, S_o, Q_o, P_o) is optimal, the regulator (3.33), (3.34) will achieve the same J.

∎

Thus, the presence of a feedback neither improves nor impairs the feedforward control quality. This makes it possible to solve the feedback and feedforward design problems separately. The transformation (3.34) was suggested by E Trulsson (1985, private communication).

With other concievable regulator structures, the achievable feedforward performance depends on the choice of feedback. For example, consider control error feedback where the feedforward signal enters through the reference input:

$$u(t) = \frac{S}{R}(y_r(t)-y(t)) \quad ; \quad y_r(t) = -\frac{Q}{P}w(t) \qquad (3.35)$$

Such regulators introduce additional zeros (the zeros of S) into the path from y_r to y. This may impair the feedforward performance, in particular if these zeros are unstable.

The same is true for regulators where a feedforward compensation signal is added to the feedback control signal:

$$u(t) = -\frac{S}{R}y(t)+u_{FF}(t) \quad ; \quad u_{FF}(t) = -\frac{Q}{P}w(t) \qquad (3.36)$$

This regulator structure introduces additional zeros (the zeros of R) into the path from u_{FF} to y.

Additional zeros can be avoided by postulating that S should be a factor of P in (3.35) or by requiring R to be factor of P in (3.36). After rearranging polynomials to avoid hidden (possibly unstable) modes inside the controller, both these measures lead to the regulator structure (3.8).

If the system is stable, and a precise model of the system is available, feedforward may be used without feedback, as in Figure 3.3.

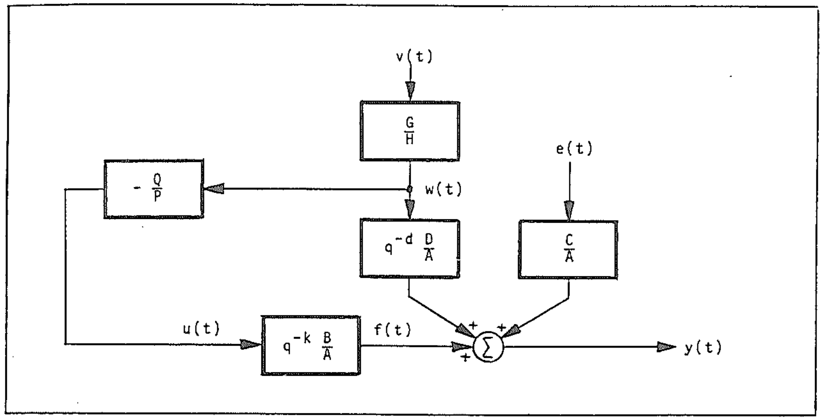


Figure 3.3 Feedforward control of the system (3.6).

<u>Corollary 3.3</u>: <u>Feedforward control of possibly non-minimum phase systems</u>

The feedforward regulator

$$u(t) = -\frac{Q(q^{-1})}{P(q^{-1})}w(t) \qquad (3.37)$$

attains the global minimum value of the criterion J for a stable system (3.7) if

- P is given by

$$P = \beta G \qquad (3.38)$$

where β is the stable spectral factor of (3.24).

- $Q_*(z^{-1})$ and $L(z)$ are the minimum degree solution of

$$z^{-d+k}BD_*G_* = r\beta Q_* + A_*H_*zL \qquad (3.39)$$

with degrees

$$nQ = \max\{na+nh-1, nd+ng+d-k\}$$
$$nL = \max\{n\beta, nb-d+k\}-1 \qquad (3.40)$$

■

Proof: Follows immediately from Theorem 3.1 with R=1, S=0 and α=A. When e(t)=0, the minimal criterion value is given by (3.30).

The calculations above are illustrated in Chapter 4 by Example 4.2.

LQG-OPTIMAL COMBINED FEEDBACK-FEEDFORWARD REGULATORS

Let us first present the optimal feedback regulator. The infinite horizon criterion (3.6) is to be minimized for the system

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t)+C(q^{-1})e(t) \qquad (3.41)$$

where C is stable and (A,B) have no unstable common factors. A feedback regulator is used:

$$R(q^{-1})u(t) = -S(q^{-1})y(t) \qquad (3.42)$$

Theorem 3.4: Optimal feedback control

Part 1

The infinte horizon criterion (3.6) for the system (3.41) is minimized by the feedback regulator (3.42) if R,S and X are the minimum degree solution with respect to X of

$$\begin{cases} r\beta R_* - z^{-k+1}B_*X = \rho\tilde{\Delta}\tilde{\Delta}_*AC_* & (3.43a) \\ r\beta S_* + zA_*X = z^kBC_* & (3.43b) \end{cases}$$

where β is the stable spectral factor defined by (3.24) and

$$nx = n\beta+k-1 \qquad (3.44a)$$

$$ns = \max\{na-1, nc-k\} \qquad (3.44b)$$

$$nr = \begin{cases} \max\{nb+k-1, nc+\deg\tilde{\Delta}\} & \text{if } \rho\neq 0 \\ nb+k-1 & \text{if } \rho=0 \end{cases} \qquad (3.44c)$$

This implies pole placement in

$$\beta C = AR+q^{-k}BS \triangleq \alpha \qquad (3.45)$$

The minimal criterion value is given by

$$2J_{FB} = \frac{\Lambda_e}{2\pi i}\oint \frac{(RR_*+\rho\tilde{\Delta}\tilde{\Delta}_*SS_*)}{\beta\beta_*}\frac{dz}{z} = \frac{\Lambda_e}{2\pi i}\oint \frac{(XX_*+\rho\tilde{\Delta}\tilde{\Delta}_*CC_*)}{r\beta\beta_*}\frac{dz}{z} \qquad (3.46)$$

Part 2

Assume that

- The closed loop system is stable

- A stationary point has been attained, i.e.

$$\frac{\partial J}{\partial r_\ell} = \frac{\Lambda_e}{2\pi i} \oint z^{\ell} \frac{SC(z^k BR_* - \rho A\Delta\tilde{\Delta}_* S_*)C_*}{\alpha^2 \alpha_*} \frac{dz}{z} = 0$$

$$; \ell=1,\dots,nr \qquad (3.47)$$

$$\frac{\partial J}{\partial s_m} = -\frac{\Lambda_e}{2\pi i} \oint z^{m} \frac{RC(z^k BR_* - \rho A\Delta\tilde{\Delta}_* S_*)C_*}{\alpha^2 \alpha_*} \frac{dz}{z} = 0$$

$$; m=0,\dots,ns \qquad (3.48)$$

- The regulator degrees nr, ns are not less than (3.44b,c).

Then the regulator must have the same closed loop behaviour (the same coprime factors) as the optimal one defined by (3.43). ∎

Proof: See Kučera (1979b) and Trulsson (1985).

The optimal feedback regulator was derived by Kučera. See Kučera (1979a,b). An alternative proof was given by Peterka (1984). When (3.43) is fulfilled, (3.47) and (3.48) hold because the integrands will be analytic inside the intergration path. The converse, namely part 2 of the theorem, was proved by Trulsson (1985). (These proofs need to be modified slightly to take the explicitly given time delay k and the polynomial $\tilde{\Delta}$ into account.) The degree conditions (3.44) are derived by inspecting the maximal powers in z and z^{-1} of the equations (3.43), as in Appendix A2, Section 1. The *implied diophantine pole placement equation* (3.45) is given

by multiplying (3.43a) with A_* and (3.43b) with $z^{-k}B_*$ and adding them. The minimal criterion value (3.46) is derived by using the expression for the closed system, (3.45) and (3.43). That $\partial J/\partial r_\ell$ and $\partial J/\partial s_m$ are given by (3.47),(3.48) is evident from (3.17), (3.18) in the case of no feedforward signal (Λ_v=0).

Remarks

LQG-optimal feedback implies pole placement in $\alpha=\beta C$. Compare with the expression $P=\beta G$ for the optimal feedforward filter denominator.

The equations (3.43a,b) may be transformed into equations in z^{-1} only by multiplying (3.43a) with $z^{-n\beta}$ and (3.43b) with $z^{-n\beta-k}$.

If A and B have no common factors, the optimal regulator may be calculated from the implied pole placement equation (3.45) directly, with polynomial degrees (3.44b,c). If A and B have stable common factors, (3.45) no longer determines the optimal regulator uniquely. There will be at least one free parameter in R and/or S. All values of these free parameters give pole placement in βC, but it is not obvious which one of them provides the optimal regulator. The use of the two coupled polynomial equations (3.43) will give the correct solution. For a further discussion of this point, see Kučera (1984).

The system (3.7) with e(t)=0 and D stable may be written in the form

$$AHy(t) = q^{-k}BHu(t)+DGv(t-d) \qquad (3.49)$$

An optimal feedback regulator could of course be calculated using A=AH, B=BH and C=DG in Theorem 3.4. Would this sometimes result in a better performance than the use of feedforward control?

Corollary 3.5

For stable systems $Ay=q^{-k}Bu+q^{-d}Dw$, where w(t) and y(t) can be measured without noise, it is never advantageous to use output feedback $u=-(R/S)y$ compared with feedforward $u=-(Q/P)w$.

Proof: Follows directly from the proof of Theorem 3.2. Let (3.32) represent the optimal feedback ($Q_o=0$) and (3.33) a feed-forward regulator (S=0,R=1).

The optimal feedforward filter is in general superior to any feedback regulator. This has a simple explanation: For nonzero disturbance time delays d, the measurement w(t) contains more timely information about the disturbance than y(t).

Example 3.1

Consider the non-minimum phase system

$$(1-0.5q^{-1})y(t) = (0.5+1.25q^{-1}+0.5q^{-2})u(t-1)+(2-1.5q^{-1})w(t-2)$$

$$w(t) = \frac{(1-0.3q^{-1})}{1-0.9q^{-1}}\,v(t)$$

where v(t) is white noise with standard deviation 1. The standard deviation of y(t) is 2.30 in open loop (u=0).

A minimum variance feedback regulator can be computed from Theorem 3.4, Part 1 for the model (3.49):

$$(1-0.5q^{-1})(1-0.9q^{-1})y(t) = (1+0.5q^{-1})(0.5+q^{-1})(1-0.9q^{-1})u(t-1)+$$
$$+(1-0.75q^{-1})(1-0.3q^{-1})\varepsilon(t)$$

where $\varepsilon(t)=2v(t-2)$. It is given by

$$u(t) = -\frac{0.317-0.199q^{-1}}{1+0.291q^{-1}-0.727q^{-2}-0.311q^{-3}}\,y(t)$$

The resulting output standard deviation is 2.04. To achieve this insignificant disturbance rejection, an input with standard deviation 0.98 has to be used.

The minimum variance feedforward regulator from Corollary 3.3 is given by

$$u(t) = -\frac{1.31-0.45q^{-1}-0.71q^{-2}+0.225q^{-3}}{1+0.7q^{-1}-0.05q^{-2}-0.075q^{-3}}\,w(t)$$

It achieves an output standard deviation of 0.76, using an input standard deviation of 1.47.

∎

The discussion above does not imply that feedback is unnecessary. Normally, feedforward has to be used in combination with feedback, for at least three reasons:

- The feedforward control principle is not robust. In contrast to feedback loops, feedforward filters cannot decrease the sensitivity of the control performance to system parameter variations or modelling errors.
- Often, significant unmeasurable disturbances have to be taken into account.
- The system may be unstable.

A method is now presented for optimizing combined feedback-feedforward regulators with the structure (3.8).

Theorem 3.6: Optimal combined feedback and feedforward

For the system (3.7), the global minimum value of the criterion (3.6) with respect to the parameters (3.9) of the regulator (3.8) is attained, if

$$P = G \tag{3.50}$$

and the regulator polynomials R, S and Q are calculated as follows:

Let β be the stable spectral factor from (3.24)

$$r\beta\beta_* = BB_* + \rho A\tilde{\Delta}\tilde{\Delta}_* A_* \tag{3.51}$$

Let $R_*(z^{-1})$, $S_*(z^{-1})$ and $X(z)$ be the minimum degree solution with respect to X of the coupled polynomial equations

$$\begin{cases} r\beta R_* - z^{-k+1}B_*X = \rho\tilde{\Delta}\tilde{\Delta}_* AC_* & (3.43a) \\ r\beta S_* + zA_*X = z^k BC_* & (3.43b) \end{cases}$$

with degrees

$$nx = n\beta + k - 1 \tag{3.44a}$$

$$ns = \max\{na-1, nc-k\} \tag{3.44b}$$

$$nr = \begin{cases} \max\{nb+k-1, nc+\deg\tilde{\Delta}\} & \text{if } \rho \neq 0 \\ nb+k-1 & \text{if } \rho = 0 \end{cases} \tag{3.44c}$$

Let $Q_*(z^{-1})$ and $L(z)$ be the minimum degree solution of the polynomial equation

$$z^{-d+1}D_*G_*X = r\beta Q_* + C_*H_*zL \tag{3.52}$$

with degrees

$$\begin{aligned} nQ &= \max\{nd+ng+d, nc+nh\}-1 \\ nL &= n\beta - 1 + g \end{aligned} \tag{3.53}$$

where

$$g \triangleq \max\{0, k-d\} \tag{3.54}$$

The use of the regulator

$$R(q^{-1})u(t) = -\frac{Q(q^{-1})}{G(q^{-1})}w(t) - S(q^{-1})y(t) \tag{3.55}$$

then achieves the criterion value

$$2J_{min} = Ey(t)^2 + \rho E(\tilde{\Delta}u(t-k))^2 = 2J_{FF} + 2J_{FB} \tag{3.56}$$

where J_{FF} is given by (3.30) and J_{FB} by (3.46).

Any stabilizing regulator (3.55) which attains this minimal criterion value with polynomial degrees no less than (3.44b,c) and (3.53), will have the same coprime factors as the filter defined by (3.43) and (3.52).

∎

Proof: See Appendix A3.

The use of optimal feedback allows us to use a lower order feedforward filter than in Theorem 3.1 and Corollary 3.3. Note especially that P is given by G, while P=Gβ is used when a feedback is absent or prespecified in an arbitrary way. The theorem describes an optimization algorithm. Let us discuss its step in words:

First, the stable spectral factor $\beta(z)$ is calculated. Then the feedback part R,S is optimized with respect to the unmeasurable disturbances e(t), as in Theorem 3.4. The poles are placed in $\alpha=\beta C$. This calculation is totally independent of the feedforward filter and transfer functions in the measurable disturbance path. This means that the regulator structure has *two degrees of freedom*. This independence has also been noted by Grimble (1986).

The feedforward filter is then calculated to suppress the measureable disturbance w(t) in an optimal way. The filter will depend on the feedback calculated in the preceeding step (X(z) enters into (3.52)). However, as was stated in Theorem 3.2, the achieveable feedforward performance is not affected by the choice of feedback with our regulator structure.

For completeness, let us introduce reference signals into the discussion. The regulator structure is extended to

$$Ru(t) = -\frac{Q}{G}w(t) - Sy(t) + \frac{T}{E}r(t) \qquad (3.57)$$

The filter T/E may be used to cancel poles and zeros to shape the response from the reference signal r(t). With the restriction that time delays cannot be shortened and non-minimum phase zeros cannot be cancelled, we are free to choose any desired response (reference model). Since R, S and Q can be designed to reject e(t) and v(t) optimally without taking this final step into account, the regulator structure has three degrees of freedom.

The use of the regulator (3.57), with pole placement in $\alpha=\beta C$ gives

$$y(t) = \frac{(q^{-d}DRG - q^{-k}BQ)}{\beta CH}v(t) + \frac{R}{\beta}e(t) + \frac{q^{-k}BT}{\beta CE}r(t) \qquad (3.58)$$

$$u(t) = -\frac{(q^{-d}DGS + AQ)}{\beta CH}v(t) - \frac{S}{\beta}e(t) + \frac{AT}{\beta CE}r(t) \qquad (3.59)$$

The characteristic polynomial is $\beta CHGE$. (G is cancelled in all transfer functions.)

Optimization of T and E is discussed in Section 3.5.

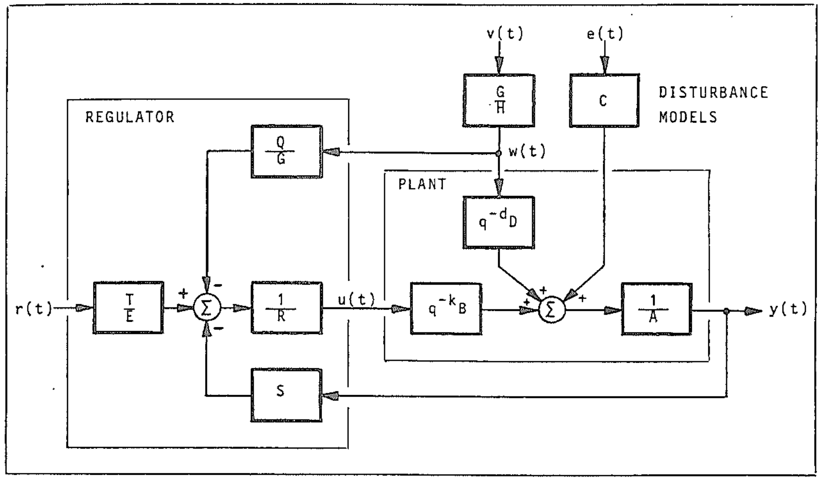


Figure 3.4 The optimal combined feedback-feedforward regulator structure with a reference input. Note the common denominator polynomial R. Also, note that the numerator G of the feedforward signal model should be cancelled by the feedforward filter. Optimal pole placement: $\beta C = AR + q^{-k}BS \triangleq \alpha$.

Notes on regulator calculation and implementation

- Some care must be taken when the regulator is implemented. (3.8) cannot be implemented as

$$u_1(t) = \frac{Q}{GR}\,w(t) \quad ; \quad u_2(t) = \frac{S}{R}\,y(t) \quad ; \quad u(t) = -u_1(t) - u_2(t)$$

  The (not necessarily stable) R-polynomial in the feedforward transfer function will then result in hidden modes.

  The recursion

$$u_1(t) = \frac{Q}{G}\,w(t)$$
$$u(t) = -r_1 u(t-1) - \ldots - r_{nr} u(t-nr) - u_1(t) - S(q^{-1})y(t) \qquad (3.60)$$

  should be used instead.

- For $n\beta \leq 2$, simple explicit expressions can be given for the spectral factor β. See Peterka (1984). For general spectral factorization algorithms, see Kučera (1979a).

- The coupled equations (3.43) can be found to give an over-determined set of simultaneous equations in the coefficients of R, S and X. With the polynomial degrees (3.44), this system will, however, have a unique solution. Some equations of the system will be linear combinations of others. A simple way to find the (exact) solution is to use the least squares method for solving over-determined systems of equations. For other approaches to the solution of (3.43) see Kučera (1979a), (1979b) and Grimble (1985).

3.3 MINIMUM VARIANCE CONTROL

In this section, we will discuss problems where $\rho=0$, i.e. minimum output variance is desired. The calculations are simple, since the spectral factor β is evident by inspection. From (3.26)

$$\beta = B_s \bar{B}_u$$

where $\bar{B}_u$ is stable, since B_u is unstable.

In this section, all polynomials have the argument q^{-1}, to simplify the notation.

MINIMUM VARIANCE FEEDFORWARD

When $\beta=B_s\bar{B}_u$, Corollary 3.3 reduces to the following result.

Corollary 3.7

The minimum variance feedforward regulator

$$u(t) = -\frac{Q(q^{-1})}{P(q^{-1})}\,w(t)$$

for a stable system (3.7) with $B=cB_sB_u$, B_s and $\bar{B}_u$ monic, is given by

$$P = B_s\bar{B}_u G \qquad (3.61)$$

and Q, L_1 solving

$$q^{-d+k-g}\bar{B}_u DG = q^{-g}cB_u Q + AHL_1 \qquad (3.62)$$

with degrees

$$nQ = \max\{na+nh-1, nd+ng+d-k\}$$
$$nL_1 = u-1+g \qquad (3.63)$$

where $u=\deg B_u$ and $g=\max\{0,k-d\}$ as in (3.54).

∎

<u>Proof</u>: See Appendix A4, Part 1.

The use of S=0, R=1, α=A, $B=cB_sB_u$, $P=B_s\bar{B}_uG$ and (3.62) reduces the expression (3.10) for the controlled output to

$$y(t) = \frac{L_1}{\bar{B}_u} v(t-\min\{k,d\}) + \frac{C}{A} e(t) \qquad (3.64)$$

As can be seen in (3.64), perfect feedforward is equivalent to L_1=0. Equation (3.62) then reduces to

$$q^{-d+k}\bar{B}_u DG = cB_u Q \qquad (3.65)$$

It is possible to find a $Q(q^{-1})$ satisfying (3.65) if and only if

1) $d \geq k$ (The input time delay k is not too large.)

and

2) B_u divides D, i.e. $D=cD_1B_u$ for some $D_1(q^{-1})$. (3.66)

(B_u cannot divide the stable polynomial $\bar{B}_uG$)

Then, the feedforward regulator

$$u(t) = -\frac{Q}{P} w(t) = -q^{-d+k} \frac{\bar{B}_u D_1 G}{B_s \bar{B}_u G} w(t) = -q^{-d+k} \frac{D_1}{B_s} w(t) \qquad (3.67)$$

achieves perfect feedforward.

Thus, there exists a class of non-minimum phase systems where perfect disturbance cancellation by feedforward is possible. It is characterized by (3.66). In such systems, the non-minimum phase dynamics is located beyond the point where the disturbance meets the control action. See Figure 3.5.

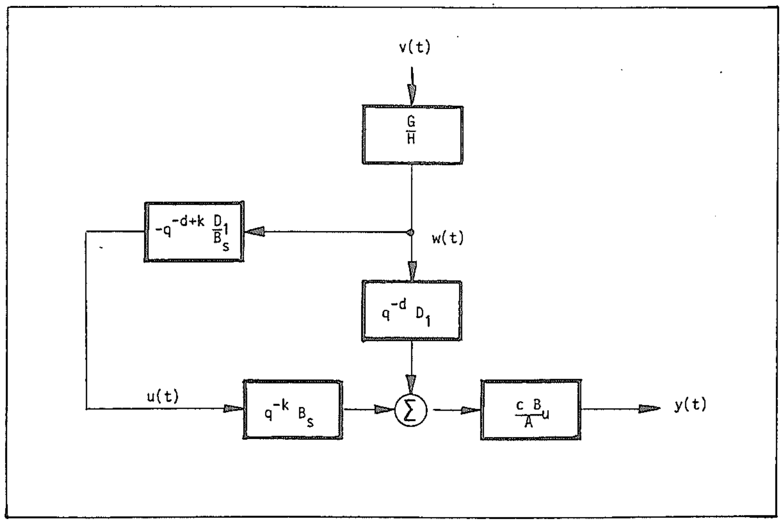


Figure 3.5 Non-minimum phase system with $d \geq k$ for which perfect feedforward is possible. The factor B_u with unstable zeros is present in both the disturbance and the input signal path.

When the system is minimum phase but k>d, the first part of (3.64) reduces to $L_1v(t-d)$. This case has a straightforward interpretation. Let v(t) be an impulse. The regulator is unable to influence the first k components of the impulse response (given by $L_1(q^{-1})v(t-d)$) but the following components will be cancelled completely.

COMBINED FEEDBACK AND FEEDFORWARD MINIMUM VARIANCE CONTROL

For simplicity, the result is presented for the case when A and B have no common factors. The feedback may then be calculated from (3.45) instead of (3.43). This gives the following corollary to Theorem 3.6.

Corollary 3.8

When A and B have no common factors, the minimum variance combined feedback-feedforward regulator for the system (3.7) is given by

o Calculation of $R_1(q^{-1})$ and $S(q^{-1})$ from

$$\bar{B}_u C = AR_1+q^{-k}cB_u S \qquad (3.68)$$

o Calculation of $Q(q^{-1})$ and $L_1(q^{-1})$ from

$$q^{-d+k-g}R_1 DG = q^{-g}cB_u Q+CHL_1 \qquad (3.69)$$

with degrees

$$nr_1 = u+k-1 \qquad (3.70a)$$

$$ns = \max\{na-1,nc-k\} \qquad (3.70b)$$

$$nQ = \max\{nc+nh,nd+ng+d\}-1 \qquad (3.71a)$$

$$nL_1 = u-1+g \qquad (3.71b)$$

where $u=\deg B_u$ and $g=\max\{0,k-d\}$.

o The use of the regulator

$$B_s R_1 u(t) = -\frac{Q}{G} w(t)-Sy(t) \qquad (3.72)$$

■

Proof: See Appendix A4, Part 2.

The use of $R=B_sR_1$, $\alpha=B_s\bar{B}_uC$, $B=cB_sB_u$, $P=G$ and (3.69) in the expression (3.10) for the controlled output immediately gives

$$y(t) = \frac{L_1}{\bar{B}_u} v(t-\min\{k,d\})+\frac{R_1}{\bar{B}_u} e(t) \qquad (3.73)$$

Compare with (3.64). Note that in the minimum variance case, the disturbance transfer function G/H is cancelled in the controlled system. G and H do not enter into (3.64) or (3.73).

COMPARISON WITH PREDICTIVE CONTROL FOR MINIMUM PHASE SYSTEMS

Let us assume that the system is minimum phase, but has a larger time delay from the input than from the disturbance (k>d). It is then possible to derive an optimal feedforward filter by minimizing the sliding short-time criterion

$$J_k = \frac{1}{2} Ey(t+k|t)^2$$

(For all t, compute u(t) such that the variance of y is minimized k steps later, when a control effort can influence y.) This is the disturbance prediction approach, discussed in Section 2.4. The self-tuning minimum variance feedback regulator, discussed in Section 5.1, is built on this principle.

For clarity, we restrict the discussion to feedforward control with e(t)=0. (A longer calculation leads to a similar result in cases with combined feedback and feedforward.)

The system (3.7) is expressed as

$$y(t+k) = \frac{B}{A} u(t)+q^{g} \frac{DG}{AH} v(t) \quad ; \quad w(t) = \frac{G}{H} v(t) \qquad (3.74)$$

where $g=k-d>0$. Thus, perfect feedforward is not possible. To derive an optimal k step ahead output predictor, solve for Q (degree $\max\{na+nh-1, nd+ng-g\}$) and L_1 (degree g-1) in

$$DG = q^{-g}Q+AHL_1 \qquad (3.75)$$

Multiply (3.75) with q^g/AH. The system (3.74) may then be rewritten as

$$y(t+k) = \frac{B}{A} u(t)+ \frac{Q}{AH} v(t)+q^{g}L_1 v(t)$$

The last term depends on $v(t+1),\dots,v(t+g)$ which are uncorrelated with all known signals at time t. The best prediction of $q^g L_1 v(t)$ is its mean value, zero.

Thus, the optimal k-step ahead predictor is

$$\hat{y}(t+k|t) = \frac{B}{A} u(t)+ \frac{Q}{AH} v(t) = \frac{B}{A} u(t)+ \frac{Q}{AG} w(t) \qquad (3.76)$$

Now, choose u(t) such that $\hat{y}(t+k|t)=0$. We get

$$u(t) = - \frac{Q}{BG} w(t) \qquad (3.77)$$

where Q is given by (3.75). This obviously minimizes J_k.

This is the same regulator as the infinite-horizon optimal one in Corollary 3.7. Compare (3.75) with (3.62) in the case $k-d=g>0$ and $B_u=1$.

The corresponding result has been proved by Åström and Wittenmark (1971) for feedback regulators. As in the feedback case, the result does <u>not</u> hold for more general criteria with input penalty. LQG control will then be superior: For a given input variance, the output variance is never higher, and often considerably lower, than when the prediction approach with a short prediction horizon is used. See Section 6.1 and Modén and Söderström (1982) for some examples.

The feedforward regulator (3.77) would of course be worthless for non-minimum phase systems. In that case, the *prediction* (3.76) is still optimal, but we can no longer derive an optimal *control action* from it in the simple way described above.

3.4 THE SCALAR DISTURBANCE DECOUPLING PROBLEM

Reconsider the system (3.1), parametrized in the form (3.4),(3.5)

$$Ay(t) = q^{-k}Bu(t)+q^{-d}Dw(t)+Ce(t) \qquad (3.4)$$

$$Hw(t) = q^{-n}Nu(t)+Gv(t) \qquad (3.5)$$

Compared to the feedforward problem discussed in Sections 3.2 and 3.3, the auxiliary output w(t) may now be influenced by the input.

One could, of course, rederive sensitivity functions, similar to (3.15)-(3.18), for this structure, and derive LQG regulators from them. Here, an alternative approach will be presented, which enables us to use all results from Sections 3.2 and 3.3 directly. An idea introduced in Section 2.4 will be used: Compensate for the influence of u(t-n) on w(t) inside the regulator. This reduces the disturbance decoupling problem ($N\neq 0$) to an equivalent feedforward problem (with N=0). Thus, let us modify the regulator (3.57) to

$$Ru(t) = -\frac{Q}{P}\underbrace{\left(w(t)-\frac{\hat{N}}{\hat{H}}u(t-n)\right)}_{\hat{w}_1(t)}-Sy(t)+\frac{T}{E}r(t) \qquad (3.78)$$

where $\hat{N}$ and $\hat{H}$ are estimates of N and H. See Figure 3.6.

To simplify the expression for the closed system, define the polynomials

$$q^{-b}\tilde{B} \triangleq q^{-k}BH+q^{-d-n}DN \qquad (3.79)$$

where b=min(k,d+n), and

$$\tilde{\alpha} = AHR+q^{-b}\tilde{B}S = AHR+q^{-k}BHS+q^{-d-n}DNS \qquad (3.80)$$

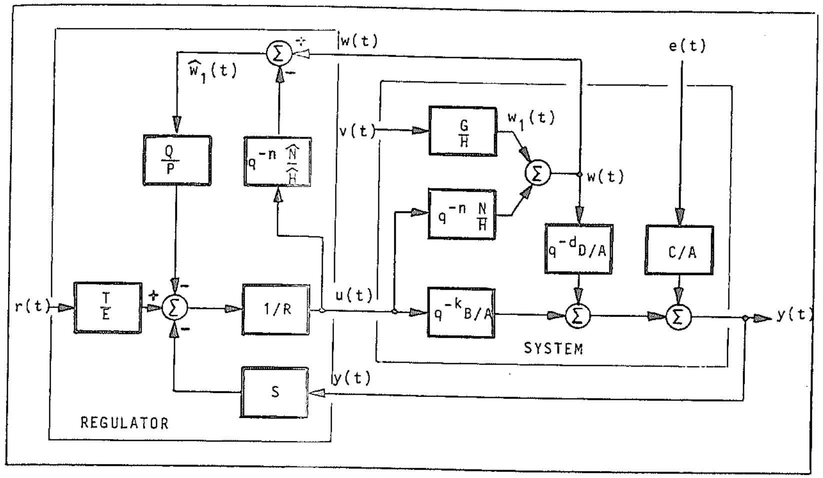


Figure 3.6 The regulator (3.78) reconstructs an estimate $\hat{w}_1(t)$ of the disturbance influence $w_1(t)$ on the auxiliary measured signal w(t). This estimate is used for feedforward. Of course, in feedforward problems where $N=\hat{N}=0$, $\hat{w}_1$ will equal w_1.

If $\hat{N}$=N and $\hat{H}$=H, the system (3.4),(3.5) combined with the regulator (3.78) is input-output equivalent to the system

$$AHy(t) = q^{-b}\tilde{B}u(t)+q^{-d}DHw_1(t)+CHe(t)$$
$$w_1(t) = \frac{G}{H}v(t) \qquad (3.81)$$

combined with the regulator

$$Ru(t) = -\frac{Q}{P}w_1(t)-Sy(t)+\frac{T}{E}r(t) \qquad (3.82)$$

See Figure 3.7.

Note that the input affects y(t) through two paths in Figure 3.6. Their effect has been combined in the transfer function $q^{-b}\tilde{B}/AH$.

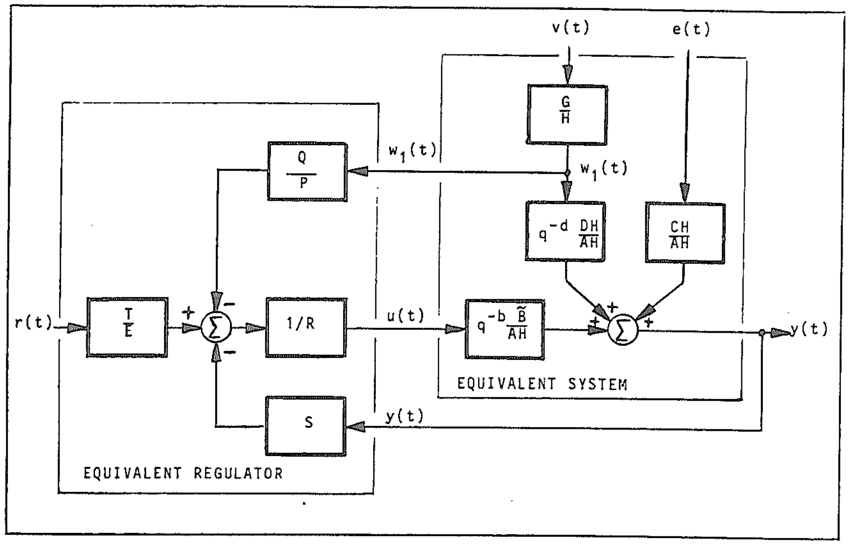


Figure 3.7 Problems where w(t) is affected by the input are reduced to equivalent feedforward problems, without such cross-couplings.

The output and input of the closed loop system are given by

$$y(t) = \frac{(q^{-d}DHPR-q^{-b}\tilde{B}Q)G}{P\tilde{\alpha}H} v(t)+ \frac{RCH}{\tilde{\alpha}} e(t)+q^{-b} \frac{\tilde{B}T}{\tilde{\alpha}E} r(t) \quad (3.83)$$

$$u(t) = - \frac{(AQ+q^{-d}DSP)G}{P\tilde{\alpha}} v(t)- \frac{SCH}{\tilde{\alpha}} e(t)+ \frac{AHT}{\tilde{\alpha}E} r(t) \quad (3.84)$$

Compare with (3.10),(3.11).

When $\hat{H}=H$ and $\hat{N}=N$, the characteristic polynomial will be $P\tilde{\alpha}\hat{H}E$. In general, the characteristic polynomial can be shown to be

$$[\tilde{\alpha}P\hat{H}+AQ(q^{-n}\hat{N}H-q^{-n}N\hat{H})]E$$

Thus, some care must be taken when this regulator is implemented. If a high gain feedforward is used ($|Q|$ large), or if the auxiliary

output depends strongly on the input ($|N|$ large), small errors in the model $\hat{N}/\hat{H}$ may cause instability. Failure to take the input influence into account ($\hat{N}=0$) is dangerous. This holds also for the adaptive regulators of Sections 5.2 and 5.3.

Thus, a regulator (3.78) minimizing the criterion (3.6) for the system (3.4),(3.5) can be found by calculating an optimal regualtor (3.82) for the equivalent system (3.81), as in Figure 3.7. Thus, all results from the previous theorems can be used. The preceeding discussion is summarized below.

<u>Theorem 3.9</u>

A regulator minimizing the infinte horizon criterion (3.6) for the scalar system (3.1), with m(t)=0, can be calculated as follows:

- Parametrize the system in the form (3.4),(3.5).

- Use any of the results from Section 3.2 or Section 3.3 to calculate optimal regulator polynomials P, Q, R and S using the following substitutions:

$$\begin{aligned} &q^{-b}\tilde{B} \text{ for } q^{-k}B \\ &\tilde{\alpha} \text{ for } \alpha \\ &AH \text{ for } A \\ &DH \text{ for } D \\ &CH \text{ for } C \end{aligned} \quad (3.85)$$

- Assuming that $\hat{N}=N$ and $\hat{H}=H$, use the regulator (3.78).

■

If the system is given in the form (3.1) from the beginning, note that from (3.3), (3.4), (3.5) and (3.79):

$$\begin{aligned} q^{-d}\frac{D}{A} &= H_{yv}H_{wv}^{-1} \\ q^{-b}\frac{\tilde{B}}{AH} &= q^{-k}\frac{B}{A} + q^{-d}\frac{D}{A}q^{-n}\frac{N}{H} = (H_{yu}-H_{yv}H_{wv}^{-1}H_{wu})+ \\ &\quad +H_{yv}H_{wv}^{-1}H_{wu} = H_{yu} \end{aligned} \tag{3.86}$$

Thus, we do not need first to calculate B from (3.3), and then $\tilde{B}$ from (3.79).

Note that $\tilde{B}$ (and not B) determines whether the plant is minimum phase.

Also note that the substitutions (3.85) in (3.39) require the Q-polynomial of the feedforward regulator (3.37) to contain the factor H. From (3.45) and (3.52) it can be seen that both Q and S in the combined feedback-feedforward regulator will contain the factor H.

Cases where perfect feedforward is possible for the equivalent system (3.81) of course correspond to exact disturbanc decoupling in the original system (3.1). We saw in Section 2.3 that such disturbance decoupling regulators are simple to calculate directly for scalar systems. With the result from this section, optimal (approximate) disturbance decoupling regulators may be designed in the numerous cases when perfect disturbance rejection is impossible. By using an input penalty, excessive input signals can be avoided.

3.5 CONTROL OF NONSTATIONARY AND DETERMINISTIC DISTURBANCES

Until now, we have assumed the disturbances to be zero mean and stationary. Such assumptions are unrealistic in many cases. In this section, we will discuss the design of optimal regulators for other types of disturbances, for example

1. Nonzero mean stationary stochastic processes
2. Drifting disturbances
3. Randomly occuring steps
4. Sinusoids with known frequency

INTEGRATING FEEDBACK-FEEDFORWARD REGULATORS

Let the disturbances e(t) and v(t) be modelled as

$$e(t) = \frac{e_1(t)}{1-q^{-1}} + e_2(t) + e_0 \tag{3.87}$$

$$v(t) = \frac{v_1(t)}{1-q^{-1}} + v_2(t) + v_0 \tag{3.88}$$

where e_0 and v_0 are constants, and e_1, e_2, v_1, v_2 are mutually uncorrelated zero mean white noises or randomly occuring impulses. As special cases, we have

1. Nonzero mean stationary disturbance. Set $e_1(t)=0$, $v_1(t)=0$.

2. Drifting disturbances. Set $e_1(t)\neq 0$ or $v_1(t)\neq 0$.

3. Steps. Set $e_2(t)=0$ and $v_2(t)=0$. Let $v_1(t)$ and/or $e_1(t)$ be Poisson processes (a train of randomly occuring impulses).

The use of integrating disturbance models would lead to a total process model where A and B have an unstable common factor $1-q^{-1}$. No stable spectral factor β could then be computed. Instead, the regulator design will be applied to a model describing y(t) with differentiated signals Δu and Δw. Differentiation of (3.4) and (3.5), and the use of (3.87),(3.88) with $\Delta \triangleq 1-q^{-1}$ gives

$$\begin{aligned} (A\Delta)y(t) &= q^{-k}B\Delta u(t)+q^{-d}D\Delta w(t)+Ce_1(t)+C\Delta e_2(t) \\ H\Delta w(t) &= q^{-n}N\Delta u(t)+Gv_1(t)+G\Delta v_2(t) \end{aligned} \tag{3.89}$$

where C, H and G are assumed stable. Assume that e(t) and v(t) are drifting or step disturbances modelled with $e_2(t)=0$ and $v_2(t)=0$. The problem is then reduced to one we have solved, namely optimal control of systems of type (3.4),(3.5). All calculations are performed with the following substitutions:

$A\Delta$ for A

$$\begin{aligned} \Delta u(t) &= u(t)-u(t-1) \quad \text{for } u(t) \\ \Delta w(t) &= w(t)-w(t-1) \quad \text{for } w(t) \end{aligned} \tag{3.90}$$

Thus, the criterion

$$J = \frac{1}{2} E[y(t)^2+\rho(\tilde{\Delta}\Delta u(t-k))^2] \tag{3.91}$$

will be minimized if the regulator

$$\begin{aligned} R(\Delta u(t)) &= -\frac{Q}{G}[\Delta w(t)-q^{-n}\frac{N}{H}(\Delta u(t))]-Sy(t) \\ u(t) &= \Delta u(t)+u(t-1) \end{aligned} \tag{3.92}$$

is used where R, S and Q are calculated from Theorem 3.6/Lemma 3.8. Use the substitutions of Theorem 3.9 when $N \neq 0$ and use $A\Delta$ as A. The output y(t) will then have zero mean and optimal variance.

The regulator may be implemented in differential form (3.92). It may also be written as

$$(R\Delta)u(t) = -\frac{(Q\Delta)}{G}[w(t)-q^{-n}\frac{N}{H}u(t)]-Sy(t) \tag{3.93}$$

It is evident that the regulator will contain an integrator. In general, if disturbance models contain unstable factors, these factors should be present in the regulator denominator. This is called the *internal model principle* (Francis and Wonham 1976). To take another application of this principle, if v(t) is a sinusoid with frequency ω_0, substitute $1-2\cos\omega_0 q^{-1}+q^{-2}$ for Δ in (3.91) and (3.93). Some other points need to be emphasized:

- When v(t)/e(t) are drifting/nonzero mean, the integrating regulator (3.92) provides a drifting/nonzero mean input to compensate for the disturbance influence on the output. This will of course work only as long as the input does not hit any bounds. When a bound is attained, the true input should be used in the regulator regression, not the calculated one. Otherwise, "integrator windup" will occur in the direct implementation (3.93).

- Note that a criterion (3.91) with differential input penalty will be used. To be able to cancel a drifting disturbance, the input must be allowed to drift. We may penalize its rate of change, but not its magnitude. Thus, the user is not completely free to choose the type of criterion to be optimized.

o If the disturbance e(t) is not drifting but nonzero mean. ($e_1=0$, $e_2\neq0$, $e_0\neq0$), the method described above does not work. The C-polynomial will be given by $C\Delta$ (see (3.89)). We cannot use pole placement in $\beta C\Delta$, since this would introduce a drifting hidden mode in the closed loop system. Instead, a suboptimal pole placement in $\beta C\Delta_1$ may be used, where Δ_1 does not have its zero close to the unit circle. The same situation occurs when $v_1=0$, $v_2=0$ and $v_0\neq0$. Then, the G-polynomial is $G\Delta$. Instead of $P=G\Delta$, use a suboptimal $P=G\Delta_1$, where Δ_1 does not have zeros close to the unit circle. The simplest choice is $\Delta_1=1$.

o An alternative way of handling nonzero mean stationary stochastic disturbances is to filter out the mean values from the measurements y(t),w(t). The mean values are then amplified separately with the correct static gains and added to u(t). With this method, high-pass filtering is used instead of differentiation. Poles may then be placed optimally. This approach is used in the case study in Section 4.2.

FEEDFORWARD CONTROL OF NONSTATIONARY AND DETERMINISTIC DISTURBANCES

Without feedback, differential models and integration in the regulators are of no help. Optimal feedforward design requires A(z) to be stable, while $A\Delta$ is used as denominator in differential models. Instead, when optimizing feedforward filters,

$$u(t) = -\frac{Q(q^{-1})}{P(q^{-1})}w(t)$$

models will be considered where H(z) has poles on the unit circle.

Let $w_1(t)$ in (3.82) be given by

$$w_1(t) = \frac{G}{H}v(t) = \frac{G}{H_s\Delta'}v(t) = \frac{G}{H_s}v_1(t) \qquad (3.94)$$

$$\Delta' v_1(t) = v(t)$$

The polynomial H_s is stable, v(t) is white and Δ' has all zeros on the unit circle. If $v_1(t)$ is nonzero constant, a drifting disturbance or a sequence of step disturbances, use

$$\Delta'(q^{-1}) = 1-q^{-1}$$

If $v_1(t)$ is a sinusoid with frequency ω_0, use

$$\Delta'(q^{-1}) = 1-(2\cos\omega_0)q^{-1}+q^{-2}$$

(For nonzero constants and sinusoids, v(t)=0. These disturbances are described by autonomus difference equations with nonzero initial values.)

It turns out that our regulator of Corollary 3.3 is optimal also for such disturbances if Δ' is a factor of either the criterion polynomial $\tilde{\Delta}$ or of D.

It is, however, not entirely trivial to show this. Because of the unstable disturbance model, the system is not stabilizable. The problem lies at the very limit of applicability of LQG design methods. The signals entering the criterion (y(t) and $\tilde{\Delta}u(t)$) are stationary, if the regulator of Corollary 3.3 is used. The signal derivatives $\partial y/\partial p_i$,$\partial y/\partial Q_j$ from (A1.4),(A1.6) will, however, be nonstationary if w(t) is nonstationary. Thus, the expressions (3.15),(3.16) for the criterion derivativs will be meaningless, even at the optimal point.

We need to establish the optimality of the feedforward filter of Corollary 3.3, without using the criterion derivative expressions. This is done for systems with nonstationary measurable disturbances in the following theorem.

Theorem 3.10

Assume that $H=H_s\Delta'$, where H_s is stable and Δ' has zeros on the unit circle. Let Δ' be a factor of either $\tilde{\Delta}$ or D, and assume e(t)=0. Then, the feedforward regulator of Corollary 3.3 minimizes the criterion (3.6). The output and $\tilde{\Delta}u(t)$ will be stationary signals with finite (and optimal) variance.

Proof: See Appendix A5.

If Δ' is a factor of D, the output of the uncontrolled system will not be drifting. If Δ' is not a factor of D, a drifting input signal is needed. To keep the criterion finite, Δ' must then be a factor of $\tilde{\Delta}$.

Note that Δ' will not be a factor of P. This is in contrast to the feedback regulator (3.93), where Δ is a denominator factor. The internal modelling principle does not apply to feedforward control.

Let us check that the feedforward filter has the correct static gain

$$\frac{Q(1)}{P(1)}\frac{B(1)}{A(1)} = \frac{D(1)}{A(1)} \quad \text{i.e.} \quad \frac{Q(1)}{P(1)} = \frac{D(1)}{B(1)}$$

when $\Delta'=1-q^{-1}$ and $\tilde{\Delta}=1-q^{-1}$. Since $D(1)=D_*(1)$ for any polynomial D, the spectral factorization (3.24) gives

$$r\beta(1)\beta(1) = B(1)B(1) + 0$$

Equation (3.38) and (3.39) give

$$P(1) = \beta(1)G(1)$$

$$B(1)D(1)G(1) = r\beta(1)Q(1)+A(1)H(1)L(1)$$

But H(1)=0 since $H=H_s(1-q^{-1})$. A combination of the expressions above confirms that

$$\frac{Q(1)}{P(1)} = \frac{B(1)D(1)G(1)}{r\beta(1)\beta(1)\ G(1)} = \frac{D(1)}{B(1)}$$

Thus, the output goes to zero asymptotically after a step disturbance. When $\rho=0$, the mean square control error of the step response is minimized. An optimal feedforward filter for step load disturbances on non-minimum phase systems was derived by Åström (1976), in continuous time. It corresponds to $\rho=0$, $\Delta'=1-q^{-1}$, $G=H_s=1$, d=0 and D=A in our formalism.

In the same way as above, the feedforward filter for sinusoid disturbances is seen to have the correct gain and phase shift at the frequency ω_0. It cancels the sinusoid completely, even for non-minimum phase systems.

When $v_1(t)$ in (3.94) is an integrated white disturbance, and $D(1)\neq0$, the design results in a nonstationary input chasing a nonstationary disturbance, trying to cancel it. The output will be nonstationary unless the regulator is optimal. At all points in parameter space except the optimal one, the criterion will be infinite. This explains why derivatives such as (3.15),(3.16) cannot be used.

When the auxiliary measurement is affected by the input, Δ' will be a common factor of H_{wu}:

$$w(t) = q^{-n}\frac{N\Delta'}{H_s\Delta'}u(t)+\frac{G}{H_s\Delta'}v(t)$$

This factor should be cancelled before the regulator (3.78) is implemented. Hidden drifting modes inside the regulator are then avoided.

SERVO FILTER OPTIMIZATION

Design of the servo filter T/E in (3.57) to optimize the step response can be seen as a feedforward control problem where Theorem 3.10 is applicable.

Let the desired response be defined by the reference model

$$y_m(t) = q^{-r}\frac{B_m}{A_m} r(t) \qquad (3.95)$$

where B_m and A_m are stable, and $r \geq k$. As a model of r(t), we consider a train of randomly occuring steps, i.e.

$$r(t) = r(t-1)+e_1(t)$$

with $e_1(t)$ being a train of impulses. When the system (3.7) is controlled with (3.57), the output becomes

$$y(t) = \frac{MG}{P\alpha H} v(t)+ \frac{RC}{\alpha} e(t)+q^{-k} \frac{cB_sB_uT}{\alpha E} r(t) \qquad (3.96)$$

where P, M, α and R are given by previous steps of the feedback-feedforward design procedure. See, for example, the Corollaries 3.7 and 3.8 with equations (3.64) and (3.73).

If the system is non-minimum phase ($degB_u>0$), the last part of (3.96) cannot be made equal to (3.95), since B_u could only be cancelled by unstable factors of E. In such cases, we may wish to minimize the influence of the non-minimum phase dynamics on the step response.

When the problem is presented as in Figure 3.8, it becomes obvious that feedforward filter optimization according to Corollary 3.7 and Theorem 3.10 may be used. (Compare with Figure 3.3.)

With obvious substitutions in Corollary 3.7, the optimal servo filter is seen to be given by

$$\frac{T}{E} = \frac{\alpha T_1}{B_s\bar{B}_uA_m} \qquad (3.97)$$

where T_1 and L_1 solve

$$q^{-r+k}\bar{B}_uB_m = cB_uT_1+A_m\Delta L_1 \qquad (3.98)$$

where $r \geq k$ has been assumed, with degrees

$$nT_1 = \max\{na_m, nb_m+r-k\}$$

$$nL_1 = degB_u-1$$

The optimal servo filter has poles in the inverse points with respect to the unit circle of non-minimum phase zeros. This property has been discussed by Shaked (1984).

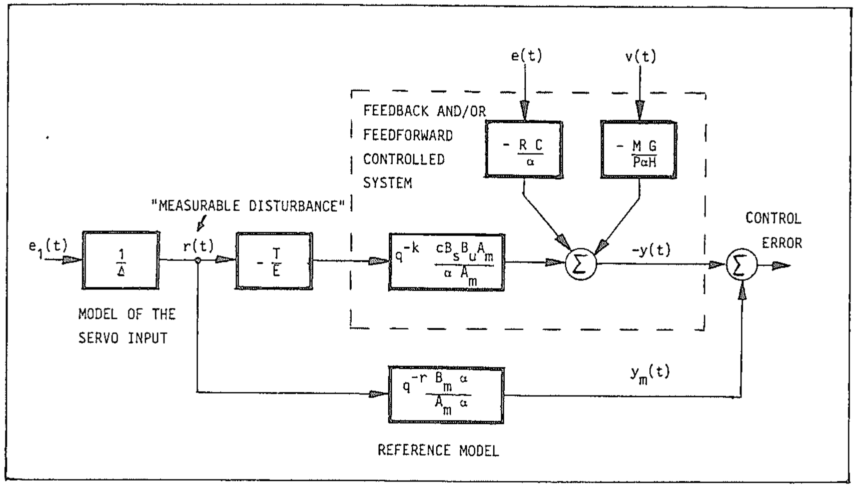


Figure 3.8 Optimization of the servo response of non-minimum phase systems. It is seen as a minimum variance feedforward regulator design problem for an integrated white disturbance.

3.6 THE CORRESPONDENCE BETWEEN FEEDFORWARD AND INPUT ESTIMATION (DECONVOLUTION)

THE INPUT ESTIMATION PROBLEM

The problem of estimating the input to a linear system from noisy measurements of the output has attracted a lot of interest, for example in seismology, hydrology and spectroscopy. In a wide sense, this is known as deconvolution. A variety of methods have been suggested. A popular starting point is a discrete time approximation of the convolution integral. This often leads to an overdetermined system of simultaneous equations. See, for example, Commenges (1984), Demoment and Reynaud (1985) or Mendel (1983). Various state space approaches have also been considered, some with an explicit model of the input, cf Crump (1974), Candy and Zicker (1982) and Ahlēn (1984),(1986).

In Ahlēn and Sternad (1985), an optimal filter in polynomial form is derived, using the same optimization method as in Appendix A2. In this section, it will be demonstrated that this solution corresponds very closely to the optimal feedforward filter of Corollary 3.3.

Consider a linear stochastic system given by

$$y(t) = q^{-k} \frac{B'(q^{-1})}{A'(q^{-1})} u(t) + \frac{M'(q^{-1})}{N'(q^{-1})} v(t) \qquad (3.99)$$

where the unknown input sequence u(t) is modelled as

$$u(t) = \frac{C'(q^{-1})}{D'(q^{-1})} e(t) \quad ; \quad Ev(t)^2/Ee(t)^2 = \rho \qquad (3.100)$$

All model parameters are assumed to be known. The polynomials A', D', N' and M' are stable and monic. C' and B' may be unstable. The white noise sequences v(t) and e(t) are stationary, zero mean and mutually uncorrelated.

The problem is to find a linear estimator of the input

$$\hat{u}(t|t-m) = \frac{Q(q^{-1})}{P(q^{-1})} y(t-m) \qquad (3.101)$$

which minimizes the mean square estimation error

$$Ez(t)^2 = E(u(t)-\hat{u}(t|t-m))^2 \qquad (3.102)$$

Depending on m, we get an input prediction (m>0), filtering (m=0) or a fixed lag smoothing problem (m<0). Obviously, the minimal achievable estimation error decreases with a decreasing m. The problem formulation includes output filtering problems (estimation of f(t) in Figure 3.9) as special cases with A'=B'=1, k=0.

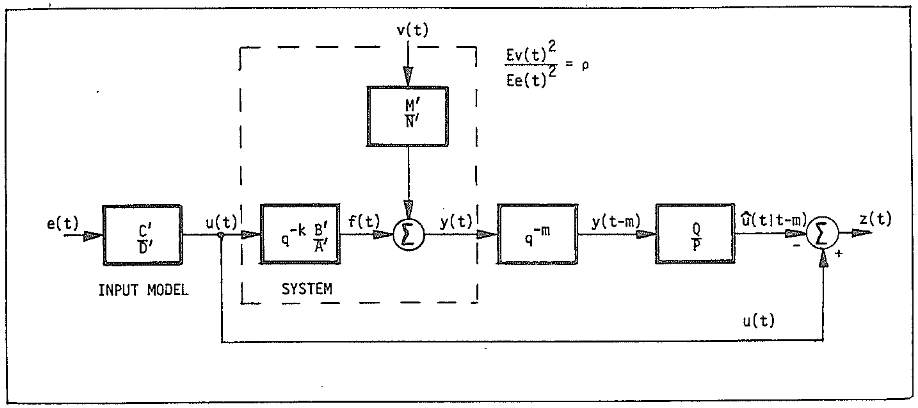


Figure 3.9 The input estimation problem. Given a noise-corrupted output measurement y(t-m), an optimal filter Q/P is to be designed. Its output should be a mean square optimal estimate of the system input u(t).

The infinite horizon optimal solution from Ahlén and Sternad (1985) is calculated in the following way.

Solve for the monic stable spectral factor β from

$$r\beta\beta_* = C'B'N'C'_*B'_*N'_*+\rho M'A'D'M'_*A'_*D'_* \qquad (3.103)$$

Let

$$\frac{Q}{P} = \frac{Q_1N'A'}{\beta} \qquad (3.104)$$

where Q_{1*} and L are the minimum degree solution of

$$z^{m+k}C'B'N'C'_* = r\beta Q_{1*}+D'_*zL \qquad (3.105)$$

Let us now transform the input estimaton problem of Figure 3.9 to an equivalent problem with two separate signal paths to z(t) from v(t) and e(t). See Figure 3.10.

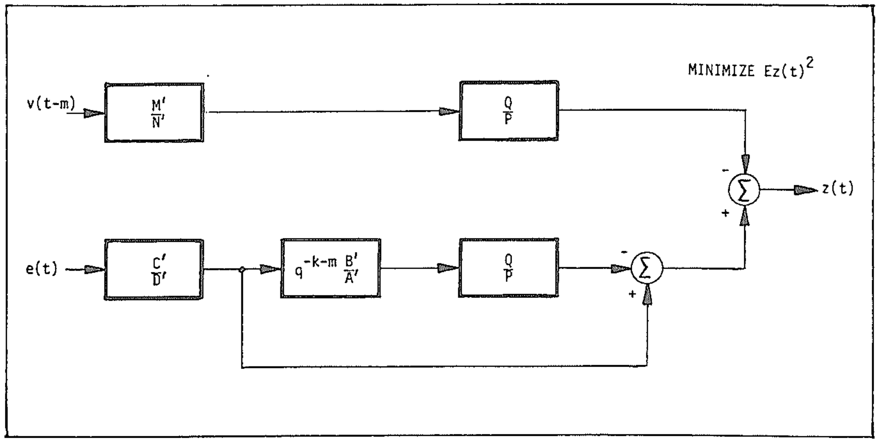


Figure 3.10 An equivalent description of the input estimation problem of Figure 3.9.

Introduce M'N' as common factors of the signal path from e(t). It may then be transformed to the structure shown in Figure 3.11 where we have introduced new signals u(t), $u_1(t)$, f(t), w(t) and y(t).

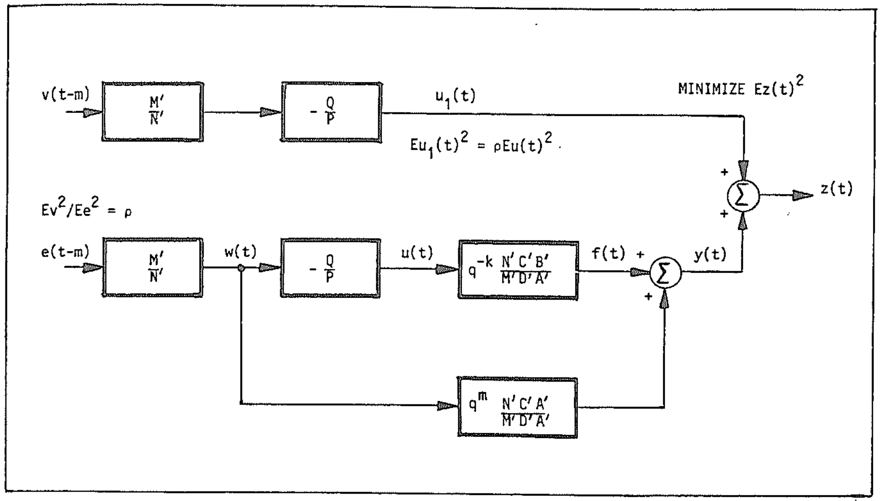


Figure 3.11 A further modified problem formulation, which is still equivalent to the input estimation problem of Figure 3.9. Note that the signal y(t) is given by a feedforward system structure, as in Figure 3.3.

In Figure 3.11, the lower signal path has been transformed to a feedforward control problem structure. Note that while the signal $u_1(t)$ in the upper signal path will be uncorrelated to both u(t) and y(t), its variance is given by $\rho Eu(t)^2$, since $Ev^2/Ee^2 \triangleq \rho$. Thus, the criterion may be written as

$$Ez(t)^2 = E(y(t)+u_1(t))^2 = Ey(t)^2+Eu_1(t)^2 = Ey(t)^2+\rho Eu(t)^2$$

This is nothing but the criterion used in the feedforward problem, with $\tilde{\Delta}(q^{-1})=1$. The upper signal path of Figure 3.11 represents the input penalty.

A comparison of the blocks in Figure 3.11 with the corresponding blocks in the feedforward problem of Figure 3.3 suggests the following result.

Theorem 3.11

The input estimation problem defined by (3.99)-(3.102) may be seen as an equivalent feedforward control problem. An optimal filter Q/P may be designed from Corollary 3.3 using the following substitutions:

- Use $Ev(t)^2/Ee(t)^2$ as ρ and set $\tilde{\Delta}=1$ in the criterion.

- When $m \leq 0$, use m as -d. (Filtering and smoothing.)

- When $m>0$, set d=0 and use k+m as k. (Prediction.)

- Use

$$\begin{aligned} &N'C'A' \text{ as } D \\ &M'D'A' \text{ as } A \\ &N'C'B' \text{ as } B \\ &M' \text{ as } G \\ &N' \text{ as } H \end{aligned} \tag{3.106}$$

∎

It is easy to verify that these substitutions will in fact convert equations (3.24), (3.37)-(3.39) into (3.103)-(3.105). In a similar way, feedforward control problems may be transformed to equivalent input estimation problems. For details see Sternad and Ahlén (1987).

The transformation discussed above helps to clarify several similarities between the control and signal processing problems. The disturbance time delay d plays the same role as the smoothing lag -m in the estimation problem. Large time delays/smoothing lags improve the control/filtering performance. Minimum variance feedforward problems ($\rho=0$) correspond to estimation problems with noise-free measurements ($v(t)=0$). Cases when perfect feedforward control is possible correspond to cases when the input may be reconstructed perfectly from output measurements. The inverse system is then used in both the control and the estimation problem.

The correspondence discussed here cannot be reduced to the well known duality between state estimation and linear quadratic state feedback problems. Note that the optimal feedforward problem is a LQG problem. In a state space formulation, it would include both state estimation and optimal feedback from the estimated states. Instead, the key to understanding the correspondence is the fact that no closed loop occurs in either problem. This is why simple transformation can be used to convert one problem into the other, as in Figures 3.9-3.11.

The two problems can be seen as two interpretations of one underlying design problem. Several other signal processing design problems can be seen as other interpretations.
Noise canceling, echo cancelation and line enhancement (cf Widrow et al 1975 or Goodwin and Sin 1984) are some examples. They can be transformed into the basic feedforward problem, with $\rho=0$, using even simpler transformations than those we have just used.

3.7 WHAT HAPPENS BETWEEN THE SAMPLING INSTANTS?

We have discussed feedforward and disturbance decoupling regulator design based on the discrete time model (3.4),(3.5). In general, the model represents a continuous time process. This section contains a short discussion of the continuous time performance of discrete time regulators. For the rest of this thesis, the complex variable z will be used for q instead of q^{-1}. (The stability area is inside the unit circle.)

HIDDEN OSCILLATIONS

Example 3.2

Consider feedforward regulator design for the continuous time system

$$y(s) = \frac{7}{(s+1)(s+7)} u(s) + e^{-0.1s} \frac{20(s-2)}{(s+1)(s+7)} w(s) \qquad (3.107)$$

Assuming u and w to be constant between the sampling instants, sampling with a period of 0.1 gives the sampled model

$$(1-1.401q^{-1}+0.4493q^{-2})y(t) = q^{-1}(0.02712+0.02079q^{-1})u(t)$$
$$+ q^{-2}(1.206-1.480q^{-1})w(t) \qquad (3.108)$$

The transfer function from u to y is minimum phase. B(z) has a zero in -0.766.

Let us test a sampling feedforward regulator which cancels the disturbance completely

$$u(t) = -\frac{Q}{P} w(t) = -q^{-1} \frac{D}{B} w(t) = -q^{-1} \frac{(44.47-54.57q^{-1})}{1+0.767q^{-1}} w(t) \qquad (3.109)$$

In Figure 3.12, a unit step disturbance w(t) enters the controlled system.

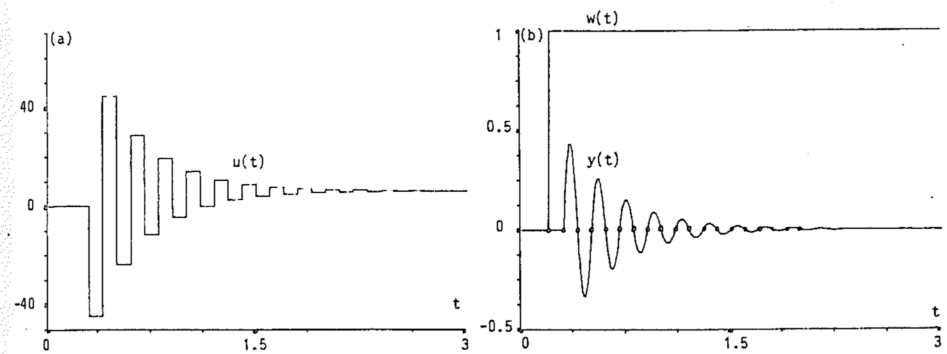


Figure 3.12 Response to a step disturbance w(t) when the perfect discrete time feedforward regulator (3.109) is used to control the system (3.107) with a sampling period of 0.1.

The disturbance is cancelled completely, but only at the sampling instants. *Hidden output oscillations*, such as in Figure 3.12b, often occur in sampled data feedback control when process zeros are cancelled. The oscillations are also called *inter-sample ripple*.

■

How to avoid inter-sample ripple

As discussed in Åström, Hagander and Sternby (1984), sampling of a system with relative degree (pole excess) $r \geq 2$ introduces r-1 additional zeros, "*sampling zeros*". (The pulse transfer function of the sampled system has relative degree 1, if no time delays are present.) These extra zeros are situated on the negative real axis for small sampling periods. Some may be outside the unit circle.

Minimum variance feedforward filters have poles in the zeros of $B_s\bar{B}_uG$. System zeros on the negative real axis result in oscillative input signals, since the feedforward filter will have poles in the region between 0 and -1. Compare with (3.109) and Figure 3.12. Zeros belonging to B_s cancel the corresponding regulator poles in the transfer function

$$q^{-k}\frac{B}{A}\frac{Q}{P} = q^{-k}\frac{B_sB_u}{A}\frac{Q}{B_s\bar{B}_uG}$$

(In general, it is evident from (3.10) and (3.13) that factors of P which are cancelled in the transfer function from v(t) to y(t) must also be factors of B, if (Q,P) are coprime.)

Thus, the corresponding oscillative modes will not affect the output at the sampling instants. If these zeros are sampling zeros, as they normally are, they do not, however, have any counterpart in the continuous time system. The continuous time output is affected by the oscillation. It will contain a Nyquist frequency oscillation which is zero at the sampling instants. The magnitude is determined by the gain at the Nyquist frequency of the system. These problems are particularily severe when fast sampling is used on systems with an even relative degree. The corresponding discrete time system then contains a zero which goes to -1 as the sampling frequency increases. See Åström, Hagander and Sternby (1984).

The intersample ripple we consider here is caused by an oscillative input. (Another type of hidden oscillation occurs when the Nyquist frequency is made exactly equal to the frequency of an oscillative mode. See Åström and Wittenmark (1984).) This indicates an obvious way to avoid the problem: Use a small nonzero input penalty whenever minimum variance control would require a real pole placed in the region between approximately -0.3 and -1. Increase the penalty until input oscillations disappear.

Example 3.2 (cont'd)

Let us optimize a feedforward filter for step disturbances, using a small input penalty.

The use of the model (3.108), $G/H=1/\Delta$, $\tilde{\Delta}=\Delta$ and $\rho=0.001$ in Corollary 3.3 gives the regulator

$$u(t) = -\frac{10.76-9.396q^{-1}-4.675q^{-2}}{1-0.735q^{-1}+0.3797q^{-2}-0.06564q^{-3}}\,w(t) \qquad (3.110)$$

with poles in 0.25 and 0.24±0.45i.

The disturbance step response is shown in Figure 3.13.

While the output is no longer zero at the sampling instants, the disturbance rejection is still good. (A unit step disturbance would result in a static output of -5.7 in the uncontrolled system.) Input oscillations and inter-sample ripple are no longer problems with this regulator.

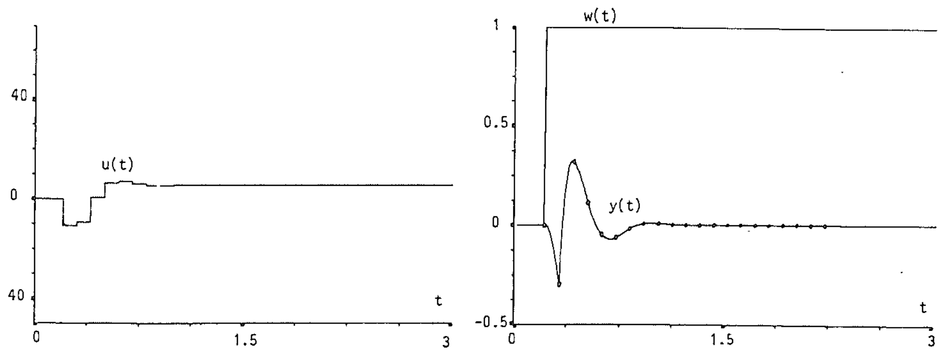


Figure 3.13 Step disturbance response when the feedforward filter (3.110), optimized with ρ=0.001 and $\tilde{\Delta}= 1-q^{-1}$, is used to control the system (3.107).

The design of feedback regulators which avoid hidden oscillations has been discussed by Zafiriou and Morari (1985) and by Lennartson (1986). They recommend that zeros with negative real parts should not be cancelled, i.e. that pole palcement in the left half of the unit circle should be avoided. This rule seems to be overly conservative. Note that only a Nyquist frequency input oscillation (changing sign each sample) can cause *hidden* oscillations. Output oscillations caused by input oscillations of lower frequency would be visible at the sampling instants. The LQG design method, which optimizes the signal behaviour at the sampling instants, would then be able to find the proper compromise between disturbance rejection and input variations.

STOCHASTIC DISTURBANCES

For stochastic disturbances, the output variance at the sampling instants is used to measure the control performance. A better measure would be the average variance of the underlying continuous time output. Lennartson and Söderström (1986) have compared these two variances for different feedback controllers. When minimum variance control is used, the continuous time variance may be considerably larger than the variance measured at the sampling instants. The mean output variations are then largest in the middle of the sampling intervals. This is a stochastic counterpart to the inter-sample ripple. A solution to this problem, both for feedback and feedforward regulators, is again the use of a positive input penalty.

The regulators we consider minimize the discrete time criterion (3.6). Let τ be an integer and h the sampling period. With $\tilde{\Delta}$=1, the criterion value can be expressed as

$$J = \lim_{N\to\infty} \frac{1}{2N} \sum_{\tau=0}^{N} E[y(\tau h)^2+\rho u(\tau h)^2] \qquad (3.111)$$

Continuous time infinite horizon LQG controllers would minimize the loss function

$$J_c = \lim_{N\to\infty} \frac{1}{2Nh} E \int_0^{Nh} (y(t)^2+\rho_c u(t)^2)dt \qquad (3.112)$$

The criterion (3.112) can be transformed into a discrete time loss function, assuming u(t) to be constant over the sampling period. (See, for example, Åström and Wittenmark 1984.) By optimizing this criterion with state space LQ theory, a sampling regulator minimizing (3.112) could be found. It would optimize the output variance of the underlying continuous time system.

The polynomial methods we have discussed, which are used to minimize (3.111), lead to simpler calculations than LQG design minimizing (3.112). One may ask, however, if minimization of (3.111) leads to significant deterioration of the (continuous time) control performance, compared to optimization of (3.112).

This problem has been investigated for feedback controllers by Lennartson (1986). Input penalties were used to fix the input variance of both types of controllers to the same value. In all design examples investigated, the output variance achieved by minimizing (3.111) was then found to be insignificantly above the optimal value. In the minimum variance control case (no input penalty), the difference may, however, be considerable, due to the inter-sample variation of the output variance.

In Lennartson (1986), the performance of several other discrete time feedback design methods was also investigated. Minimization of sliding short time criteria and several pole placement rules performed considerably worse than minimization of (3.111).

Let us summarize the points made in this section.

- Minimum variance feedforward control may lead to hidden oscillations and significant inter-sample variations of the output variance.

- The use of an input penalty large enough to prevent Nyquist frequency input oscillations will remove such problems.

- When such an input penalty is used, the control performance (measured in continuous time) will, in general, be close to the best that can be achieved by a discrete time regulator.

3.8 CONLUDING DISCUSSION

How does the method developed in this chapter compare to the previous approaches discussed in Chapter 2?

Compared to the manual tuning of simple filters, a systematic optimization method indicates the best achievable performance. In many cases, it is worthwhile to investigate if simple filters come close to this performance. If they do, they may be implemented instead of the optimal ones, for cost or convenience reasons. An example of such an investigation is found in Section 4.2.

Geometric and algebraic disturbance decoupling methods cannot be used in the numerous cases when perfect cancellation of disturbances is impossible. For such problems with a single input, our method provides the optimal solution. In addition, trade-offs between input variations and disturbance rejection are possible.

In contrast to the disturbance prediction approach, the method works for non-minimum phase systems.

What is gained by performing a linear quadratic Gaussian design with polynomial methods instead of state space formulations and algebraic Riccati equations? In addition to the absence of problems when $\rho=0$, there are three main advantages:

1. Polynomial methods give a good insight into the frequency domain properties of optimal filters.

2. Certain problems where the system is undetectable or unstabilizeable, as in Theorem 3.10, can be solved with polynomial methods. See also Kučera and Šebek (1985).

3. The calculations become simpler. This is especially evident for systems with significant time delays. Delays increase the dimension of the state vector and the computational burden in solving Riccati equations. Compare this to equations (3.24) and, for example, (3.39). The spectral factorization is unaffected by the time delays k and d. The order of the polynomial equation is affected only by the *difference* k-d.

While the advantages, compared to the alternative approaches, are evident, the design method is certainly not without limitations. Let us revisit Section 3.1, and point out some of the basic assumptions stated there.

- The method has been developed only for single input systems. (Multiple auxiliary measurements can be handled.)

- Sampled models are used to describe a continuous time reality. As discussed in Section 3.7, the regulators achieve close to the best performance possible for discrete time regulators. This performance may however be considerably below the performance of a continuous time LQG regulator, if the sampling rate is low. Let J_{cd} and J_{cc} be the minimal values of (3.112) when discrete time and continuous time control is used, respectively. It can be shown that J_{cd} depends on the sampling period h as

  $$J_{cd} = J_{cc} + O(h^2)$$

  to a first approximation. (Åström,1963.)

  For system without time delays, results corresponding to Theorems 3.1-3.10 can be formulated in continuous time. For systems with time delays, the situation is more complicated. The disturbance decoupling problem for delay-differential systems is far from solved. Some partial results have been presented by Pandolfi (1986). The ease of treating time delays in sampled data systems is a main reason, apart from the prevalence of computer control, to consider feedforward filter design primarily in discrete time.

- Systems are assumed to be linear. This will be an approximation, valid only around an operating point. One advantage of adaptive regulators is their ability to retune regulators at different operating points. The subject of nonlinear feedforward control is not discussed in this thesis. It should be said, however, that static nonlinear functions may often be useful as feedforward links. A method which, among other applications, can be used to tune static nonlinear feedforward links adaptively, has been suggested by Åström (1985).

- The system dynamics and disturbance characteristics are assumed to be time invariant. Disturbances with time-varying dynamics are often encountered in practice. One possibility in the case of large time variations is to abstain from using any information about the disturbance, i.e. to set G=H=1 in the polynomial equations. Another possibility is the use of an adaptive regulator, which retunes the controller parameters whenever a new type of disturbance appears.

- No measurement noise m(t) on w(t) was assumed to be present. With measurement noise, the design method would include another spectral factorization. The more an auxiliary output w(t) is corrupted by noise, the less a regulator should depend on it. The adaptive algorithms discussed in Chapters 5 and 6 have this property.

- The dynamics of output sensors has not been taken into account. The signal to be controlled, y(t), is assumed to be directly measurable in the feedback design.

4. EXAMPLES AND A CASE STUDY

This chapter illustrates the use of the results from Chapter 3 for off-line design of feedforward regulators.

The two numerical examples of Section 4.1 illuminate the following points:

- The simplicity of the calculations used for designing optimal feedforward filters.

- How the achievable control performance depends on system parameters.

- The use of an input penalty to achieve a tradeoff between disturbance rejection and input energy.

The case study of Section 4.2 describes the optimization of load feedforward filters used to control hydro power stations. The methods developed in Chapter 3 are well suited to solve this problem: The plant is non-minimum phase. A tradeoff between disturbance rejection and input rate is involved. Theorem 3.1 is relevant to the problem, since an optimized digital feedforward filter is to complement an existing analog feedback regulator. The following topics are also discussed:

- How does the choice of regulator structure influence the numerical sensitivity (finite word length characteristics) of the feedforward filter?

- How robust is the design? The system is affected by a time-varying parameter: The total electrical load.

- Is it really worthwhile to perform an optimization? The optimal regulator is compared to a very simple first order feedforward filter.

4.1 EXAMPLES

EXAMPLE 4.1

The performance of minimum variance feedforward control

$$u(t) = -\frac{Q}{P}w(t)$$

as a function of the parameters b and d in the system

$$y(t) = (1+bq^{-1})B_s(q^{-1})u(t-k)+(1+dq^{-1})w(t-k)$$

$$w(t) = v(t)$$

is investigated.

$B_s(q^{-1})$ is a stable monic polynomial. A=1, G=1, H=1 and d=k. When $|b|<1$, perfect feedforward is possible. When $|b|=1$, a stable minimum variance feedforward filter cannot be designed. Now assume $|b|>1$, i.e. $B=cB_uB_s$, c=b, $B_u=(1/b)+q^{-1}$ or $\bar{B}_u=1+(1/b)q^{-1}$. Equations (3.61)-(3.63) of Corollary 3.7 then give

$$P = B_s\bar{B}_uG = B_s(q^{-1})(1+(1/b)q^{-1})$$

$$\bar{B}_uDG = cB_uQ+AHL_1 \qquad (g=0,d=k)$$

with degrees nQ=1 and $nL_1=0$.

$$(1+(1/b)q^{-1})(1+dq^{-1}) = (1+bq^{-1})(Q_o+Q_1q^{-1})+L_1$$

The solution is

$$Q_o = ((b^2-1)d+b)/b^3 \quad ; \quad Q_1 = d/b^2 \quad ; \quad L_1 = (b^2-1)(b-d)/b^3$$

Thus, the optimal feedforward regulator is

$$u(t) = -\frac{Q}{P}w(t) = -\frac{\frac{((b^2-1)d+b)}{b^3}+\frac{d}{b^2}q^{-1}}{B_s(q^{-1})(1+\frac{1}{b}q^{-1})}w(t)$$

According to (3.64), with e(t)=0, the controlled output becomes

$$y(t) = \frac{L_1}{\bar{B}_u}v(t-k) = \frac{(b^2-1)(b-d)}{b^3(1+\frac{1}{b}q^{-1})}v(t-k)$$

The minimal output variance is

$$Ey(t)^2 = \frac{(b^2-1)^2(b-d)^2}{b^6(1-(1/b)^2)}Ev(t)^2 = \frac{(b^2-1)(b-d)^2}{b^4}Ev(t)^2$$

The output variance without regulator was

$$Ey_o(t)^2 = (1+d^2)Ev(t)^2$$

Introduce

$$I \triangleq \frac{Ey(t)^2}{Ey_o(t)^2} = \frac{(b^2-1)(b-d)^2}{b^4(1+d^2)}$$

as a measure of feedforward control quality. Figure 4.1 is a plot of the variance ratio I(b,d).

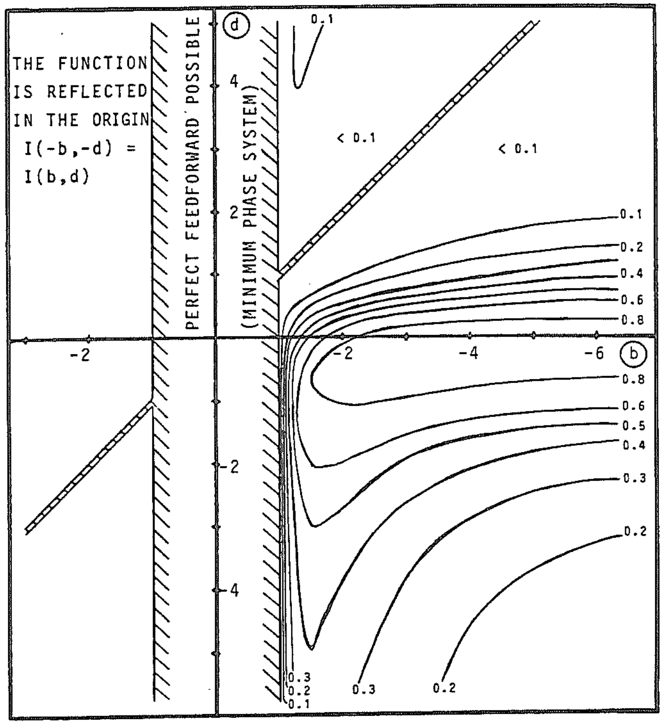
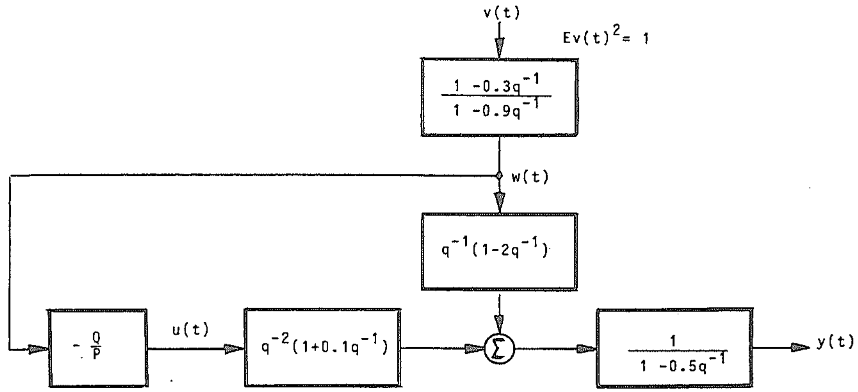


Figure 4.1 Output variance reduction with feedforward control as a function of the system parameters b and d.

It is evident that considerable disturbance rejection can be achieved for non-minimum phase systems. The feedforward is least effective (variance reduction <20%) in the region $|b|>1.5$, small $|d|$. Along the line b=d, i.e. $cB_u=D$, perfect feedforward is possible. The non-minimum phase dynamics is then present in both the disturbance and the input signal path. Cf Figure 3.5.

EXAMPLE 4.2



This example illustrates

- Feedforward regulator design for a system with a time delay k>d, making complete cancellation impossible.

- How the performance is influenced by an input penalty ρ.

System: $k=2;\ d=1;\ B_u=1;\ B_s=1+0.1q^{-1};\ A=1-0.5q^{-1};\ D=1-2q^{-1};$

$G=1-0.3q^{-1};\ H=1-0.9q^{-1};\ e(t)=0.$

Criterion: $J = \frac{1}{2}\left(Ey(t)^2+\rho Eu(t)^2\right)$

In open loop, the output variance is

$$Ey_o(t)^2 = 11.58 = 2J$$

Minimum variance feedforward (Corollary 3.7)

When $\rho=0$, Corollary 3.7 can be used:

$$P = BG = 1-0.2q^{-1}-0.03q^{-2}$$

$$q^{-1+2-1}DG = q^{-1}Q+AHL_1 \quad (d=1,k=2,g=1,B_u=1)$$

where Q and L_1 have degrees 1 and 0, respectively. Using numerical values, the polynomial equation is

$$1-2.3q^{-1}+0.6q^{-2} = q^{-1}(Q_o+Q_1q^{-1})+(1-1.4q^{-1}+0.45q^{-2})L_1$$

with solution $Q(q^{-1}) = -0.9+0.1q^{-1}$ and $L_1=1$.

With $B_u=1$ and $L_1=1$, the controlled output (3.64) becomes $y(t)=v(t-1)$. Thus, in spite of the time delay difference k-d=1, the output variance can be decreased by 91% from 11.58 to 1 in this example.

Spectral factorization

When $\rho>0$, r and β from the spectral factorization (3.24)

$$r\beta\beta_* = BB_*+\rho AA_*$$

have to be calculated.

$$r(1+bz)(1+bz^{-1}) = (1+0.1z)(1+0.1z^{-1})+\rho(1-0.5z)(1-0.5z^{-1})$$

Equating terms with equal power of z on both sides leads to

$$r(\rho) = 0.505+0.625\rho+0.495(1+2.9844\rho+0.5739\rho^2)^{1/2} \qquad (4.1)$$

$$b(\rho) = (0.1-0.5\rho)/r(\rho) \qquad (4.2)$$

When $\rho=0$, then r=1 and b=0.1 ($\beta(z)=B_s(z)$).

When $\rho\to\infty$, then $r\to\rho$ and $b\to-0.5$ ($\beta(z)\to A(z)$)

For illustration, let us perform the regulator design calculation for one value of ρ.

Regulator calculation for $\rho=2$ using Corollary 3.3

The expressions (4.1) and (4.2) give r=3.26 and $\beta(z)=1-0.276z$ when $\rho=2$. The filter denominator thus becomes

$$P = G\beta = (1-0.3q^{-1})(1-0.276q^{-1})$$

$Q_*(z^{-1})$ and L(z) are calculated from (3.39). In this case, nQ=1 and nL=1

$$z^{-1+2}(1+0.1z)(1-2z^{-1})(1-0.3z^{-1}) = 3.26(1-0.276z)(Q_o+Q_1z^{-1})+$$

$$+(1-0.5z^{-1})(1-0.9z^{-1})z(\ell_o+\ell_1z)$$

Multiplication with $z^{nQ}=z$ results in an equation with positive powers of z only. A system of simultaneous equations is given by considering terms with equal power of z.

$$\begin{pmatrix} 3.26 & 0 & 0.45 & 0 \\ -0.9 & 3.26 & -1.4 & 0.45 \\ 0 & -0.9 & 1 & -1.4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_o \\ \ell_o \\ \ell_1 \end{pmatrix} = \begin{pmatrix} 0.6 \\ -2.24 \\ 0.77 \\ 0.1 \end{pmatrix} \qquad (4.3)$$

The solution is $Q_1=0.1147$, $Q_o=-0.4535$, $\ell_o=0.5018$ and $\ell_1=0.10$. Numerator and denominator of the feedforward filter

$$u(t) = -\frac{Q}{P}w(t) = -\frac{(-0.4535+0.1147q^{-1})}{1-0.576q^{-1}+0.0828q^{-2}}w(t)$$

have no common factors. It is the filter of lowest degree minimizing $2J=Ey(t)^2+2Eu(t)^2$. For the controlled system, $Ey(t)^2=2.15$ and $Eu(t)^2=1.15$.

It is evident from Figure 4.2 how different values of input penalty affect the control performance.

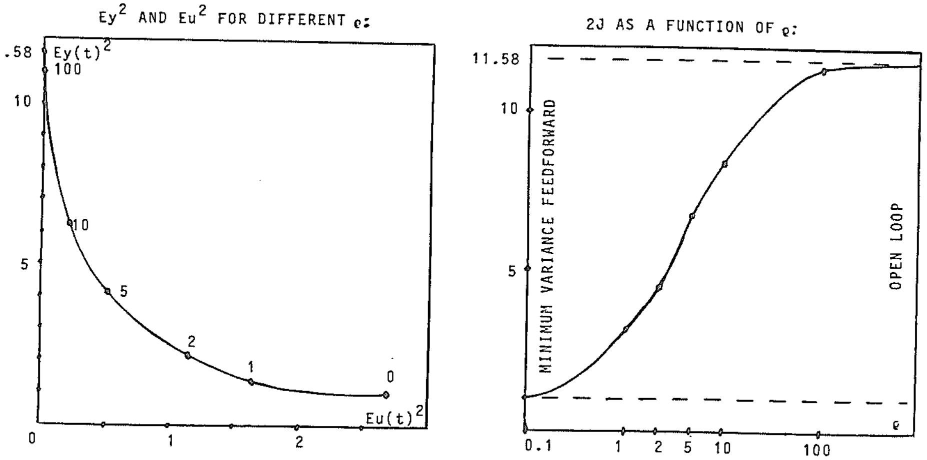


Figure 4.2 The control performance as a function of the input penalty ρ. Note that for all ρ<∞, 2J is less than the open loop value 11.58. This is not surprising: We could always achieve 2J=11.58 by just turning off the feedforward regulator.

4.2 LOAD FEEDFORWARD FREQUENCY CONTROL OF HYDRO POWER STATIONS

If the load in a power system increases, the network frequency will decrease. The power balance is temporarily maintained by "borrowing" stored kinetic energy from the rotating generators. Large variations in turbine speed and electrical frequency reduce the useful life of the turbines. Normally, the electrical frequency can be held within acceptable limits with frequency feedback control. In some systems, feedback control may, however, be insufficient. This is the case in the Argentinian Interconnected System, which motivate this study.

Since the electrical load can be measured, load feedforward may be applied. The problem considered is the design of an optimal digital feedforward filter, acting on all hydro power stations of the system. It will complement the existing analog hydro turbine power-frequency feedback controllers. The disturbance rejection needs to be improved in the frequency range 0.001-0.1 Hz. In this design problem, the system is non-minimum phase and the input is rate-limited. The choice of regulator structure turns out to be an important problem. It affects the numerical sensitivity of the digital filter coefficients. A more detailed discussion of the modelling and the simulation results can be found in Milocco and Sternad (1987).

THE INTERCONNECTED SYSTEM MODEL

In the Argentinian Interconnected System, power is produced by five hydro units, with 60% of the capacity, and several small thermal units. The generating stations are coupled together tightly and form a coherent group. In such coherent areas, the network frequency is the same throughout in static as well as in dynamic conditions. Modelling of interconnected electrical systems is described, for example, in Elgerd (1971). In the frequency range corresponding to the range of time constants

3s-3000s, the thermal stations, with their power frequency regulators, are modelled adequatly by a first order linear system. All hydro units working in parallel are modelled as one equivalent system with a governor:

$$\text{Hydraulic dynamics:}\quad P_W = \frac{1-s\lambda T_W}{1+s(\lambda T_W/2)}\, u \qquad (4.4)$$

$$\text{Speed controller:}\quad u = \frac{1}{E_H}\,\frac{(1+sT_I)}{(1+sT_I')}\,\Delta\omega \qquad (4.5)$$

This is reasonable, since time constants, load factors and regulator parameters are approximately similar in all hydro power stations. Figure 4.3 describes the total linearized model.

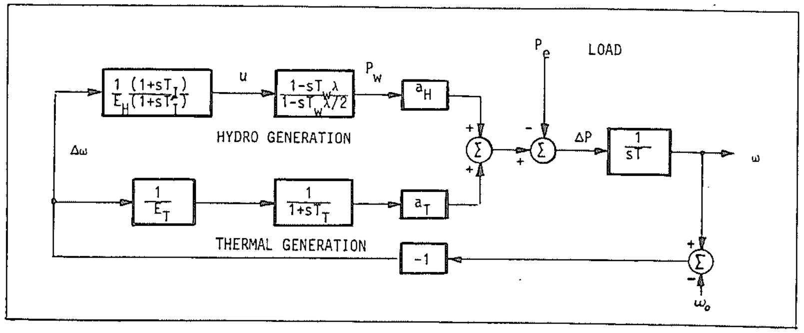


Figure 4.3 A model describing the power frequency deviations in the Argentinian Interconnected System. The fractional contributions of hydro and thermal production are a_H and a_T with $a_H+a_T=1$.

Notations

P_W The turbine power deviation from a nominal operating point pu[MW].

λ The operating point (1=full load).

P_e The load deviation (measurable disturbance) pu[MW].

ω is the system frequency (turbine speed) [Hz] while $\Delta\omega=\omega_0-\omega$ represents the deviation from a reference ω_0 pu[Hz].

T is a constant representing the system inertia [s].

u is the deviation of the total control signal (feedwater gate position) from the operating point pu[MW].

T_W Hydraulic time constant, also called water starting time [s].

T_T Thermal generator time constant [s].

$1/E_H$, $1/E_T$ Hydraulic and thermal regulator gains [pu MW/pu Hz].

("Pu" means "per unit". For example, the value 0.01 represents a 1% increase from the base value. The base values are the full load power production 4000 MW and the nominal network frequency 50 Hz.)

Equation (4.4) is the classical model for describing the linearized hydraulic dynamics. It represents a non-minimum phase system with an inverse response: An opening of the feedwater gate initially results in a *decrease* of the turbine power. (A physical explanation of the inverse response, and a derivation of (4.4) can be found in Milocco and Sternad (1987)). Turbine and load self-regulation has been neglected in (4.4). Ideal turbines have been assumed. Note that the dynamics depends on the operating point λ. The input u is rate-limited: Feedwater gates cannot be moved with arbitrary speed.

Parameter values for the Argentinian system are given in Table 4.1. With E_H=0.04, T_I=10 and T_I'=135, the hydraulic speed governor is a rather weak proportional plus integrating feedback. The rejection of load disturbances is unsatisfactory. The use of higher regulator gain or derivative action is prevented by the presence of oscillatory modes, which give stability problems. These oscillatory modes are not present in our simplified model, since they are faster (0.5-2 s) than the frequency range of interest here.

E_H	:	0.04 pu MW/pu Hz
T_I	:	10s
T_I'	:	135s
T_w	:	4s
λ	:	Varies in the range [0.4,1].
E_T	:	0.3 pu MW/pu Hz
T_T	:	0.35s
T	:	10s
a_H	:	0.6
a_T	:	0.4

Table 4.1. Parameter values.

Figures 4.4 and 4.5 describe how the system rejects load disturbances when λ=0.5. Figure 4.4 is the Bode diagram and Figure 4.5 describes the step response, with the load P_e as input and the frequency deviation $\omega-\omega_0$ as output.

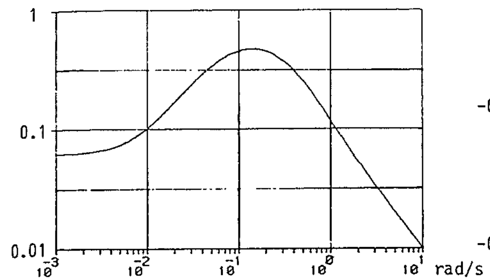


Figure 4.4 Bode plot of the transfer function from disturbance P_e to output $\omega-\omega_0$.

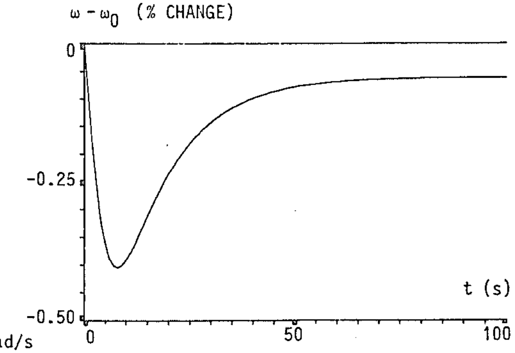


Figure 4.5 Response of the frequency to a positive 1% step load change. (With ω_0=50 Hz, the maximal deviation of -0.4% corresponds to 49.8 Hz.)

As is evident from the Bode plot, the disturbance rejection is worse in the frequency band [0.01 rad/s,1rad/s] than at other frequencies. This corresponds to disturbance time constants in the range 6s-600s. Step load changes result in a large transient frequency deviation for the first ~30s. This kind of load disturbance step response is typical for power systems.

The thermal generators cannot be used to improve the disturbance rejection. Their short time constant T_T=0.35s corresponds to situations with enough steam in the boilers. The stored steam is, however, inadequate for sustaining positive step changes of the operating point. Sustained load changes can be accomodated only by a change in steam production. This is a very slow process, with a time constant of several minutes.

NORMAL LOAD DISTURBANCES

The upper curve in Figure 4.6 shows a typical daily load register. The load can be described by a stochastic process superposed on a daily trend. A weak integrating outer feedback loop compensates for the daily trend. It changes the set point of the previously described control loop (ω_0 in Figure 4.3). The remaining variation, a zero mean stochastic process plotted as the lower curve in Figure 4.6, represents the load deviation P_e.

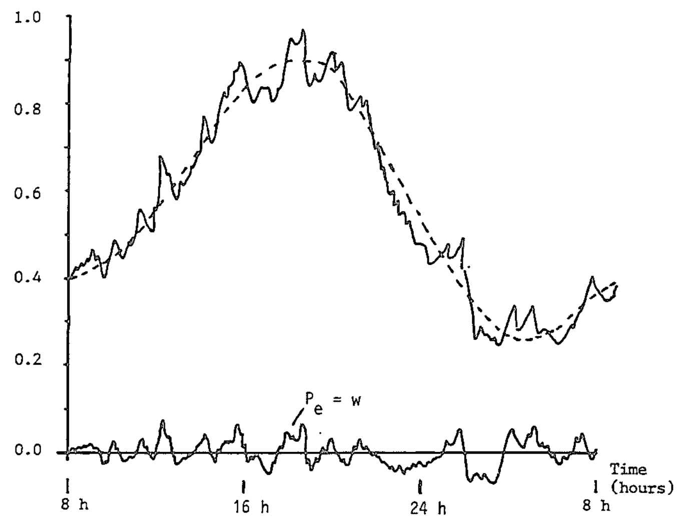


Figure 4.6 The daily load variation. The daily trend is compensated by an outer cascade feedback loop. The remaining variation (lower curve) constitutes the load deviation P_e.

In the discrete time feedforward filter design, a simple discrete time model of the disturbance has been used. Using a sampling time of 1s, an AR model with time constant 1000

$$w(t) = 0.999w(t-1)+v(t) \quad ; \quad Ev(t)^2 = 5\cdot 10^{-6} \qquad (4.6)$$

describes the disturbances with sufficient accuracy. In (4.6), v(t) is white noise and $w(t)=P_e(t)$. The standard deviation of P_e is approximately 0.05 pu, or 5% of the full load. The signal $w=P_e$ is measured by a high-pass filtering the total load, to remove the daily trend.

DESIGN OF AN OPTIMAL FEEDFORWARD FILTER

An improved rejection of the measurable load disturbance P_e is desired. It has been explained why better frequency feedback control cannot solve the problem effectively. Improved control using the thermal units is not technically feasible. This leaves load feedforward acting on one or several hydraulic units as the remaining possible solution. We will consider feedforward control acting on all hydro stations simultaneously here. Other possibilities are discusssed in Milocco and Sternad (1987).

As a feedforward filter design problem, the case studied here has the following characteristic features:

- o The plant has an unstable inverse.

- o The input is rate-limited. To achieve a tradeoff between input rate and disturbance rejection, a differential input penalty $\tilde{\Delta}=1-q^{-1}$, is used. Regulators using input rates with standard deviation less than 12 MW/s are deemed to be acceptable.

- o The load disturbance P_e is (almost) a random walk process. This is another motivation for the use of a differential input penalty, as was explained in Chapter 3.5.

- o The feedforward filter is to complement a preexisting feedback regulator. Theorem 3.1 gives the solution to such problems.

- o The system contains a (slowly) time-varying parameter λ. The feedforward control quality should be robust with respect to variations of λ in the range [0.4,1].

Let us discretize the feedback regulator (4.5). A sampling time of 1s is reasonable, compared with the time scales of interest (3s-3000s). Around the sampling frequency, the system has low gain, cf Figure 4.4. Thus, it is a good approximation to assume $\Delta\omega$ to be constant between the sampling instants. With this assumption, and using $T_I=10$, $T_I'=135$ and $E_H=0.04$, the discrete time model of the feedback becomes

$$\underbrace{(1-0.99262q^{-1})}_{R(q^{-1})}u(t) = \underbrace{(1.85818-1.66735q^{-1})}_{S(q^{-1})}\underbrace{\Delta\omega}_{-y(t)} \tag{4.7}$$

The rest of the system in Figure 4.3 has been sampled separetely, by making the slight approximation that u(t) and P_e=w(t) are constant between the sampling instants.

In this application, the optimal regulator strucutre (3.8) reduces to one where *the feedforward signal is added to the feedback control signal*. To see this, let $\Delta \triangleq 1-q^{-1}$, and make the slight approximations $R=\Delta$ (cf (4.7)) and $H=\Delta$ (cf (4.6)).

The use of $R=\Delta$, $\tilde{\Delta}=\Delta$, $H=\Delta$ and G=1 in (3.28) gives

$$(B(1-z^{-1})-\rho z^{-k}A(1-z)(1-z^{-1})S_*)z^{-d+k}D_* = r\beta Q_*+\alpha_*(1-z^{-1})zL$$

Since $1-z^{-1}$ is a factor of the other three terms, it must be a factor of $r\beta Q_*$. It cannot be a factor of β, which is stable. Consequently,

$$Q_* = Q_{1*}(1-z^{-1}) \tag{4.8}$$

Thus, equation (3.28) reduces to

$$(B-\rho z^{-k}A\Delta S_*)z^{-d+k}D_* = r\beta Q_{1*}+\alpha_* zL \tag{4.9}$$

with polynomial degrees

$$nQ_1 = nQ-1 = \max\{n\alpha-1, nd+d-k, nd+d+ns\}$$
$$nL = \max\{n\beta, nb-d+k\}-1 \tag{4.10}$$

The use of $P=\beta$ from (3.27), $R=\Delta$ and $Q=Q_1(1-q^{-1})$ in the regulator (3.8) gives

$$(1-q^{-1})u(t) = -\frac{Q_1(q^{-1})(1-q^{-1})}{\beta(q^{-1})}w(t)-S(q^{-1})y(t)$$

Cancelling the common factor $1-q^{-1}$ in the feedforward signal path, we arrive at the optimal regulator structure

$$u(t) = -\frac{Q_1(q^{-1})}{\beta(q^{-1})}w(t)-\frac{S(q^{-1})}{R(q^{-1})}y(t) \tag{4.11}$$

where $R(q^{-1})=1-q^{-1}$ and $S(q^{-1})$ are prespecified and β is the stable spectral factor from $r\beta\beta_*=BB_*+\rho A\Delta\Delta_* A_*$.

In this application, an implementation using Theorem 3.1 and the structure (3.8) directly cannot be recommended. Since Q has a zero close to +1, the static gain Q(1)/P(1)R(1) of the feedforward filter Q/PR would be extremely sensitive to the polynomial coefficients of $Q(q^{-1})$. Solution and implementation according to (4.9)-(4.11) reduces such problems. The filter can be implemented using 3 significant decimal places. If lower precision is used, it is advisable to implement the filter in δ-operator form. See Middleton and Goodwin, (1986).

Let us mention an alternative approach which works well in many other cases, but not in this applicaton. The whole feedback controlled system could be represented by one sampled system $Ay=q^{-k}Bu+q^{-d}Dw$, with u being a feedforward control signal added to the feedback action. An optimized feedforward filter could then be calculated from Corollary 3.3. This would avoid separate sampling of system and feedback, which introduces a small approximation. However, the B-polynomial would then contain the regulator pole (practically an integrator) as a factor. Since $\tilde{\Delta}=\Delta$ is used, the spectral facorization (3.24) would contain zeros almost on the unit circle for all ρ. The solution of the spectral factorization would become numerically ill-conditioned.

SIMULATION RESULTS

The system is studied at full load ($\lambda=1$) and half load ($\lambda=0.5$). Optimal feedforward filters Q_1/β for the regulators (cf (4.7), (4.11))

$$u(t) = -\frac{Q_1(q^{-1})}{\beta(q^{-1})}w(t)-\frac{(1.8518-1.66735q^{-1})}{1-0.99262q^{-1}}y(t) \qquad (4.12)$$

have been calculated for different input penalties ρ. When $\rho>0$, optimal feedforward filter are of third order.

Performance has been compared with feedforward control using simple suboptimal first order filters:

$$u(t) = \frac{(1-p)1.667}{1-pq^{-1}}w(t)-\frac{(1.8518-1.66735q^{-1})}{1-0.99262q^{-1}}y(t) \qquad (4.13)$$

The low pass filter pole p is used as a parameter. The factor $1.667=1/a_1$ gives a correct static gain, so that load step disturbances are cancelled asymptotically. Filters with the structure (4.13) were considered because of their simplicity.

Figures 4.7 and 4.8 describe the stationary performance of the optimal filters when $\lambda=1$ and $\lambda=0.5$, respectively. This performance is compared to that obtained by (4.13). The robustness of the design is tested by applying filters optimized for $\lambda=0.5$ on the system with full load, and the other way around. σy and $\sigma\Delta u$ are the standard deviations of $\omega-\omega_0$ and u(t)-u(t-1), respectively. Values of $\sigma\Delta u$ below 12 MW/s are acceptable.

When the load is nominally maximal ($\lambda=1$), the feedwater gates are not assumed to be completely open: There is enough extra generation capacity to make control of normal load deviations around this operating point possible.

In Figure 4.9, typical plots of y(t) and u(t) are shown for the full load case.

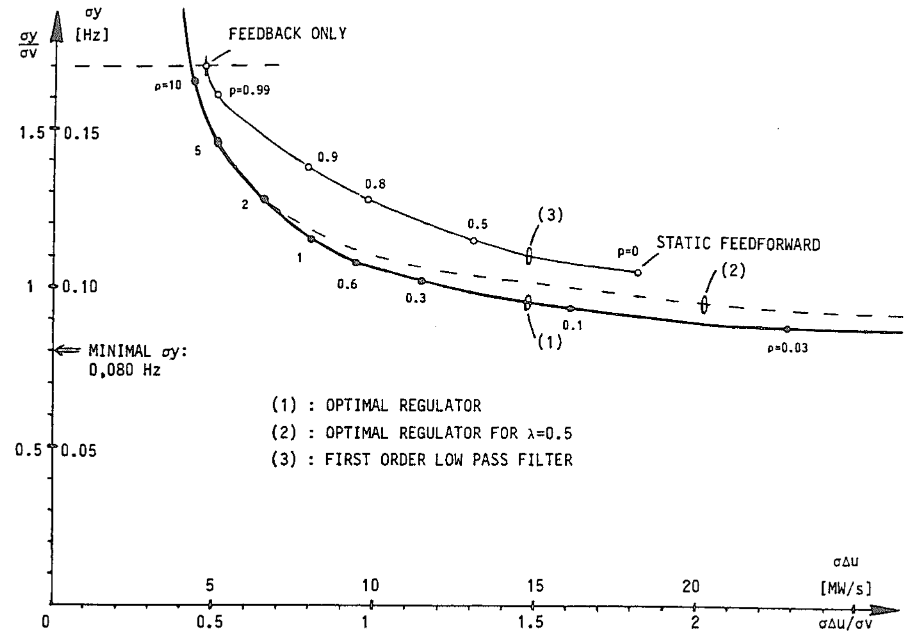


Figure 4.7 Achievable disturbance rejection and input rate at full load (λ=1).

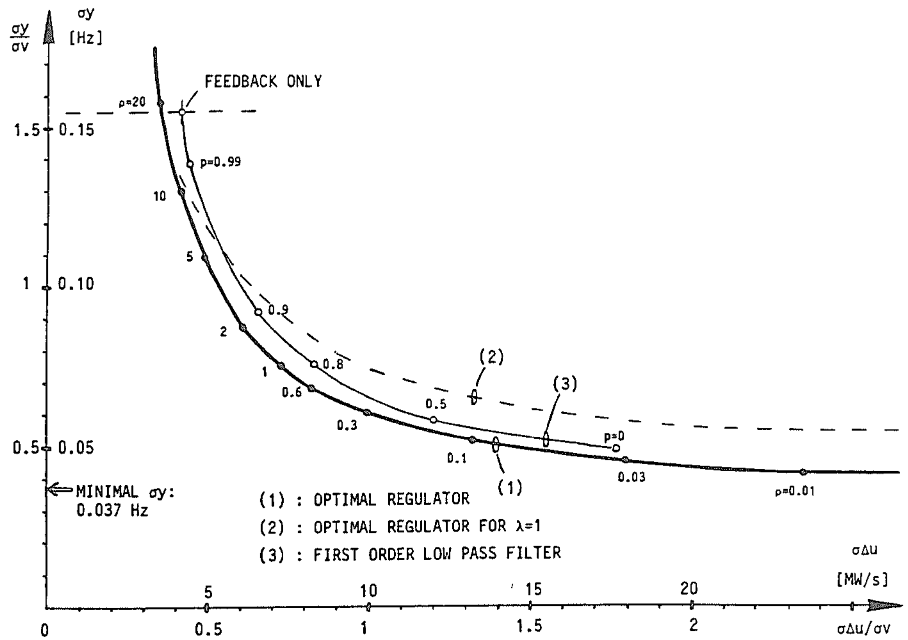
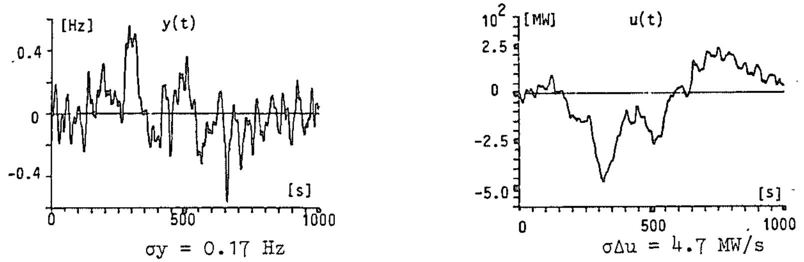
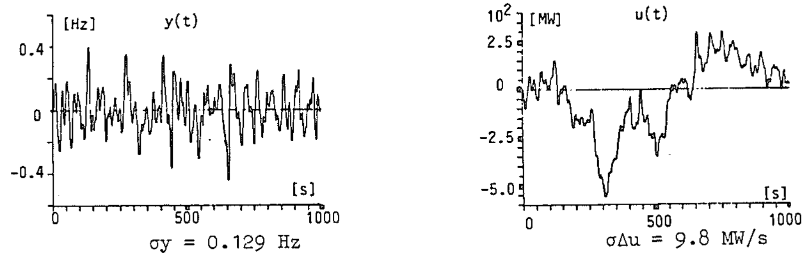


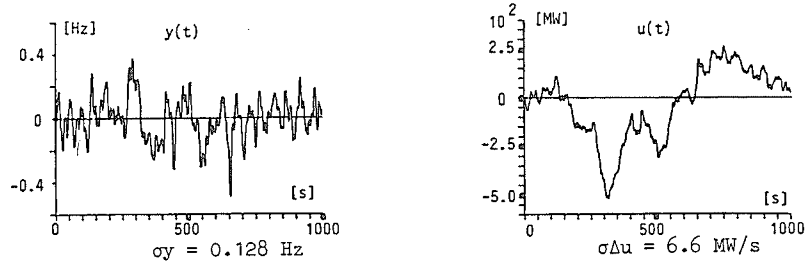
Figure 4.8 Achievable disturbance rejection and input rate at half load (λ=0.5).



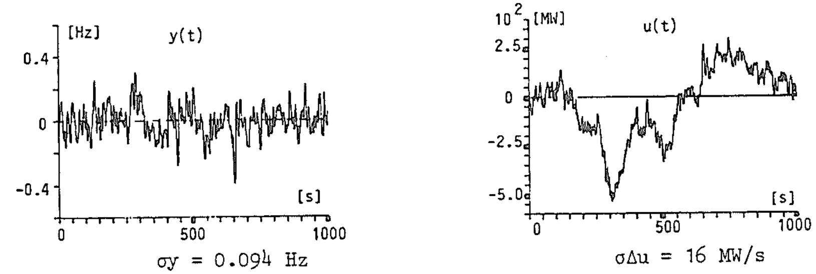
4.9a: Feedback only.



4.9b: Feedback + first order feedforward, p=0.8.



4.9c: Feedback + optimal feedforward, ρ=2.



4.9d: Feedback + optimal feedforward, ρ=0.1.

Figure 4.9 Input and output at full load (λ=1), using different control strategies.

The load step response has also been investigated. The step response indicates how the system recovers from larger transients. It was explained in Chapter 3.5 that the load step response is optimized when the disturbance model $w(t)=w(t-1)+v(t)$ is used. Figures 4.10 and 4.11 show the response to a 1% positive load step when optimal filters are used. Figure 4.12 displays the step response for $\lambda=0.5$ using a first order low pass feedforward filter (4.13) with p=0.8.

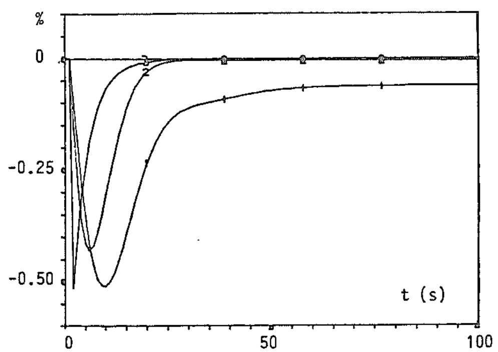


Figure 4.10 Load step response at full load ($\lambda=1$).

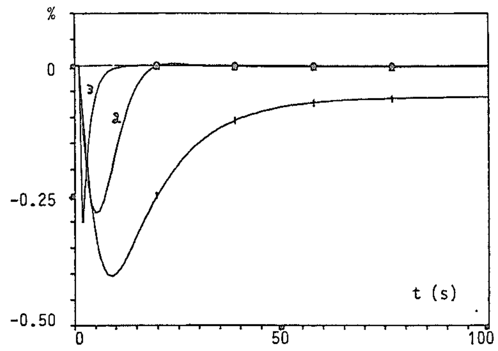


Figure 4.11 Load step response at half load ($\lambda=0.5$).

1: Feedback only.
2: Optimal feedforward for $\rho=0.6$.
3: Optimal feedforward for $\rho=0$.
(Minimal $\Sigma(\omega-\omega_o)^2$.)

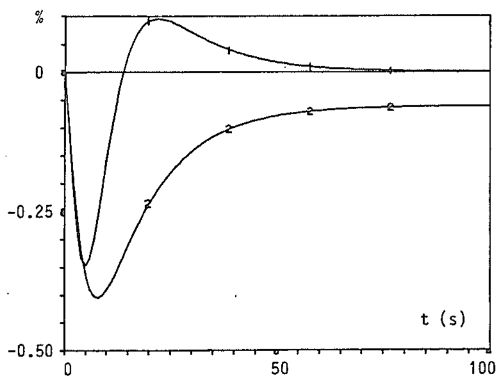


Figure 4.12 Load step response for $\lambda=0.5$.
1: Using first order low pass feedforward control (4.13) with p=0.8.
2: With output feedback only.

CONCLUSIONS

The following conclusions can be drawn from Figures 4.7-4.12:

- o A very large control effort is needed to attain the maximal disturbance rejection ($\rho=0$). Feedforward control can, however, reduce the power frequency standard deviation by 25-60%, with only a moderate increase of the required input rate. An input penalty in the range 0.3-2 seems reasonable.

- o Feedforward filters designed for one load λ behave well in the whole range of load conditions. With filters designed for $\lambda=0.5$, with $\rho>0.3$, we practically obtain an optimal performance also when $\lambda=1$.

- o The load step response is improved. The use of feedforward filters optimized with $\rho=0$ is unrealistic. However, the use of filters optimized with ρ in the range 0.3-2 improve the response considerably. The static error is eliminated. The recovery time is decreased significantly. The peak deviation is reduced. This should decrease the risk of power failures.

- o First order feedforward filters are, as expected, inferior to optimal third order filters. To achieve a given disturbance rejection, they need a larger control effort. For stochastic disturbances, the difference is, however, not overwhelming (when $\lambda=0.5$, first order filters actually perform better than optimal filters designed for $\lambda=1$). In the load step response, the difference is more pronounced. Compare curve 2 in Figure 4.11 to curve 1 in Figure 4.12.

The improved load step response is a compelling reason to choose optimal third order feedforward filters, rather than suboptimal first order filters.

5. ALTERNATIVE ADAPTIVE CONTROL STRATEGIES

The advantages (and problems) of adaptive feedback regulators have been discussed extensively over the last two decades. When feedforward/disturbance decoupling problems are considered, the arguments for adaption become even stronger than for feedback problems. While simple manually tuned feedforward links sometimes improve control considerably, design of a good feedforward compensator in general requires accurate process and disturbance models. The corresponding modelling effort is time-consuming and expensive. Since the underlying system will often be nonlinear, regulators based on linear design may have to be retuned at different operating points. Unless perfect cancellation is possible and desired, the optimal regulator depends on the disturbance properties. Therefore, the ability to retune the regulator when new types of disturbances enter is valuable.

Adaptive feedback regulators with feedforward terms have been proposed by several investigators. Some variants are available commercially (see Section 5.1). These regulators are mostly based on minimum variance or extended minimum variance control. While they are effective in many applications, the need for improved design principles is apparent. It can be seen, for example, in a study of different adaptive feedforward control strategies by Schumann and Christ (1979). Seven methods, among them minimum variance, extended minimum variance and deadbeat feedforward control were tested. For stochastic disturbances on non-minimum phase systems, only static feedforward proved to be acceptable.

Regulators based on the polynomial LQG design of Chapter 3 are obvious candidates for improved adaptive feedforward control. Explicit process models are then identified recursively on-line.

The optimal regulators are computed from these models. In Section 5.2, an algorithm is presented in which feedback, feedforward or both can be optimized. The scalar disturbance decoupling problem is handled, and servo control is included. Methods for decreasing the computational load are discussed. The control performance will be investigated in Section 6.2.

In Section 5.3, an algorithm based on explicit criterion minimization is discussed. It can be seen as an alternative way of computing polynomial LQG regulators recursively. Instead of spectral factorization and polynomial equations, a recursive stochastic minimization method is used for optimizing the regulator. One reason for considering this method is the possibility that it might be more robust than controller calculation based exclusively on an identified model. Another reason is its ability to optimize low order regulators for high order systems. The control performance is studied by simulation in Section 6.1.

For comparison, a simple extended minimum variance self-tuner is discussed in Section 5.1. It is shown that auxiliary outputs affected by the input can be handled without special problems by this alorithm. Exact disturbance decoupling can be attained by the simplest of all self-tuning feedforward regulators: The minimum variance self-tuner with a feedforward term.

Adaptive feedback controllers based on LQG design and on explicit criterion minimization have been investigated previously by several authors. The main novelty in this chapter is that these regulators are enabled to make optimal use of auxiliary feedforward signals.

The reader is assumed to be familiar with the subject of recursive identification. A good reference is Ljung and Söderström (1983).

All the adaptive methods are based on the assumption that the main output y(t) is measureable. Thus, they are not applicable to inferential control problems.

5.1 SELF-TUNING REGULATORS BASED ON RECEDING HORIZON CRITERIA

Adaptive regulators designed to minimize receding horizon criteria have received a lot of interest. See for example Åström and Wittenmark (1985) and Clarke and Gawthrop (1975),(1979). It was already pointed out in Åström and Wittenmark (1973), that their main output feedback is easily complemented with a feedforward term, measuring w(t). Implementation is simple: A k step ahead predictor is identified recursively using Recursive Least Squares (RLS). Only trivial operations are needed to compute the regulator parameters. In many variants, the regulator parameter vector θ_r (containing polynomial coefficients) is identified directly.

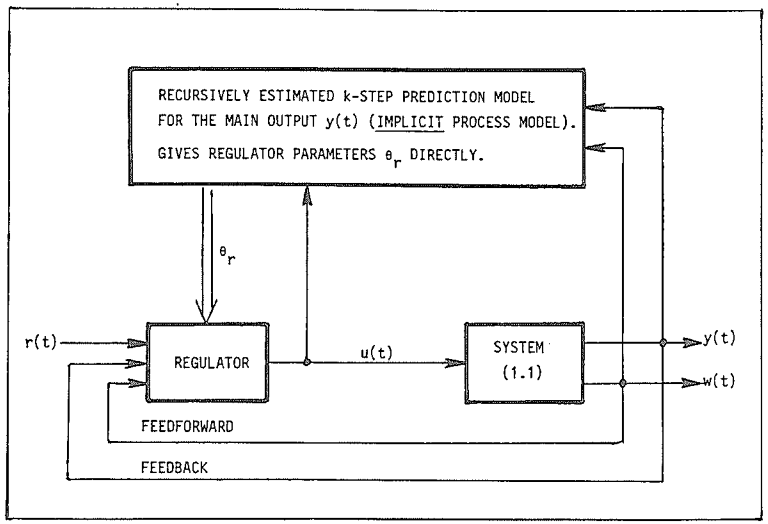


Figure 5.1 Self-tuning regulators in implicit form optimize a predictor model recursively, using input and output measurements. Predictor model parameters are used directly in the controller.

Self-tuning regulators employing feedforward, for example Novatune/ Masterpiece 170T from ASEA are available commercially, cf Bengtsson and Egardt (1984). Several successful applications have been reported, for example by Allidina et al (1981). It is of interest to compare the feedforward performance of this regulator type to the performance of adaptive LQG controllers. It is also of interest to investigate if the presence of an input influence on the measurable disturbance (as in (3.5)) causes any difficulties.

A simple extended minimum variance algorithm (Clarke and Gawthrop 1975) has been tested on discrete time systems defined by (3.4), (3.5).

The sliding k-step horizon criterion

$$J_k = \frac{1}{2} E[(y(t+k)-y_r(t))^2+\rho(\tilde{\Delta}(q^{-1})u(t))^2] \qquad (5.1)$$

is minimized, with $y_r(t)$ being a reference input. The simplest form of the algorithm is obtained by assuming C=1 and $d \geq k$. A straightforward way to derive a k-step ahead output predictor for the system

$$Ay(t) = q^{-k}Bu(t)+q^{-d}Dw(t)+e(t) \qquad (5.2)$$

is then to introduce a monic polynomial F of degree k-1 and a polynomial S (of degree na-1) which solve

$$1 = AF+q^{-k}S \qquad (5.3)$$

The use of (5.2) and (5.3) give

$$y(t+k) = AFy(t+k)+Sy(t) = BFu(t)+DFw(t+k-d)+Fe(t+k)+Sy(t) \qquad (5.4)$$

The term Fe(t+k) does not depend on any signals known at time t. By predicting it with its mean value zero, the k step predictor which is optimal in a mean square sense is obtained

$$\hat{y}(t+k|t) = BFu(t)+DFw(t+k-d)+Sy(t) \qquad (5.5)$$

As explained in Clarke and Gawthrop (1975), the criterion can be differentiated with respect to the control signal u(t) which is to be determined. The minimum of (5.1) is obtained when

$$[\hat{y}(t+k|t)-y_r(t)]b_o+\rho\tilde{\Delta}u(t) = 0$$

where b_o is the leading coefficient of B (the instantaneous gain). The corresponding control action is

$$(BF+\frac{\rho}{b_o}\tilde{\Delta})u(t) = -Sy(t)-DFw(t+k-d)+y_r(t) \qquad (5.6)$$

The regulator (5.6) has been implemented adaptively in the following way.

<u>Algorithm 5.1 (Clarke)</u>

Tuning parameters: ρ, $\tilde{\Delta}(q^{-1})$, the prediction horizon k and the orders of the prediction model polynomials.

1. Measure y(t) and w(t).

2. Update the polynomials BF, $Q \triangleq q^{-d+k}DF$ and S of the prediction model (5.5) using the recursive least squares method with parameter vector

$$\theta_r^T = (s_o,\ldots,s_{ns}, BF_1,\ldots,BF_{nr}, Q_o,\ldots,Q_{nQ})$$

regression vector

$$\varphi(t-k)^T = (y(t-k),\ldots,y(t-k-ns), u(t-k-1),\ldots,u(t-k-nr), w(t-k),\ldots,w(t-k-nQ))$$

and prediction error

$$\varepsilon(t) = y(t)-\hat{y}(t|t-k) = y(t)-\theta_r^T(t)\varphi(t-k)-u(t-k)$$

3. Calculate $R \overset{\Delta}{=} BF+\rho\tilde{\Delta}$.

4. Compute the control action u(t) from

$$Ru(t) = -Sy(t)-Qw(t)+y_r(t)$$

5. Shift the data vectors, and go to step 1.

■

The leading coefficient of BF has been fixed to 1, to avoid an over-determined prediction model. UD-factorization has been used in the least squares algorithm. If d-k is known by the user, the first d-k coefficients of Q may be set to zero.

To avoid convergence problems with this algorithm, it is important that the instantaneous gain of the true system (which is assumed to be 1) is positive, and that the time delay k is not underestimated.

The use of a differential input penalty is not sufficient for attaining integration. When differentiated signals are used in the prediction model

$$\hat{y}(t+k|t) = BF(\Delta u(t))+DF(\Delta w(t+k-d))+Sy(t) \quad (5.7)$$

a variant of the regulator with integration

$$(BF+\frac{\rho}{b_o}\tilde{\Delta})(\Delta u(t)) = -Sy(t)-DF(\Delta w(t+k-d))+y_r(t)$$
$$u(t) = u(t-1)+\Delta u(t) \quad (5.8)$$

is however easily obtained.

Nothing has been said up to now about the properties of the disturbance w(t). Feedfoward signals may be used in an opportunistic way in this adaptive regulator (as in the other regulators introduced in Sections 5.2 and 5.3): If auxiliary measurement signals are present, they are included in the regression vector and the regulator. If they do improve the output prediction, they will also improve the control performance. If not, the corresponding feedforward polynomial Q will converge to zero. This ability to experiment with different measurement signals is extremely valuable.

The properties of the controlled system will be derived. The predictor (5.5) is assumed to have converged to an optimum, i.e. (5.3) is satisfied. The signal w(t) is described by (3.5)

$$Hw(t) = q^{-n}Nu(t)+Gv(t)$$

Let $\lambda \overset{\Delta}{=} \rho/b_o$, and recall the definition (3.79)

$$q^{-b}\tilde{B} = q^{-k}BH+q^{-d-n}DN$$

Note that b=k in this case since $d \geq k$ is assumed. The use of (5.2), (5.3), (5.6) and (3.5) in straightforward but somewhat tedious calculations result in the following expressions defining the closed loop system

$$(\tilde{B}+\lambda\tilde{\Delta}AH)y(t) = q^{-d}\lambda\tilde{\Delta}DGv(t)+(F\tilde{B}+H\lambda\tilde{\Delta})e(t)+q^{-k}\tilde{B}y_r(t) \quad (5.9)$$

$$(\tilde{B}+\lambda\tilde{\Delta}AH)w(t) = G(B+\lambda\tilde{\Delta}A)v(t)-q^{-n}NSe(t)+q^{-n}NAy_r(t) \quad (5.10)$$

$$(\tilde{B}+\lambda\tilde{\Delta}AH)u(t) = -q^{-d+k}DGv(t)-SHe(t)+AHy_r(t) \quad (5.11)$$

From (5.9)-(5.11), it is seen that N≠0 presents no special problems. The characteristic polynomial is $\tilde{B}+\lambda\tilde{\Delta}AH$. When N=0, it reduces to $(B+\lambda\tilde{\Delta}A)H$. The input penalty can be seen as a root locus parameter.

When $\rho=0$ (i.e. $\lambda=0$), we have the minimum variance regulator of Åström-Wittenmark (1973). For minimum phase systems ($\tilde{B}$ stable) with adequate disturbance time delay ($d \geq k$), control with $\lambda=0$ achieves complete decoupling of the disturbance v(t) from the output y(t) at the sampling instants. Similar results can be obtained for multiple input systems using a multivariable version of the regulator, for example Johansson (1983). Thus, the design of regulators achieving exact disturbance decoupling is surprisingly simple. If the problem is solvable at all, the very simplest self-tuning feedforward regulator will mostly solve it. Cases when exact cancellation is possible in systems with unstable inverses (cf Figure 3.5 in Section 3.3) are exceptions. The minimum variance self-tuner then becomes unstable. Explicit criterion minimization or LQG control can handle these cases as well. (The use of a unity forgetting factor in the estimation algorithm is assumed whenever perfect cancellation is discussed. The loss of excitation because of the cancellation would otherwise result in estimator windup.) It is evident from (5.9) that when differential input penalty ($\tilde{\Delta}=1-q^{-1}$) is used, step disturbances and nonzero mean values in v(t) are cancelled asymptotically. In addition, the static gain from y_r to y is unity. To handle step disturbances in e(t), the integrating regulator (5.8) is required.

While the properties just described are attractive, there are some major complications with the algorithm 5.1.

- The prediction horizon k must not be lower than the time delay of the true system. This complicates the design of regulators for systems with time-varying delays.

- The characteristic polynomial $\tilde{B}+\lambda\tilde{\Delta}AH$ may be unstable, especially when $\tilde{B}$ is unstable. This is in marked contrast to the LQG design of Chapter 3, where the controlled system is stable for all ρ if the model is correct. The use of a sufficiently large input penalty, increase of the sampling period or extension of the prediction horizon often leads to stable control of non-minimum phase systems, cf Åström and Wittenmark (1985). These modifications do however fail in some cases. Optimization of sliding multi-step criteria has also been investigated (Mosca et al, 1984, Ydstie 1984, Peterka 1984). As the number of steps goes to infinity, LQG control is approached, but the algorithms become increasingly complicated. With short prediction horizons, stability cannot be guaranteed.

- A constant offset in y(t) introduces a bias in the estimation of the predictor (5.5). This problem is avoided by using the differential form (5.7). Differentiation of the regressors does however emphazise the high-frequency properties of the predictor. This may cause problems if short sampling periods are used.

- As has been pointed out by Modén and Söderström (1982), control performance (measured by the stationary input and output variances) may be considerably worse than the optimal one, achieved by LQG regulators.

- Finally, the predictor structure (5.5) is not correct for systems where $C\neq 1$ (when $\lambda>0$) and/or $d<k$. If $n+d\geq k$, i.e. $b=k$, the optimal k-step predictor can be shown to be

$$\hat{y}(t+k|t) = \frac{S}{C}\, y(t)+ \frac{\tilde{B}R_1}{CH}\, u(t)+ \frac{Q}{CG}\, w_1(t) \qquad (5.11)$$

where S, R_1 and Q satisfy (3.68) and (3.69) with $B_u=1$, and $w_1(t)$ is defined in (3.78). By selecting u(t) such that $\hat{y}(t+k|t)=y_r(t)$ the corresponding minimum variance regulator for minimum phase systems is obtained

$$\tilde{B}R_1 u(t) = -SHy(t)- \frac{QH}{G}\, w_1(t)+CHy_r(t) \qquad (5.12)$$

(This regulator can also be derived by applying Theorem 3.9 on Corollary 3.8 when $B_u=1$.) Note the factor G and the signal $w_1(t)$

in (5.12), which would have to be obtained from an explicit model of the signal w(t). Simulations have shown that the simpler algorithm 5.1 behaves reasonably well in examples where d<k. (The system no 7 in Chapter 6, also discussed in Example 4.2, is a case where this is true.) A variant of the algorithm designed to cope with time-varying k and d, and d<k, has been presented by Tahmassebi et al (1985).

5.2 ADAPTIVE LQG REGUALTORS

This section presents adaptive LQG regulators in polynomial form, based on the results of Chapter 3. Explicit algorithms based on the certainty equivalence principle are considered. At each time step, models of y(t) and w(t) are updated recursively. Assuming these models to be correct, regulator polynomials are computed from a spectral factorization and one or several linear polynomial equations. The regulator may be redesigned at each sample, or with larger intervals.

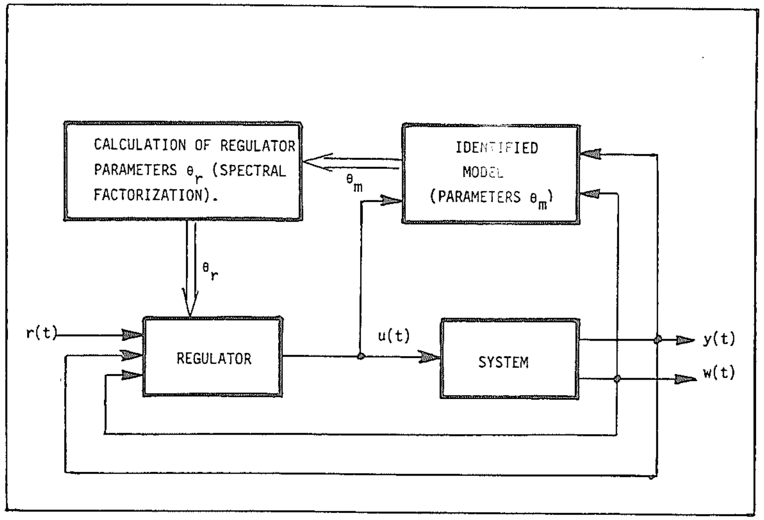


Figure 5.2 Adaptive LQG regulators are difficult to write in an implicit form, as in Figure 5.1. Algorithms based on explicit models are straightforward: With the polynomial approach, model polynomials are used in a spectral factorization and in linear systems of equations to calculate θ_r.

LQG self-tuners using state space methods, based on an iterative solution of the Riccati equation, have been proposed by different investigators. See Åström (1974), Bartolini et al (1982) and

Clarke et al (1985). The polynomial approach to adaptive LQG feedback control has been investigated by Åström and Zhao-Ying (1982), Grimble (1984), Peterka (1984) and Hunt et al (1986). Peterka (1984) has added a feedforward signal to the control law, using the disturbance model structure $H(q^{-1})w(t)=v(t)$. A drawback with the method in Peterka (1984) is that the least squares method is used to estimate a model

$$A'y = q^{-k}Bu'+q^{-d}Dw'+\varepsilon \quad ; \; Hw' = v$$

where A', u' and w' may be normal or differentiated polynomials/signals, cf (3.89). If the disturbances do not fit into this description (C, N or G-polynomials are present in a system with the structure (5.4),(5.5)), the adaptive predictor converges to a biased model. Good control cannot be guaranteed. A remarkable property of minimum variance regulators based on LS-models is that optimal control of systems with coloured noise can be achieved, (Åström and Wittenmark 1973). This result does, however, not hold for more general infinite horizon criteria (Casalino et al, 1987). A straightforward way to avoid these kinds of problems is to use the Extended Least Squares (ELS) or the Recursive Prediction Error Method (RPEM). These methods are capable of delivering consistent estimates of systems of type (3.4) and (3.5).

In the following, delays, i.e leading coefficients which are zero, are included in the model polynomials.

SUMMARY OF THE ALGORITHM

Based on recursive prediciton error identification and Theorems 3.1, 3.6 and 3.9 of Chapter 3, an adaptive regulator has been developed. It is designed to minimize the criterion (3.6):

$$J = \lim_{N\to\infty} \frac{1}{2N} \sum_{t=0}^{N} Ey(t)^2+\rho E(\tilde{\Delta}(q^{-1})u(t-k))^2$$

In this criterion, y(t) and u(t-k) denote signals resulting from regulator action. The servo action has been subtracted away. As special cases, the follwing adaptive regulators are included:

- Optimal feedback control, without measurable disturbances

- Optimal feedback and feedforward

- Optimal feedforward without feedback

- Optimal feedforward with a (non-adaptive) prespecified feedback

A servo filter is also designed.

Algorithm 5.2 (LQG)

Initial parameters:

- Polynomial degrees in the model structure (3.4),(3.5), including leading coefficients: na,nb,nd,nc,nh,nn,ng

- A known prespecified feedback $R(q^{-1}),S(q^{-1})$, if present.

- The input penalty ρ and the polynomial $\tilde{\Delta}(q^{-1})$.

- A servo reference model $B_m(q^{-1})/A_m(q^{-1})$.

- A flag determining if feedback, feedforward or both are to be optimized.

1. The signals y(t) and w(t) are measured.

2. Models of y(t) and w(t) with the structure

$$Ay = Bu+Dw+C\varepsilon_y \; ; \;\; Hw = Nu+G\varepsilon_w$$

are updated, using two RPEM routines for single output systems. Parameters of the C, H and G-polynomials are projected into stable regions. See Appendix A6 for details.

3. If nn>0 (an input influence on w(t) is suspected), the substitutions of Theorem 3.9 are performed. The signal $\hat{w}_1(t)$ from (3.78) is computed.

4. The spectral factor β and r from (3.24) is calculated, using the algorithm of Kučera (1979).

5. If required, an optimal feedback R, S (and X) is calculated from (3.43). These polynomial equations give an over-determined linear system of equations, with an exact solution. The system is solved using the least squares method.

6. If required, an optimal feedforward filter P, Q is calculated.

   (i) $P=G\beta$ and (3.28) are used if a feedback is absent or prespecified and constant.

   (ii) P=G and (3.52) are used if the feedback has been optimized in step 5.

   Equations (3.28),(3.52) are solved as a linear system of equations. (Cf Example 4.2, Section 4.1.)

7. A servo filter T/E is designed to cancel poles and zeros, so that the servo response corresponds to the reference model for minimum phase systems. From (3.79), (3.80), (3.83)

$$q^{-b}\frac{\tilde{B}T}{\tilde{\alpha}E} = q^{-b}\frac{B_m}{A_m}$$

   (if N=0, then $\tilde{B}=B$ and $\tilde{\alpha}=\alpha=AR+q^{-k}BS$)

   (i) Test the stability of $\tilde{B}$.

   (ii) If $\tilde{B}$ is unstable, then $\frac{T}{E} = \frac{\tilde{\alpha}B_m/B(1)}{A_m}$, else $\frac{T}{E} = \frac{\tilde{\alpha}B_m}{\tilde{B}A_m}$.

8. Compute the control action u(t) from (3.78)

$$Ru(t) = -\frac{Q}{P}\hat{w}_1(t)-Sy(t)+\frac{T}{E}r(t)$$

9. Shift the data vectors and go to step 1.

A variant of the algorithm has also been developed where adaptive pole placement feedback (α prespecified) is combined with optimal feedforward control.

The integrating variant of the regulator, proposed in Section 3.5, has also been implemented. Servo filter optimization for non-minimum phase system, described in Section 3.5, has not been tested.

STABILITY AND CONVERGENCE

Consider the model structure defined by (3.4),(3.5), in the case of nn=0, and assume that convergence has occured to a time-invariant, but not necessarily correct, model

$$Ay = Bu+Dw+C\varepsilon_y$$

$$Hw = G\varepsilon_w$$

If only the feedforward filter $Q/G\beta$ is optimized for a stable system, stability is assured, since β is stable by construction and stability of G is monitored.

If both feedback {R,S} and feedforward {Q,P} are adapted, good estimates of A and B are needed to assure stability. Errors in the model polynomials D, C, H and G will affect the control performance, but they cannot cause instability. (Since stability of C is monitored, pole placement in βC results in a stable system, if the model polynomials A and B are estimated correctly.)

For explicit LQG feedback algorithms based on state space methods, global convergence has been proved by, for example, Moore (1984) and Chen and Guo (1986). ELS estimation, a positive real condition on $C - \frac{1}{2}$, a time invariant system, and known upper bounds on na, nb and nc have been assumed. Chen and Guo (1986) add a small disturbance tending to zero to the input signal. It assures identifiability in closed loop (Gustavsson et al, 1977) and consistent parameter estimation without affecting the infinite horizon criterion value. With similar assumptions, Grimble has recently proved global convergence of the polynomial LQG feedback regulator based on Theorem 3.4 (Hunt et al, 1986, Grimble 1986b).

REDUCING THE COMPUTATION TIME

An algorithm based on two recursive identification routines, one spectral factorization and one or two linear systems of equations requires a significant amount of computation. This has been the main argument against the use of LQG methods in adaptive systems. For large classes of applications, recent technical developments are rapidly decreasing the relevance of this argument. As discussed in Section 6.3, application to audio signal processing problems seem possible with the latest generation of signal processors. Still, methods to decrease the computational load are an important area of investigation.

Spectral factorization problems can be solved iteratively using the algorithms of Kučera (1979) or Ježek and Kučera (1985). A few steps can be taken at each sampling interval, using the result from the last sample as an initial condition. The solution obtained at each iteration is guaranteed to be stable.

Efficient routines for solving polynomial equations have been derived by Kučera (1979) and Ježek (1982). The computational load increases as n^2, where n is the number of unknown polynomial coefficients in the algorithm of Jezek. Solution of the polynomial equations can be distributed over several sampling intervals. This will, however, delay the attainment of optimal control after a system parameter change. Several alternative methods are possible:

1. If a real time operating system is used, the algorithm can be divided in a natural way into two tasks. Task 1 (the foreground program) handles identification and the control signal calculation. It is started at the sampling instant. Task 2, the calculation of new controller parameters, uses slices of several sampling intervals, and delivers a new regulator perhaps every 10'th sampling interval. (This regulator will then be based on the model of 10 samples ago.) A variant of this approach is mentioned in Chapter 7.

2. Polynomial equations can be formulated as systems of simultaneous equations

$$Ax = B \tag{5.18}$$

where the mxn matrix A and the m-vector B contain known coefficients. (Equation (3.43) becomes a system with $m=n+n\beta$, but with an exact solution.) The system (5.18) can be solved using recursive least squares: At each time t, update A and B. Let

$\varphi(t)$ = the j'th row of A

$y(t)$ = the j'th element of B

where $j=(t \bmod m)+1$

Then, update the parameter vector x of the model

$$\varphi(t)x-y(t) = 0$$

by taking one step with the recursive least squares algorithm. If A and B are constant, the solution is obtained after n steps, if rank A=n. The method has been tested with a forgetting factor of 0.98. It works satisfactory when model parameters change slowly. The x-vector will, however, be significantly more "noisy", compared with complete solution of (5.18) at each sample. When large model parameter changes occur, the feedback control may be unstable for a short period after the changes. These problems may possibly be reduced if A and B are updated when j=1, and then held constant for m samples.

3. Wellstead et al (1979) propose to solve the Diophantine equation

$$A(q^{-1})X(q^{-1})+B(q^{-1})Y(q^{-1}) = D(q^{-1}) \tag{5.19}$$

with respect to X and Y by generating a white noise n(t) internally in the computer and calculating the signals $s_1(t)=A(q^{-1})n(t)$, $s_2(t)=B(q^{-1})n(t)$ and $s_3(t)=D(q^{-1})n(t)$.

By multiplying (5.19) with n(t), it is transformed to

$$X(q^{-1})s_1(t)+Y(q^{-1})s_2(t)-s_3(t) = 0 \tag{5.20}$$

Equation (5.20) can be solved with the RLS method, taking one step each sampling instant. Note that no noise is introduced into the estimation since s_1, s_2 and s_3 are known signals. As with method 2, the exact solution is provided after a finite number of steps if A, B and D are time-invariant.

The recursive methods 2 and 3 above introduce a measure of uncertainty into the calculations. They work well if model parameters change slowly, but may behave in an unacceptable way for large variations. For this reason, only method 1 (slow but exact controller calculation) can be recommended.

If most or all of the sampling interval is needed for regulator calculations, the control signal should be changed immediately before the next sampling of y and of w. This increases the total system delay by one sample. Thus, k+1 and n+1 should be used instead of k and n in all calculations.

5.3 EXPLICIT CRITERION MINIMIZATION

This method has been developed by Trulsson (1983), see also Trulsson and Ljung (1985), based on earlier ideas by Tsypkin (1973). Motivated by the problem of finding optimal controllers of restricted complexity, Goodwin and Ramdage (1979) developed the approach independently. Assume a criterion J and a regulator of fixed structure to be given. Let θ_r be a vector of regulator parameters. The main idea is to seek a local minimum (where the gradient $dJ/d\theta_r=0$) by using a stochastic minimization algorithm.

In our problem, the method is applied to a linear system with a quadratic criterion, and the minimum is attained by a LQG-optimal regulator if e(t) and v(t) are white and stationary. The general idea does, however, have much wider applications, for example nonlinear regulators, nonquadratic criteria and static optimization problems. This is discussed by Trulsson (1983).

ADAPTIVE CONTROL BASED ON EXPLICIT CRITERION MINIMIZATION: AN OUTLINE

Consider the criterion (3.6). Assume ergodic signals, and include a reference $y_r(t)$, which is not a function of the regulator parameters θ_r. The criterion is then given by

$$J = \frac{1}{2} E((y(t)-y_r(t))^2+\rho(\tilde{\Delta}(q^{-1})u(t-k))^2) \tag{5.21}$$

Differentiation with respect to θ_r gives

$$\frac{dJ(\theta_r)}{d\theta_r} = E\left(\frac{dy(t,\theta_r)}{d\theta_r}(y(t,\theta_r)-y_r(t))+\rho\frac{d\tilde{\Delta}u(t-k,\theta_r)}{d\theta_r}\tilde{\Delta}u(t-k,\theta_r)\right) \tag{5.22}$$

Note that for stochastic systems this gradient, being an expected value, cannot be observed directly. It can, however, be estimated recursively.

In (5.22), $y(t,\theta_r)$ and $u(t-k,\theta_r)$ denote the signals that would result from the closed loop system if the regulator parameters were held constant. The *signal derivative* vectors $dy(t,\theta_r)/d\theta_r$ and $d\tilde{\Delta}u(t-k,\theta_r)/d\theta_r$ are given by filtered measureable signals. The filters will, however, depend on the true system, which is unknown. As an approximation, parameters from a recursively identified model are used. This seems intuitively reasonable, as long as the model describes the true system well, and model (and system) parameters change much more slowly than the signals. The direct minimization of a criterion, using the sensitivity function (5.22), resembles the well-known MIT rule for deterministic model reference adaptive control. An important difference is that identification of an explicit process model is used in the present algorithm, which is summarized below

Algorithm 5.3 (Critmin)

Repeat the following steps each sampling period:

1. Update a recursively estimated model with parameter vector θ_m.

2. Compute approximate signal derivative filters, using $\theta_m(t)$ and the regulator parameter vector $\theta_r(t-1)$.

3. Update the regulator parameters $\theta_r(t)$ towards a local minimum where $dJ/d\theta_r=0$. Steps 2 and 3 are explained in more detail in the following.

4. Compute the control signal $u(t)$, using $\theta_r(t)$.

∎

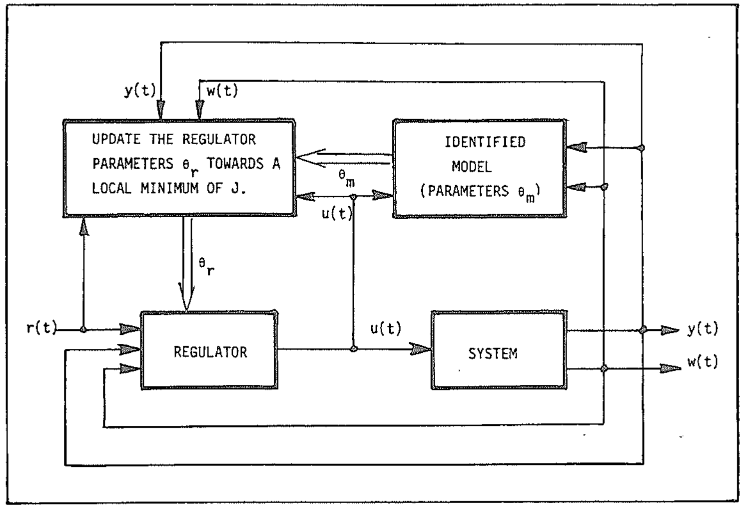


Figure 5.3 Combined feedback-feedforward control using explicit criterion minimization. Compared with the LQG regulator structure in Figure 5.2, calculation of θ_r has been replaced by a recursive stochastic minimization routine.

The algorithm 5.3 consists mainly of two coupled recursions: One for updating the model parameters θ_m and one for optimizing θ_r. An interesting property is that signals from the system affect the regulator calculation directly. This might improve robustness, compared to the LQG algorithm 5.2. Another interesting property is that the regulator can have any order. Using a known high order model the method can, for example, be used off-line to optimize a low order regulator.

CONVERGENCE

Since regulator parameters are continuously adjusted on-line, only approximations of the signals in (5.22) will be observed. These approximations, and the use of parameters from an identified model, raise questions about the possible convergence of the algorithm. In Trulsson (1983), local convergence is proven under the following main assumptions:

- All signals stay bounded.
- Disturbance and reference inputs can be described by zero mean stochastic processes with rational spectral density.
- The parts of θ_m used in the signal derivative filters converge to their true values w.p.1.

Then, under mild conditions on J and on the updating method for θ_r, the algorithm 5.3 will converge to a local minimum of J w.p.1.

Assume furthermore

- A linear system and an infinite horizon quadratic criterion, such as (5.21), to be given.
- A linear regulator with the optimal number of parameters. This implies the existence of a unique global minimum point $J(\theta_r)$ and the nonexistence of other local minima.

Then, θ_r will converge to this unique minimum, which coincides with the LQG solution. It should be noticed that non-minimum phase systems present no special problems. In Trulsson (1983), convergence of θ_m, and consequently of the whole scheme, is proven assuming feedback control, the use of Instrumental Variable identification, possible injection of extra disturbances to assure identifiability and the use of a model with correct polynomial orders.

It is of course of interest to investigate the behaviour of algorithm 5.3 if some of the rather restrictive conditions above not are satisfied. This will be illustrated by some simulation experiments in Section 6.1.

The algorithm will now be adapted to our problem structure. What is new, compared to Trulsson (1983), is the use of a feedforward signal, and the successful use of a stochastic Newton method for minimizing criteria with an input penalty.

THE REGULATOR

A system with the structure (3.4),(3.5) is assumed. As with the LQG regulator described in Section 5.2, the regulator structure (3.78) is used. The servo behaviour is, however, included in the optimization. Thus, the regulator is given by

$$Ru(t) = -\frac{Q}{G}w_1(t)-Sy(t)+Ty_r(t) \tag{5.23}$$

where

$$w_1(t) = w(t)-\frac{N}{H}u(t) \tag{5.24}$$

and the reference is shaped by a low pass filter

$$y_r(t) = \frac{E(1)}{E(q^{-1})}r(t) \tag{5.25}$$

Note that P=G, and that G, H and N are model polynomials, with leading zero coefficients included in N. Model identification is performed in the same way as in LQG control, with two RPEM routines. See Appendix A6 for a description.
The R, Q, S and T-polynomials are to be optimized, with a parameter vector defined by

$$\theta_r \triangleq (r_1,\dots,r_{nr},\ s_0,\dots,s_{ns},\ Q_{DelQ},\dots,Q_{nQ},\ T_0,\dots,T_{nt})^T \tag{5.26}$$

If a time delay is known to be needed in the feedforward path, the dimension of the parameter vector can be reduced by setting a number (DelQ in (5.26)) of leading $Q(q^{-1})$-coefficients to zero. In most cases, DelQ=0. When nQ=0, (5.23) reduces to the feedback regulator studied by Trulsson and Ljung (1985). Stanković and Radenković (1984) have presented a simulation study where it was found to behave well.

Explicit criterion minimization has also been tested with the regulator

$$u(t) = -\frac{Q}{P} w_1(t) + Ty_r(t) \tag{5.27}$$

which uses feedforward only. It was found to be considerably less reliable than (5.23). Additional discussions of this regulator can be found in Sternad (1986b).

UPDATING THE REGULATOR PARAMETERS

The criterion (5.21) is to be minimized with respect to θ_r, given by (5.26). Let us rewrite (5.21) in the following way:

$$J = \frac{1}{2} E(y(t)-y_r(t), \tilde{\Delta}u(t-k)) \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} y(t)-y_r(t) \\ \tilde{\Delta}u(t-k) \end{pmatrix}$$

$$\triangleq \frac{1}{2} E\varepsilon(t)^T \Lambda^{-1} \varepsilon(t) \tag{5.28}$$

The criterion (5.28) has the same algebraic structure as the type of criteria minimized in recursive prediction error identification for multiple output systems. Using this analogy, an identification algorithm from Ljung and Söderström (1983) is used. It updates θ_r in an approximate Gauss-Newton direction. Compared to stochastic approximation algorithms, which update θ_r in the gradient direction, this improves the speed of convergence.

Define the $\dim\theta_r \times 2$ matrix

$$\psi(t,\theta_r) \triangleq -\frac{d[\varepsilon(t,\theta_r)^T]}{d\theta_r} = \left(-\frac{dy(t,\theta_r)}{d\theta_r} \, , \, -\frac{d\tilde{\Delta}u(t-k,\theta_r)}{d\theta_r} \right) \tag{5.29}$$

The elements of (5.29) (the signal derivatives) are given by

$$\frac{\partial y(t)}{\partial r_\ell} = -\frac{q^{-b}\tilde{B}}{\tilde{\alpha}} u(t-\ell) \quad ; \quad \frac{\partial \tilde{\Delta}u(t-k)}{\partial r_\ell} = -\frac{AH\tilde{\Delta}}{\tilde{\alpha}} u(t-k-\ell)$$

$$\frac{\partial y(t)}{\partial s_m} = -\frac{q^{-b}\tilde{B}}{\tilde{\alpha}} y(t-m) \quad ; \quad \frac{\partial \tilde{\Delta}u(t-k)}{\partial s_m} = -\frac{AH\tilde{\Delta}}{\tilde{\alpha}} y(t-k-m)$$

$$\frac{\partial y(t)}{\partial Q_j} = -\frac{q^{-b}\tilde{B}}{\tilde{\alpha}G} w_1(t-j) \quad ; \quad \frac{\partial \tilde{\Delta}u(t-k)}{\partial Q_j} = -\frac{AH\tilde{\Delta}}{\tilde{\alpha}G} w_1(t-k-j)$$

$$\frac{\partial y(t)}{\partial T_\tau} = +\frac{q^{-b}\tilde{B}}{\tilde{\alpha}E} r(t-\tau) \quad ; \quad \frac{\partial \tilde{\Delta}u(t-k)}{\partial T_\tau} = +\frac{AH\tilde{\Delta}}{\tilde{\alpha}E} r(t-k-\tau)$$

$$\tag{5.30}$$

where $\ell=1,\ldots,nr$, $m=0,\ldots,ns$, $j=DelQ,\ldots,nQ$, $\tau=0,\ldots,nt$, and $\tilde{B}$, $\tilde{\alpha}$ are defined by (3.79) and (3.80).

These signals are obtained by using P=G and the substitutions of Theorem 3.9 ($q^{-k}B \to q^{-b}\tilde{B}$, $\alpha \to \tilde{\alpha}$, $A \to AH$) on the expressions (A1.6)-(A1.8), (A1.10), (A1.13) and (A1.15) in Appendix A1. The derivatives with respect to $T(q^{-1})$ are evident from (3.83) and (3.84).

The algorithm for updating θ_r can be expressed as

$$S(t) = \lambda(t)\Lambda+\psi(t)^T P(t-1)\psi(t) \quad ; \quad S(0) = \lambda(0)\Lambda \qquad (5.31a)$$

$$P(t) = \frac{1}{\lambda(t)} [P(t-1)-P(t-1)\psi(t)S(t)^{-1}\psi(t)^T P(t-1)] \qquad (5.31b)$$

$$\theta_r(t) = \theta_r(t-1)+P(t-1)\psi(t)S(t)^{-1}\varepsilon(t) \quad ; \quad \theta_r(0) = 0 \qquad (5.31c)$$

Note that the matrix $P(t-1)\psi(t)S(t)^{-1}$ in (5.31c) is already computed in (5.31b). When $\rho=0$, a scalar version of the algorithm is used.

This algorithm should be implemented in factorized form. In (5.31), $S(t)$ is a 2x2 matrix, while $P(t)$ is the $\dim\theta_r \times \dim\theta_r$ covariance matrix of the estimates $\theta_r(t)$. $\lambda(t)$ is the forgetting factor of the regulator parameter optimization. It is initialized at 0.90-0.95 and goes to a final value between 0.98 and 1 exponentially. A value below 1 is needed to track time-varying systems. The value 0.98 mostly gives the best behaviour. Updating of the regulator is discontinued if the standard deviation of the control error decreases below a preset bound, or if the trace of P increases above a bound. Regulator updating is resumed (with P reset to I·100) if the control error standard deviation increases a factor of 2, compared with the value when updating was discontinued. In addition, the regulator optimization must be deactivated when the input attains a bound. The filters in (5.30) must be stable. As Trulsson (1983) has pointed out, it is important to monitor the stability of $\tilde{\alpha}(q^{-1})$ both before the filterings (5.30) are performed and when a new regulator update has been made. Regulator updates that would lead to unstable $\tilde{\alpha}$ are rejected and the old value of θ_r is retained. (The regulator is frozen.) A more detailed description of this "safety net" can be found in Sternad (1986b). The signal vectors are filled in the first 20 samples, which are used for open loop identification. This is a simple and effective way of improving the initial transient behaviour of θ_r.

From simulations, the use of an input delay $k>0$ in (5.21), (5.30) and $\varepsilon(t)$ in (5.31c) has been found to be important. With $k=0$, the algorithm diverges, when large input penalties are used. If $k\geq 1$, no problems occur. Under-estimation of the delay has created no problems.

6. PERFORMANCE OF THE ADAPTIVE REGULATORS

Different aspects of the control performance will be discussed in this chapter and illustrated by simulated examples. In Section 6.1, adaptive control based on explicit criterion minimization is studied, and compared to extended minimum variance control. This section is a somewhat modified and shortened version of Sternad (1986b). The study is the most extensive investigation presented so far on the behaviour of explicit criterion minimization for linear systems. In Section 6.2 the LQG algorithm 6.2 is considered. Attention is focused on the robustness of this method, compared to explicit criterion minimization. How different methods to reduce the computational load can affect the control performance is also illustrated. The relative merits of the three algorithms are compared in Section 6.3.

Approximately 25 different discrete time models have been used in simulations. The behaviour of the algorithms will be exemplified by experiments on twelve models, presented below. Since simulation studies never can be made exhaustive, the conclusions must be regarded as tentative. Still, it is believed that the main properties, advantages and weaknesses of the adaptive regulators proposed in Chapter 5 are illustrated rather well on the following pages.

THE SYSTEM MODELS USED

<u>System 1</u>

$$(1-q^{-1}+0.25q^{-2})y(t) = (1-0.2q^{-1})u(t-1)+2v(t-1)$$

$$(1-q^{-1}+0.25q^{-2})w(t) = u(t-1)+(1+0.5q^{-1})v(t)$$

The disturbance $v(t)$ can be cancelled perfectly by a regulator (5.27) with $Q=2-2q^{-1}+0.5q^{-2}$ and $P=1+0.3q^{-1}-0.1q^{-2}$.
The system may be expressed in the form (3.4):

$$(1-0.5q^{-1}-0.25q^{-2}+0.125q^{-3})y(t) =$$
$$= q^{-1}(1-1.7q^{-1}-0.1q^{-2})u(t) + q^{-1}(2-2q^{-1}+0.5q^{-2})w(t)$$

System 2

$$(1-0.5q^{-1})y(t) = q^{-1}(0.5+1.25q^{-1}+0.5q^{-2})u(t)+q^{-2}(2-1.5q^{-1})w(t) +(2-1.5q^{-1})e(t)$$

$$(1-0.9q^{-1})w(t) = (1-0.3q^{-1})v(t)$$

If $v(t)$ is white noise, the minimum variance feedforward regulator for this non-minimum phase system, calculated from Corollary 3.8, achieves $\sigma y(t)=0.76\sigma v(t)$ when $e(t)=0$. It is given by (3.55) with

$$R(q^{-1}) = 1 +0.85q^{-1}+0.175q^{-2}$$
$$S(q^{-1}) = -0.2$$
$$Q(q^{-1}) = 1.31 -0.78q^{-1}-0.36q^{-2}+0.16q^{-3}$$

System 3

$$(1-0.95q^{-1})y(t) = q^{-2}(1+2q^{-1})u(t)+q^{-3}w(t)+(1+0.7q^{-1})e(t)$$

System 4

$$(1-0.8q^{-1})y(t) = q^{-2}(1+0.8q^{-1})u(t)+q^{-3}w(t)+e(t)$$

$$(1-0.9q^{-1})w(t) = 2q^{-1}u(t)+(1-0.3q^{-1})v(t)$$

This system, with $w(t)$ affected by $u(t-1)$, is non-minimum phase, since $\tilde{B}=1-0.1q^{-1}+1.28q^{-2}$ has zeros in $-0.05\pm1.13i$.

System 5

$$(1-q^{-1}+0.41q^{-2})y(t) = q^{-1}(1-0.95q^{-1})u(t)+(1+0.7q^{-1})e(t)$$

System 6

$$(1-0.8q^{-1})y(t) = q^{-1}(1-0.1q^{-1}-0.29q^{-2}+0.549q^{-3})u(t) +q^{-1}(1+0.5q^{-1})w(t)+e(t)$$

The system is minimum phase. Perfect feedforward is possible.

System 7

$$(1-0.5q^{-1})y(t) = q^{-2}(1+0.1q^{-1})u(t)+q^{-1}(1-2q^{-1})w(t)+e(t)$$

$$(1-0.9q^{-1})w(t) =(1-0.3q^{-1})v(t)$$

Perfect feedforward is not possible because of the time delays (k=2, d=1). See also Example 4.2, Section 4.1.

System 8

$$(1-1.6q^{-1}+0.75q^{-2})y(t) = q^{-1}(1+0.9q^{-1}+0.95q^{-2})u(t) +(1+1.5q^{-1}+0.75q^{-2})e(t)$$

This example is used in Trulsson (1983). The combination of B-polynomial and non-positive real C-polynomial may lead to failure of convergence if the Extended Least Squares method is used for system identification.

System 9

$$(1-2q^{-1}+1.5q^{-2})y(t) = q^{-1}(1+2q^{-1}+2q^{-2})u(t)+q^{-2}(1+0.5q^{-1})w(t)+e(t)$$

An unstable non-minimum phase system with poles in $+1\pm0.71i$ and zeros in $-1\pm i$.

System 10

$$A(q^{-1})y(t) = q^{-1}B(q^{-1})u(t)+q^{-1}D(q^{-1})w(t)+C(q^{-1})e(t)$$

with

$$A(q^{-1}) = 1-1.113q^{-1}-0.642q^{-2}+0.757q^{-3}$$
$$B(q^{-1}) = 0.061-0.0032q^{-1}-0.057q^{-2}$$
$$D(q^{-1}) = -0.063+0.017q^{-1}+0.060q^{-2}$$
$$C(q^{-1}) = 1-0.317q^{-1}-0.726q^{-2}+0.101q^{-3}$$

This is a model of the bottom temperature in a glass furnace, discussed in Wertz (1986). Rather fast sampling is used, compared with the dynamics of the process (poles in 0.978, 0.95 and -0.815, zeros in 0.993 and -0.941). As reported by Wertz (1986), a GMV self-tuner based on an explicit process model has been installed to control this process. From the model, a static feedforward D(1)/B(1) is computed. It improves the control behaviour.

System 11

$$y(t) = -u(t-1)-u(t-2)-,\dots,-u(t-7)+w(t-7)+e(t)$$

A moving average process with all zeros on the unit circle. This is a reasonable model for the final temperature or the output moisture content in crossflow grain-driers. w(t) is then a temperature or moisture measurement of the incoming grain, used for feedforward. It is discussed in Nybrant (1986)

System 12

$$(1-0.6q^{-1}+0.73q^{-2})y(t) = q^{-1}(1+2q^{-1})u(t)+q^{-1}w(t)$$

$$(1-0.9q^{-1})w(t) = q^{-1}(-1.1+2.44q^{-1})u(t)+(1+0.8q^{-1})v(t)$$

Perfect decoupling of the disturbance v(t) is possible since $\tilde{B}=1+0.64q^{-2}$ is stable and k=d.

Some properties of the system are summarized in the table below.

System no	Perfect disturbance decoupling possible	Non-minimum phase	Unstable	Auxiliary output w affected by u	The system is discussed in
1	X			X	Fig 6.1, ex 1
2		X			Fig 6.2, ex 4,5 8
3		X			Fig 6.3, ex 2,7,9
4		X		X	Fig 6.4
5					
6	X				ex 6,10
7					ex 13
8					ex 12
9		X	X		ex 11
10	X				
11		(X)			
12	X			X	ex 3

In the examples, the reference $y_r(t)$ will either be zero or a square wave r(t) filtered by $0.7/(1-0.3q^{-1})$.

SOME NOTATIONS USED

na, nb, np etc: Polynomial degrees, including leading zeros. (Example: If $q^{-k}B=q^{-2}(1-2q^{-1})$, then nb=3.)

σy, σu, σv etc: Standard deviation of signals.

σy_T: Standard deviation of y(t), measured from time T to time 1000.

σy/σv: Normalized standard deviation of y(t).

ρ: Input penalty

ρ_Δ: Differential input penalty.($\tilde{\Delta}(q^{-1})=1-q^{-1}$)

6.1 EXPLICIT CRITERION MINIMIZATION

6.1.1 HOW AN INPUT PENALTY AFFECTS THE REGULATOR PERFORMANCE

The achievable tradeoff between control performance and input magnitude is illustrated for four typical cases below. In all cases, v(t) is white noise and $y_r(t)=0$. The output standard deviation is plotted as a function of the input standard deviation for different input penalties for the following regulators:

LQG: Time-invariant LQG-control, calculated for a known system.

FF: The adaptive feedforward regulator (5.27).

FF+FB: The adaptive combined feedback-feedforward regulator (5.23).

EMV: An extended minimum variance self-tuner, algorithm 5.1.

For all regulators, correct model orders and optimal regulator orders have been used. σy_{500} and σu_{500} were measured. For comparison, the performance of the following regulators were also calculated.

MV: Minimum variance self-tuner with extended prediction horizon k. Suggested in Åström and Wittenmark (1985) as a method to handle non-minimum phase systems.

FB: Minimum variance feedback regulators. $Ru(t)=-Sy(t)$.

SFF: Optimal static feedforward $u(t)=-Q_0w(t)$, where the gain Q_0 is calculated with criterion minimization.

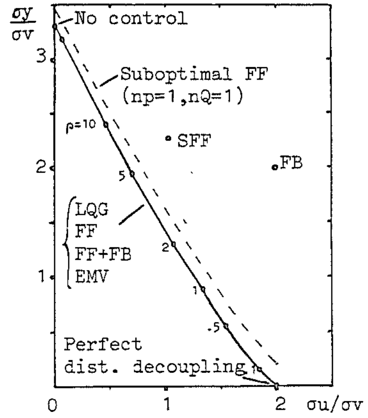


Fig 6.1 System 1. Perfect disturbance decoupling (σy=0) is possible

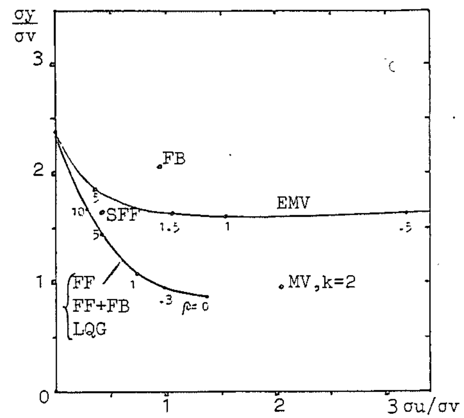


Fig 6.2 System 2 with e(t)=0. Non-minimum phase system.

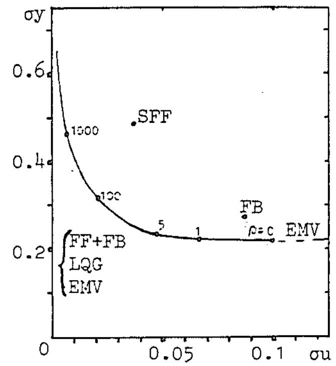


Fig 6.3 System 3. Non-minimum phase system with an additional unmesurable disturbance e(t). σe=0.1, σv=0.1

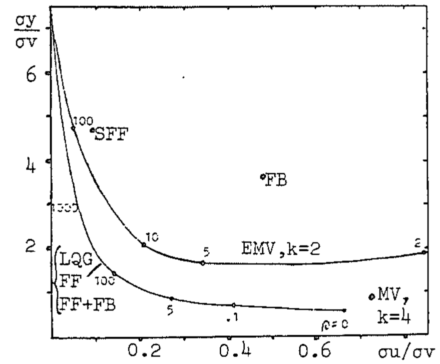


Fig 6.4 The nonminimum phase system 4.

Adaptive control based on explicit criterion minimization, FF(+FB), behaves as expected: It achieves the LQG-optimal solution.

Minimum variance feedforward control (the right end-point of the LQG curves) is often "expensive" in terms of input energy. By increasing ρ, σu can often be decreased markedly with only a slight increase in σy.

The extended minimum variance self-tuner with feedforward is unstable for non-minimum phase systems when $\rho=0$. By increasing the input penalty (thus decreasing the regulator gain) it can be stabilized for stable systems. Sometimes LQG-optimal control performance is achieved (fig 6.1 and 6.3), sometimes not (fig 6.2 and 6.4).

The simulations have confirmed that self-tuning minimum variance controllers with extended prediction horizon often achieve good performance for non-minimum phase systems. They are far from reliable, however. For the system 4, use of k=3 resulted in instability, while k=4 gave a good result.

Feedforward/disturbance decoupling regulators may improve control performance drastically, compared with minimum variance feedback regulators (FB). As is evident from fig 6.2 and 6.4, this holds also for non-minimum phase systems. In fig 6.3, the improvement is rather small, since the non-measurable disturbance e(t) dominates.

In fig 6.1, an under-parametrized adaptive regulator of type (5.27) achieves almost the optimal performance. Many other such examples have been found.

Figures 6.1-6.4 illustrate the asymptotic behaviour of adaptive control based on explicit criterion minimization. The transient behaviour is illustrated by the following two examples.

EXAMPLE 1: DISTURBANCE DECOUPLING

The regulator (5.23) is used on *system 1*. A square wave is used as r(t). The disturbance v(t) is white noise with $\sigma v=0.1$. A

correct model parametrization and optimal regulator parametrization with nr=2, ns=0, nQ=2 and nt=1 is used. The convergence of some model parameters, starting from $\theta_m=0$, is shown in Figure 6.5.

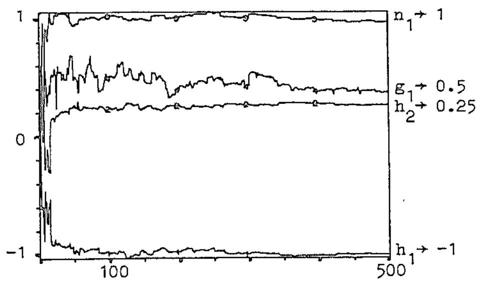


Fig 6.5a Parameters of the w(t)-model.

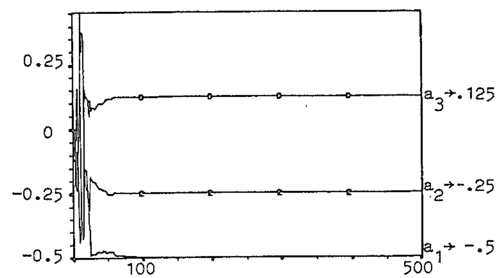


Fig 6.5b Estimates of the A-polynomial coefficients.

The estimated w(t)-model parameters of fig 6.5a are used directly in the regulator. The other regulator coefficients are adapted by criterion minimization and converge more slowly. See fig 6.6.

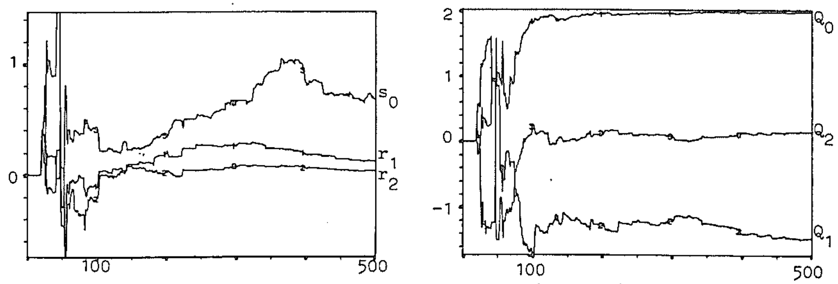


Fig. 6.6 Convergence of $R(q^{-1})=1+r_1q^{-1}+r_2q^{-2}$, $S(q^{-1})=s_o$ and $Q(q^{-1})=Q_o+Q_1q^{-1}+Q_2q^{-1}$. Exact disturbance decoupling is achieved when $R(q^{-1})=1+0.3q^{-1}-0.1q^{-1}$ and $Q(q^{-1})=2-2q^{-1}-0.1q^{-2}$. The value of s_o will then no longer affect the control performance, as long as stability is assured. The drift of s_o is caused by this overparametrization.

Parameters to which the disturbance rejection is sensitive, such as Q_0, settle down quickly. This is why the control performance, shown in fig 6.7 is good already from t=100. At t=400, virtually complete disturbance decoupling has been achieved.

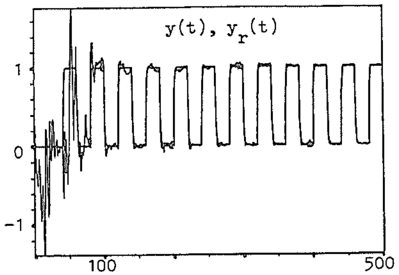


Fig 6.7 Disturbance decoupling of the main output y(t).

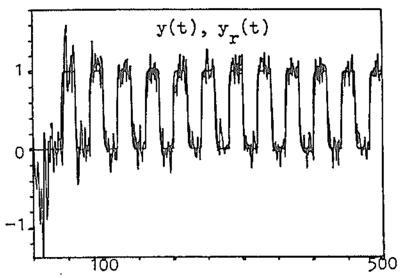


Fig 6.8 Control performance with an input penalty $\rho_\Delta=0.5$.

Fig 6.8 shows the effect of using a small differential input penalty: The speed of convergence is increased, but a considerable part of the disturbance is no longer cancelled. ∎

EXAMPLE 2: OPTIMAL FEEDFORWARD CONTROL OF A NON-MINIMUM PHASE SYSTEM

The regulator (5.23) is used on *system 3*. This is a non-minimum phase system affected both by the measurable disturbance w(t) and unmeasurable disturbances e(t). In this example, w(t) and e(t) are both white noises with $\sigma v=0.1$ and $\sigma e=0.1$. No reference signal is used. For white disturbances, the minimum variance regulator $Ru(t)=-Qw(t)-Sy(t)$ is given by

$$R(q^{-1}) = 1+2.15q^{-1}+1.62q^{-2}$$

$$S(q^{-1}) = 0.77$$

$$Q(q^{-1}) = 0.254+0.67q^{-1}+0.81q^{-2}$$

It achieves the output standard deviation $\sigma y=0.222$.

Figures 6.9 and 6.10 show the convergence of model and regulator parameters when $\rho=0$.

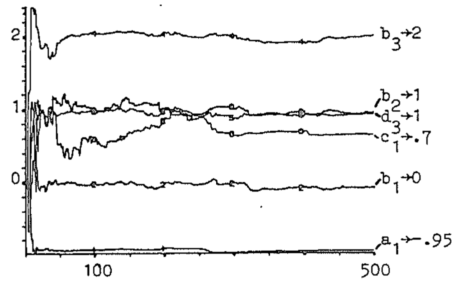


Fig 6.9 Model parameters for system 3.

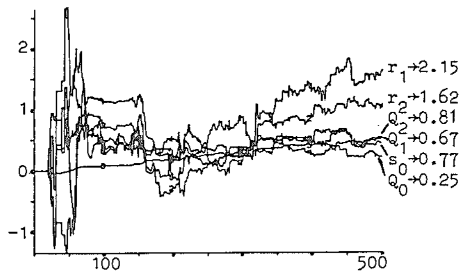


Fig 6.10 Minimum variance regulator parameters.

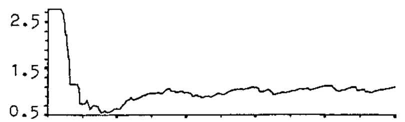


Fig 6.11 Log(trace(P))

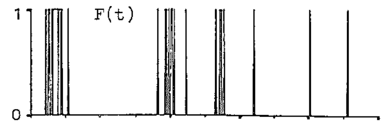


Fig 6.12 Indicator of frozen regulator updates.

Figure 6.11 illustrates the P-matrix behaviour of the regulator recursion (5.30). A final forgetting factor value λ_∞ of 0.98 is used in this and the other examples. After t=200, the magnitude of P (and thus of the parameter updating gain in (5.31c) is approximately constant. Whenever a new regulator would result in an unstable closed model, the update is rejected and the old stabilizing regulator is retained. F(t)=1 in fig 6.12 indicates when this happened. The frequency of F(t)=1 in this example is rather typical for the simulations performed so far.

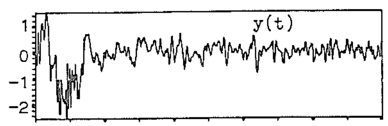


Fig 6.13 Output with minimum variance control. σy_{500}=0.224.

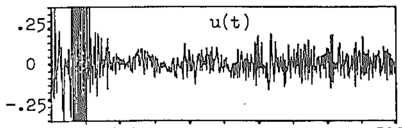


Fig 6.14 Input with minimum variance control. σu_{500}=0.089.

Figures 6.13 and 6.14 show the control performance with ρ=0. In fig 6.3 it is seen that an input penalty may decrease the input variation markedly without any appreciable increase of the output standard deviation. (This was not possible in

example 1, cf fig 6.1). Figures 6.15-6.16 show the control performance with input penalty ρ=5.

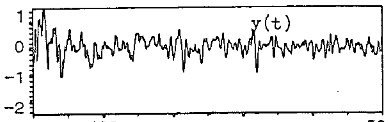


Fig 6.15 Output when ρ=5 is used. σy_{500}=0.230.
Optimal value:
σy = 0.224

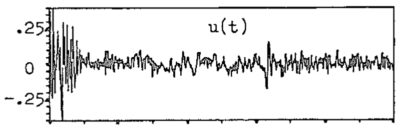


Fig 6.16 Input when ρ=5 is used. σu_{500}=0.049.
Optimal value:
σu = 0.046

As in example 1, use of an input penalty improves the transient performance of the regulator. It moderates the initial excursion of y(t) and u(t). ∎

6.1.2 COMPARISION WITH EXTENDED MINIMUM VARIANCE SELF-TUNERS

In general, extended minimum variance self-tuners converge faster than regulators optimized with explicit criterion minimization. This is especially noticable in examples where exact disturbance decoupling is possible.

EXAMPLE 3: PERFECT DISTURBANCE DECOUPLING POSSIBLE

Adaptive criterion minimization and minimum variance self-tuning control are compared on *system 12*, where v(t) is an integrated noise with σ(v(t)-v(t-1))=0.1. In both regulators, nr=2, ns=0 and nQ=2 are used.

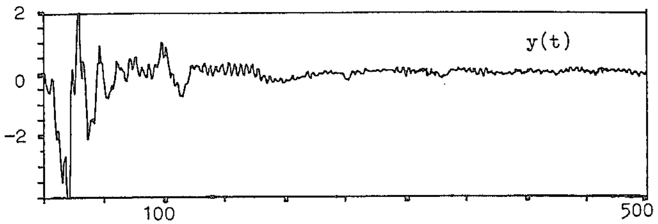


Fig 6.17 Control by explicit criterion minimization, without input penalty.

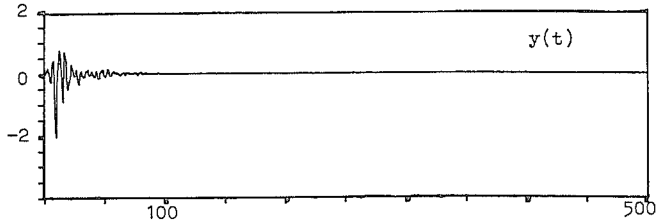


Fig 6.18 Control performance with a minimum variance self-turner. Exact disturbance decoupling is attained at t=100.

Of all simulation examples tested so far, this was the one where criterion minimization performed worst, compared with EMV.

∎

If maximum disturbance rejection is desired and the system is known to be minimum phase, there is really no need to use a more complicated regulator than the original self-tuning controller of Åström and Wittenmark (1973). It will perform better than explicit criterion minimization, as in Example 3.

On the other hand, criterion minimization is often superior when input penalties have to be used, or when systems are non-minimum phase. System no 2 is such a case.

EXAMPLE 4: A NON-MINIMUM PHASE SYSTEM

Adaptive criterion minimization and extended minimum variance self-tuning control are compared for the non-minimum phase *system no 2*. v(t) is white noise with σv=0.1. Polynomial degrees nr=2, ns=0, and nQ=3 are used by both regulators.

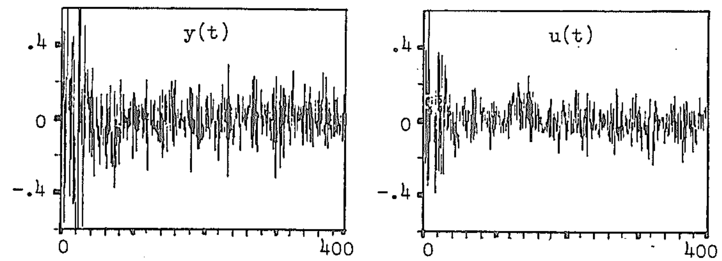


Fig 6.19 Control of system 2 with explicit criterion minimization. ρ=1. σy_{500}=0.108, σu_{500}=0.073.

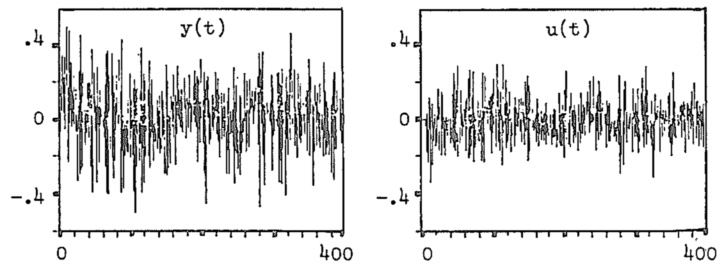


Fig 6.20 Control of system 2 with an extended minimum variance self-tuner using ρ=1.5. σy_{500}=0.163, σu_{500}=0.106.

Both the input and the output standard deviations are decreased when criterion minimization is used instead of EMV. Also compare with fig 6.2.

∎

Simulation experiments comparing EMV and explicit criterion minimization (Critmin) are summarized in table 6.1. For each of the systems 1-12, a somewhat subjective judgement about the relative merit of the two methods is given. In all examples, EMV and Critmin use Q, R and S-polynomials with optimal degrees. Critmin uses models with correct parametrization.

Table 6.1 Summary of simulation experiments comparing EMV with explicit criterion minimization.

System no	Critmin best	Same performance	EMV best	Comment
1			X	Perfect disturbance decoupling is achieved much faster with the minimum variance self-tuner.
2	X			Cf fig 6.2, 6.19 and 6.20.
3		X		Identical asymptotic performance, cf fig 6.3. Similar transient behaviour.
4	X			Cf fig 6.4.
5		X		Identical performance. Both converge quickly.
6	X			A penalty on Δu was used. Critmin has much better asymptotic performance. EMV converge somewhat faster.
7		X		Identical performance.
8	X			EMV has clearly worse asymptotic performance, and its input signal often contains "bursts".
9	X			Unstable non-minimum phase system. No stabilizing EMV regulator was found.
10		X		EMV converges somewhat faster. Critmin has somewhat better asymptotic performance.
11		X		EMV converges somewhat faster. Critmin has somewhat better asymptotic performance.
12			X	Cf fig 6.17 and 6.18

While there are cases such as Example 3, where EMV performs better, Critmin clearly has an edge: Cases when the use of an infinite horizon criterion gives superior performance compared to the use of a k-step ahead criterion are quite common. Other advantages are good output behaviour in continuous time and no need to guess the plant time delay correctly. These advantages may well motivate the use of criterion minimization, despite of its significantly higher complexity and somewhat worse transient performance.

6.1.3. ROBUSTNESS: UNDER-PARAMETRIZATION

FULL ORDER MODELS AND UNDER-PARAMETRIZED REGULATORS

System identification has worked reliably with correctly parametrized models. In the transient phase, the signal derivatives are incorrect because of model errors. As these errors diminish, the mean updating direction of the regulator parameters becomes nearly correct, and a local minimum of (5.21) is soon attained. Regulators with low (suboptimal) polynomial degrees have caused no special problems. This suggests another use of the explicit criterion minimization approach: On-line or off-line optimization of parameters in low order regulators. For example, given a complex high-order plant model, the parameters of a three-term PID-controller may be optimized. Such problems are hard to solve analytically. For the systems presented in the introduction, it was investigated how much performance deteriorated if regulator polynomial degrees were suboptimal. Almost LQG-optimal control behaviour could be achieved in most examples when polynomial degrees were decreased by 1 from their optimal values. Such an experiment for system 1, with the regulator (5.27), was presented in Figure 6.1.

UNDER-PARAMETRIZED MODELS (AND REGULATORS)

Under-parametrization will result in model coefficient errors in the signal derivative filters. No theoretical results are known on the magnitude of acceptable errors. As long as the angle between the true Newton direction and the average updating direction is small, steps will be taken in a "downhill" direction and the method converges. If the angle is large, the method either diverges or gets stuck at a θ_r which is not even a local minimum. In the simulation experiments performed so far, the method has performed well with moderately under-parametrized models. Severe under-parametrization, preventing the models from reflecting important dynamical aspects of the true system, may lead to divergence.

The algorithm normally fails to converge if the model structure has any of the following (rather extreme) deficiencies:

- When the degree of $q^{-k}\hat{B}(q^{-1})$ is so low that non-minimum phase zeros cannot be modelled. (Example: k=2, $B=1-2q^{-1}$ in the system, nb=2 in the model.)

- When the degree of $q^{-k}\hat{B}(q^{-1})$ is so low that the true time delay cannot be modelled. (Example: k=2 in the system, while nb=1 in the model.)

- When the degree of $\hat{A}(q^{-1})$ is so low that the dominating system time-constant cannot be modelled.

- When the auxiliary output w(t) is affected by the input, while $\hat{N}(q^{-1})=0$ in the model, and feedforward from w(t) is used.

The examples below illustrate the behaviour of the algorithm when models are under-parametrized.

EXAMPLE 5: EXPLICIT CRITERION MINIMIZATION COMPARED WITH LQG CONTROL BASED ON A MODEL OF REDUCED ORDER

Control of *system 2* is attempted with the following model and regulator degrees:

Model	Regulator
na = 1	ns = 0
nb = 2 instead of 3	nr = 1 instead of 2
nd = 2 instead of 3	nQ = 1 instead of 3
nc = 0 instead of 1	nt = 1
ng = 0 instead of 1	
nh = 0 instead of 1	

This is an under-parametrized model of least squares type (nc=0). No model of w(t) is identified. The regulator degrees are optimal for the given model order. The disturbances v and e are white with $\sigma v=0.1$ and $\sigma e=0.02$. A square wave is used as r(t). $\rho=0$. At time t=1000 the model has converged to:

$$(1-0.54q^{-1}) = q^{-1}(0.65+1.42q^{-1})u(t)+q^{-1}(-0.29+1.036q^{-1})w(t)+e(t) \qquad (6.1)$$

Though all parameters are wrong, the model (6.1) approximates the main dynamic aspects of the system: The static gain 4.5, the pole at +0.5 and the non-minimum phase zero at -2. The adaptive algorithm has no problem finding an optimal restricted complexity regulator. Fig 6.21 and 6.22 show the output with full parametrization and under-parametrization, respectively. After convergence, the control error standard deviation $\sigma(y-y_r)$ is practically the same in the two cases.

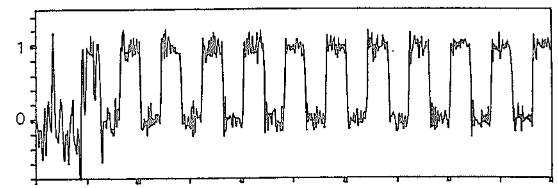


Fig 6.21 Criterion minimization control performance for system 2, using a full order model and regulator (ns=0, nr=2, nQ=3), ρ=0. $\sigma(y-y_r)_{500}$=0.113.

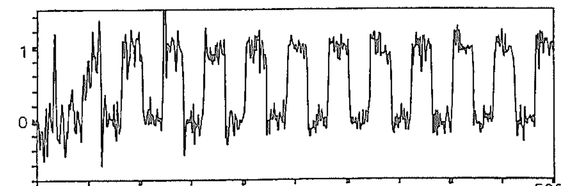


Fig 6.22 Criterion minimization control performance for system 2, with under-parametrized model and regulator (ns=0, nr=1, nQ=1), ρ=0. $\sigma(y-y_r)_{500}$=0.123.

It is instructive to compare this performance with LQG optimal control, based on the model (6.1).

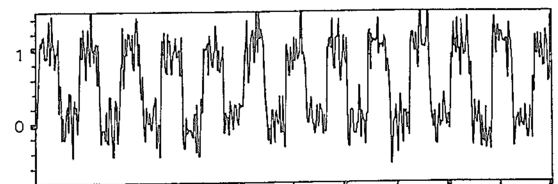


Fig 6.23 A time-invariant LQG optimal regulator, based on the low-order model (6.1) is used on the true system. Regulator orders ns=0, nr=1, nQ=1 and input penalty ρ=0 are used. $\sigma(y-y_r)_{500}$ = 0.260

That a regulator is optimal for a reduced order model does not imply optimal (or even good) control of the true system. As will be evident in Sections 6.2 and 6.3, there does not, however, exist a clear case for the use of explicit criterion minimization instead of adaptive LQG control.

∎

EXAMPLE 6:

Adaptive criterion minimization with an under-parametrized model (na=1, nb=2, nd=1) and a corresponding under-parametrized regulator (ns=0, nr=1, nQ=0) is tested on *system 6*, where w(t) is white noise with variance 1. The control performance as a function of the input penalty is shown in fig 6.24, and compared to full order control. The difference is considerable for ρ=0 (where perfect feedforward is achieved with a full order regulator) but decreases with increasing ρ. The accumulated loss function for ρ=1 is compared in fig 6.25.

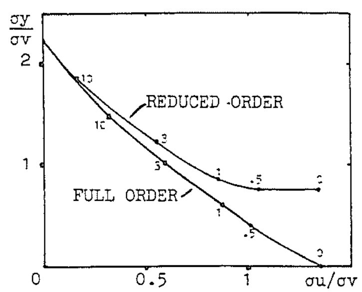


Fig 6.24 Control performance on system 6 as a function of ρ for full order and reduced order regulators.

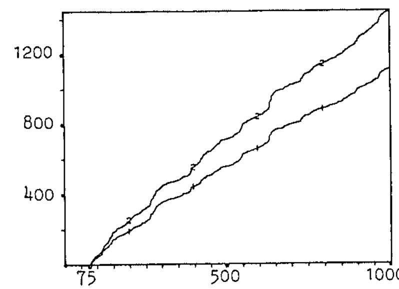


Fig 6.25 The accumulated loss function for ρ=1 $\sum_{75}^{t} y(t)^2+u(t)^2$ for full order (1) and reduced order (2) control.

6.1.4 ROBUSTNESS: OVERPARAMETRIZATION

Monitoring of the matrices P(t) of the RPEM routines is needed to assure good behaviour for over-parametrized models and regulators: When forgetting factors<1 are used, model identification and regulator optimization sometimes has to be deactivated temporarily, as described in Section 5.2. In over-parameterized regulators (5.23), R and S may contain common factors. These factors are assured to be stable, since stability of $\tilde{\alpha}$ is monitored. (Factors common to R and S will be factors of $\tilde{\alpha}=AHR+q^{-b}\tilde{B}S$.)

EXAMPLE 7: OVER-PARAMETRIZED MODEL

The *system 3* is controlled by adaptive criterion minimization with the following features:

- An over-parametrized model (na=2, nb=4, nd=4, nc=2) is used.
- w(t) is white noise, but since we do not know this a priori, a model (3.5) for w(t) with nn=1, nh=1 and ng=1 is estimated.
- A correspondingly over-parametrized regulator, with ns=1, nr=3 and nQ=4 is used.

As in example 2, v(t) and e(t) are white disturbances with σv=0.1, σe=0.1. An input penalty ρ=5 is used. Figures 6.26 and 6.27 show the transient control performance. Compare with the correctly parametrized control of fig 6.15 and 6.16. From t≈300, the performance of over-parametrized and correctly parametrized regulators are essentially equal.

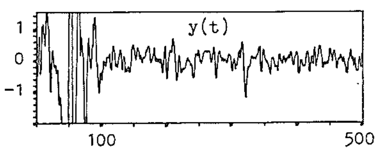


Fig 6.26 Output of system 3 with ρ=5. σy_{500}=0.253.

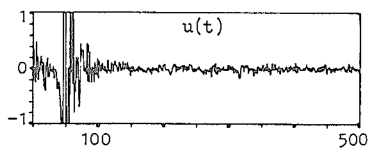


Fig 6.27 Input of system 3 with ρ=5. σy_{500}=0.042.

6.1.5 ROBUSTNESS: NONSTATIONARY DISTURBANCES, TIME-VARYING SYSTEMS AND UNCERTAIN MEASUREMENTS

The behaviour of the algorithm is best demonstrated by some examples.

EXAMPLE 8: A TIME-VARYING SYSTEM WITH A NONSTATIONARY DISTURBANCE

System 2 is disturbed by an integrated white noise v(t), with σ(v(t)-v(t-1))=0.01, and a white noise e(t), σe=0.1. The regulator is used with forgetting factors 0.98 in the identification routines, regulator degrees nr=2, ns=0, nQ=2, nt=1, and ρ=0. At t=300, $q^{-k}B(q^{-1})$ changes from $0.5q^{-1}+1.25q^{-2}+0.5q^{-3}$ to $0.5q^{-1}+0.3q^{-2}$. The system now becomes minimum phase, making perfect feedforward of the disturbance w(t) possible. From t=500, e(t)=0 so that the feedforward performance on the output can be seen clearly. Figures 6.28-6.30 show the disturbance w(t), the output and the input respectively. In fig 6.31, coefficients of the model polynomial $B(q^{-1})=b_1q^{-1}+b_2q^{-2}+b_3q^{-3}$ are compared with the true values. For large t, the system identification will eventually be turned off, since no information is gathered about the C-polynomial coefficients when e(t)=0. The corresponding P-matrix diagonal elements grow exponentially. At time 500, the regulator has compensated for the change in the system, and achieves almost perfect control.

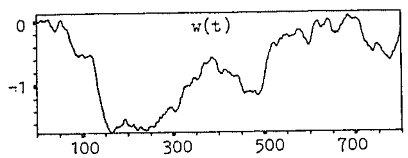


Fig 6.28 The disturbance w(t) entering system 2.

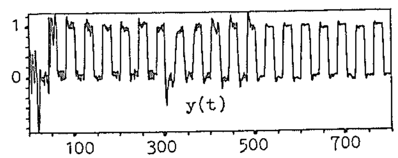


Fig 6.29 The output. At t=300, the system becomes minimum phase. From t=500, e(t)=0. Almost perfect disturbance decoupling has been achieved by then.

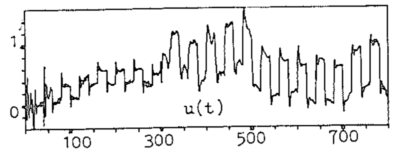


Fig 6.30 The input. Since the static gain of the system decreases at t=300, the input amplitude has to increase.

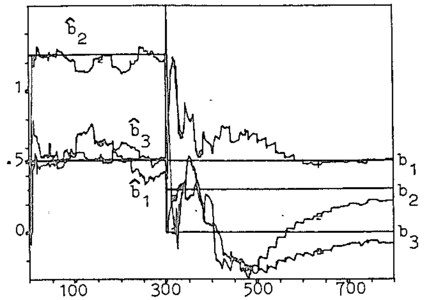


Fig 6.31 Coefficients of the model B-polynomial.

EXAMPLE 9: STEP DISTURBANCES

The disturbance w(t) acting on *system 3* in this example is a square wave with amplitude 1 added to white noise with standard deviation 0.1. It is shown in fig 6.32. A fully parametrized regulator (5.23) is used, with differential input penalty 1 and identification forgetting factor 0.98. At t=300, b_2 changes from 1 to 3. While the model coefficient, shown in fig 6.33, converges to the new value, the closed system is unstable for a short time around t=350, but quickly recovers.

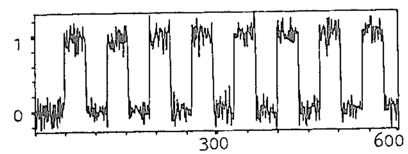


Fig 6.32 The disturbance w(t) entering system 3.

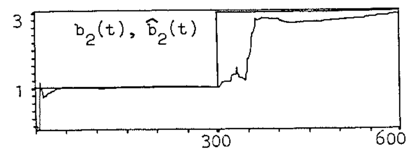


Fig 6.33 The time-varying parameter b_2 and its estimate.

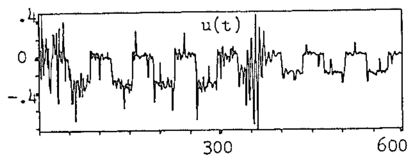


Fig 6.34a The input.

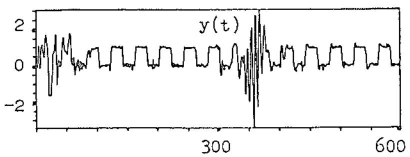


Fig 6.34b The output. r(t) is a square wave with amplitude 1 and period 40.

EXAMPLE 10: A NOISE-CORRUPTED AUXILIARY MEASUREMENT w(t)

An adaptive regulator should decrease its feedforward gain if the feedforward information is unreliable. This is demonstrated in a simulation on *System 6* below. w(t) is white noise with $\sigma w = 0.1$. $\sigma e = 0.02$. From time 300, the measurement of w(t) is corrupted by white noise with standard deviation 0.1. The feedforward parameters Q_0 and Q_1 become smaller. At time 600, the D-polynomial of the system is set to zero. The disturbance w(t) no longer affects y(t). The regulator reacts by turning off the feedforward action.

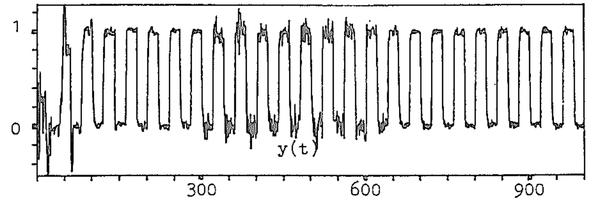


Fig 6.35 From t=150 to 300, feedforward cancels the disturbance w(t) on system 6. From t=300, the measurement of w(t) becomes corrupted by noise. This forces the regulator to decrease the feedforward gain, and allow w(t) to partially affect the output. From t=600, w(t) no longer affects the system. In this simulation, an unmeasurable disturbance e(t) with standard deviation 0.02 has also been added.

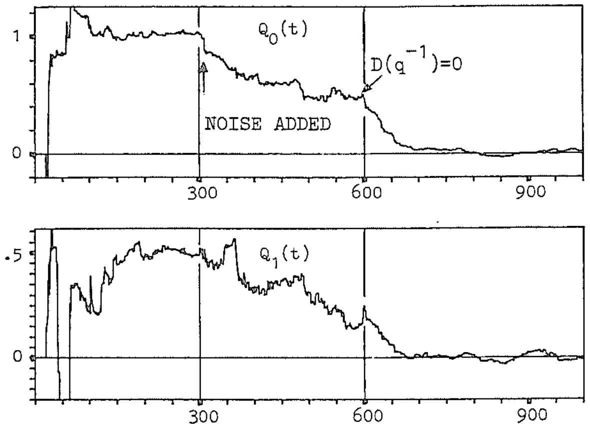


Fig 6.36 When w(t) becomes corrupted by noise, the regulator reduces the feedforward action $-Q_0w(t)$ $-Q_1w(t-1)$ towards a new optimum. When $D(q^{-1})=0$ after t=600, feedforward is deactivated, since w(t) is now useless for control of y(t).

6.2 THE ADAPTIVE LQG REGULATOR

The LQG algorithm 5.2 has been found to work well. The introductory example in Chapter 1 gives an impression of its performance. When models are parametrized correctly, the main difference, compared to explicit criterion minimization, is that the LQG regulator converges faster. This is not surprising: If the model is correct, the correct regulator is calculated immediately by the LQG algorithm, while explicit criterion minimization has to perform a recursive search.

EXAMPLE 11: CONTROL OF AN UNSTABLE NON-MINIMUM PHASE SYSTEM

The *system 9* is controlled below with the LQG and Critmin algorithms. Poles of the uncontrolled system are located in $+1\pm0.71i$ and zeros in $-1\pm i$. LQG provides very good control as soon as the model is accurate, at time 30. Explicit criterion minimization attains the same servo and regulator performance as the LQG regulator from time 400, but its behaviour prior to this time is inferior.

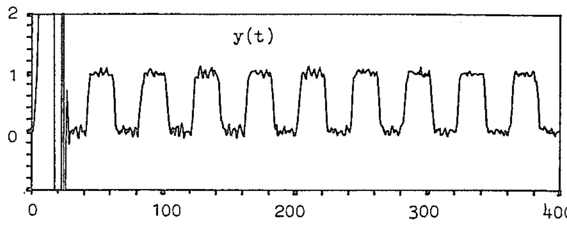


Figure 6.37 The unstable non-minimum phase system 9 controlled with the LQG algorithm 5.2. The model is parametrized correctly. $\rho=0$. $\sigma v=0.1$ and $\sigma e=0$. $r(t)$ is a square wave and the reference model is $y_r(t)=\frac{0.7}{1-0.3q^{-1}}r(t)$.

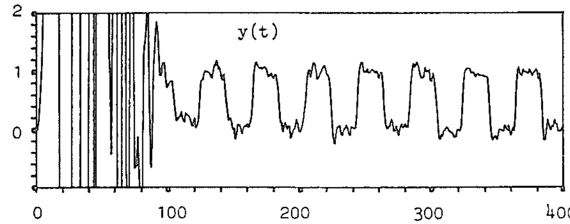


Figure 6.38 Control with explicit criterion minimization nr=2, ns=1, nQ=2, nt=1, $\rho=0$.

In most examples, criterion minimization has attained acceptable control after 70-80 samples. In some cases, however, the convergence is considerably slower.

EXAMPLE 12

When controlling *system 8*, explicit criterion minimization cannot compete with LQG control even if a known model is used in the algorithm.

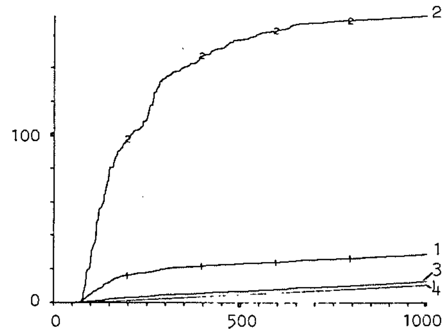


Figure 6.39 Accumulated criterion value $\sum_{75}^{t} y(t)^2$ when adaptive minimum variance feedback control is applied on the minimum phase system 8 with $\sigma e=0.1$.

1: Critmin, based on a known model.
2: Critmin, based on an identified model of correct order.
3: LQG control, based on an identified model of correct order.
4: Optimal criterion value.

It should be mentioned that minimum variance control is extremely sensitive to modelling errors in the A and B-polynomials in this example. If model parameters drift because forgetting factors less than 1 are used, infrequent short bursts of instability can occur. As usual, the use of an input penalty improves the robustness. ∎

The LQG regulator performs as it should when the auxiliary output w(t) is corrupted by measurement noise. For example, in the experiment described in Example 10, LQG behaves in the same way as Critmin: The feedforward action is decreased when w(t) becomes corrupted by noise. The mechanism leading to this behaviour is, however, different in the two algorithms. When w(t) becomes corrupted by noise, this signal becomes less valuable for predicting y(t). Consequently, the D-polynomial coefficients of the model decrease. In the LQG algorithm, this leads, via equation (3.52), to a decrease of the Q-polynomial coefficients. The D-polynomial is not used in the Critmin algorithm, cf (5.30). Instead, the regulator retuning is based on direct measurements of y(t), w(t) and u(t).

As in the case of explicit criterion minimization, adaptive LQG feedforward control becomes more robust when it is complemented with an adaptive feedback. When adaption of feedforward filters without feedback is considered, the LQG algorithm is more robust than explicit criterion minimization.

For regulators adapting both feedback and feedforward, it is hard to say which algorithm is more robust. Results from simulation experiments are ambiguous: In some cases, criterion minimization has superior robustness properties. In other cases, LQG works better. Table 6.2 below summarizes the experience from 31 tests. These tests were similar to Examples 5, 6, 8 and 9. The performance with under-parameterized models in combination with non-zero mean, drifting and deterministic disturbances was investigated.

LQG performs best	5
LQG better, both good	4
Both behave well	6
Critmin better, both good	4
Critmin performs best	5
Both algorithms failed	7

Table 6.2 *Summary of results from experiments comparing the robustness of adaptive feedback-feedforward regulators based on algorithm 5.2 and 5.3.*

For both algorithms, it is important that the model polynomial degrees na and nb are not so low that important dynamical aspects of the systems (dominating poles, delays, non-minimum phase zeros) cannot be modelled.

When an input influence on w(t) is present, it should be modelled, i.e. nn=0 cannot be used. High-pass filtering of the regressors (cf Appendix A6) should be used to prevent models from becoming biased because of non-zero mean disturbances. Most of the bad control performance in the experiments, summarized in Table 6.2, was caused by breaking one of these simple rules. With the qualifications above, both adaptive control methods display a robust behaviour.

In Section 5.2, different ways of reducing the computational load were discussed. The effect of two of these methods on the control performance is illustrated in the next example.

EXAMPLE 13: RECURSIVE CONTRA SLOW BUT EXACT SOLUTION OF POLYNOMIAL EQUATIONS

LQG feedback + feedforward control with a differential input penalty $\rho_\Delta=0.5$ is used on *system 7*. The disturbance v(t) is a square wave with period 60. Figure 6.40 shows the measurable disturbance w(t). At time 300, the static gain is halved. $B(q^{-1})$ changes from $1+0.1q^{-1}$

to $0.5+0.05q^{-1}$. Figures 6.41 and 6.42 show the result of adaptive LQG control with a forgetting factor 0.98.

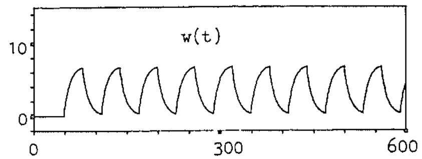


Figure 6.40 The measurable disturbance w(t).

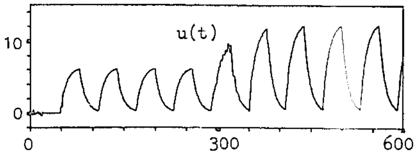


Figure 6.41 The input u(t).

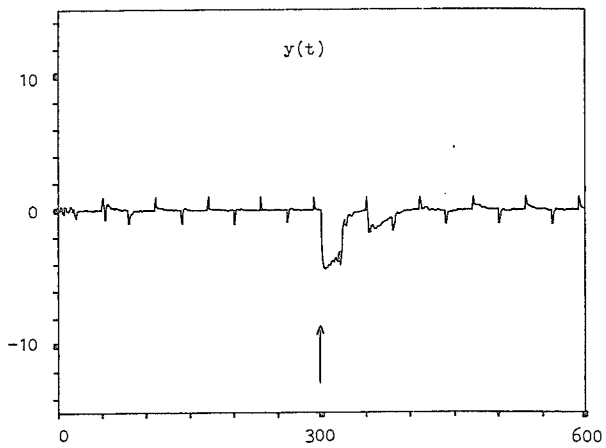


Figure 6.42 The output y(t). The disturbance can be cancelled almost completely. The time delay difference k-d=1 prevents perfect cancellation. At t=300, the system gain is halved. At t=400, the control performance has recovered.

We now consider the two first of the methods discussed in Chapter 5.2 for reducing the computational load. Their effect on the output behaviour is described by Figures 6.43 and 6.44.

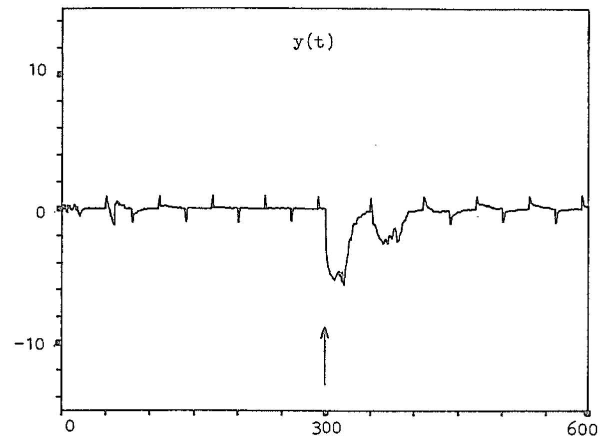


Figure 6.43 Control behaviour when the regulator is recalculated every 5'th sample, based on the model of 5 samples ago. The performance deterioration is insignificant compared to Figure 6.42 where the regulator was recomputed at each sampling instant.

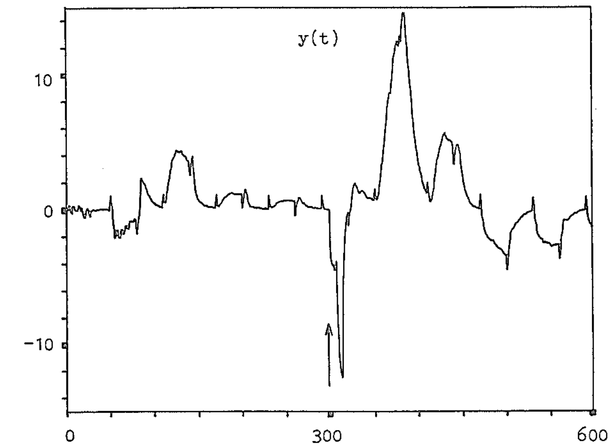


Figure 6.44 Control behaviour when the polynomial equations, written as a system (5.18) of simultaneous equations, are solved by RLS. A new step of the recursive solution is taken at each sampling instant. The regulator parameters have only just attained their correct values at time 300, when the change in the system occurs. The control performance eventually recovers after t=600.

Infrequent computation of new regulators is a safe and effective method of reducing the computational load. Recursive computation has been found to work acceptably in most simulations, but it may sometimes result in a behaviour similar to Figure 6.44. A possible way to improve the behaviour would be to use a fixed model for a whole solution pass: A, B would be updated only when j=1, and held constant in-between. The regulator would be held constant, and updated only when the LS-algorithm had reached the lowest row of A, i.e. when j=m. The x-vector should then have converged to the correct solution of Ax=B. Unless such modifications are made, the use of the recursive methods discussed in Section 5.2 cannot be recommended.

6.3 CONCLUDING DISCUSSION

The table below summarizes the relative merits of extended minimum variance self-tuners, adaptive LQG controllers and explicit criterion minimization. The conclusions are based on the discussion in Chapter 5, the simulation experiments of Chapter 6 and a significant number of additional simulation experiments.

ALGORITHM	5.1 (EMV or Clarke)	5.2 (LQG)	5.3 (Critmin)	REMARKS
	Ranking in order of performance			
Asymptotic performance (correct parametrization)	2	1	1	
Performance for non-minimum phase systems	2	1	1	
Speed of convergence	1	1	2	
Control of unstable systems	2	1	3	The larger transient of 5.3 makes it unsuitable for such problems. Bounded inputs lead to divergence.
Servo behaviour	3	2	1	5.3 shows a slightly better behaviour 5.2. Low T-polynomial orders can be used in 5.3.
Required computation time, relative to 5.1	1	4-10	10-20	New LQG regulator computed every 5'th sample
		10-40		each sample
Feedforward behaviour				
Speed of convergence to a perfect disturbance decoupling regulator	1	2	3	
Handling of feedforward signals corrupted by noise	1	1	1	All perform equally well
Handling of nonstationary and deterministic measureable disturbances	2	1	1	

Table 6.3 *The relative merits of the adaptive control strategies.*

Extended minimum variance control, algorithms 5.1, is a very simple strategy with a low computational load. It is superior to the other algorithms in achieving perfect cancellation of disturbances for minimum phase systems. A reasonable control performance can be attained for many non-minimum phase systems by using an extended prediction horizon. In general, the performance is, however, inferior to LQG control.

Adaptive LQG control, algorithm 5.2, attains the best over-all performance. There are really few reasons not to use this method. One reason, the large computational load, is commented on below.

Explicit criterion minimization, algorithm 5.3, also performs well. The speed of convergence is, however, lower than for LQG control. A main reason for considering this method was the possibility that it might be more robust with respect to under-parametrized models. This has not turned out to be the case. Critmin and LQG have about the same robustness properties. In general, they behave well, but severely under-parametrized models can lead to failure. The use of a robust identification method is important in both algorithms. Thus, there remain few strong reasons for using explicit criterion minimization instead of LQG control. One reason is, however, that the regulator order is not determined by the model order. Low order polynomials can be used in the regulator. For example, the use of first order T-polynomials has resulted in excellent servo behaviour. The servo filter calculation in the LQG algorithm 5.2 is based on cancellation of poles and zeros. It leads to almost as good behaviour as with criterion minimization, but results in high order servo filters.

An algorithm, which combines the best properties of LQG and criterion minimization can be conceived: Feedback and feedforward polynomials are calculated with the LQG method. A low order servo filter is optimized, using explicit criterion minimization. Since few parameters are optimized, convergence will be fast. The properties of the servo filter would be attuned to the properties of the reference signal.

It is evident that the computational load is larger for LQG control than when simple EMV regulators are used. The relevant question here is: Can sufficient sampling frequencies be attained with reasonably inexpensive equipment? Two main areas of application will be discussed briefly.

I Process control. Sampling intervals from 0.2s up to several minutes are common. Let us consider an implementation on an IBM AT computer with a real time operating system and a floating point processor for such applications.

II Signal processing in the audio range. The correspondence of feedforward control to several signal processing problems was mentioned in Section 3.6. The use of adaptive algorithms similar to 5.2 for these problems is under study. Sampling frequencies in the range 10-40 kHz would be required. An implementation on two Motorola DSP56000 signal processors working in parallel can be concieved: One handles filtering and identification and the other computes a new filter.

The computation of a feedforward filter with 9 parameters is considered in Table 6.4. It shows the measured computation time per sample on a Norsk Data ND500 minicomputer and estimates of the computation times with the two implementations discussed above. In all cases, the identified models are updated every sample while a new regulator is computed every 5'th sample. If required, more infrequent regulator recomputation could be used to increase the maximal sampling frequency.

	ND500 minicomputer	IBM AT	Two DSP56000 signal processors
Computation time	6.0ms	90ms	60μs
Maximal sampling frequency	170 Hz	11 Hz	16 kHz

Table 6.4 *Estimated computation times for an adaptive feedforward filter based on Corollary 3.3.*

Remarks: The ND500 time is a measured value. The computation time on an IBM AT has been estimated by comparing the required computation time of EMV self-tuners implemented on ND500 and IBM AT. For LQG control, the personal computer would need a real time operating system with regulator recalculation as a background process. The DSP56000 handles a 24 bit multiplication and a 56 bit accumulation in one 0.1μs machine cycle. The LQG computation time has been estimated by comparing this value to the 140μs needed for floating point multiplication followed by addition on the IBM AT, 6 MHz with 80287 coprocessor.

To summarize: The computational load may have been a relevant argument against the use of explicit LQG/Wiener filtering algorithms 10-15 years ago. It no longer is.

7 CONCLUSIONS

A design technique for discrete time feedforward and scalar disturbance decoupling problems has been developed. It is based on linear quadratic optimization using input-output models in polynomial form. Optimal regulators for non-minimum phase systems can be designed in a simple way. The technique is useful for off-line design of time invariant regulators. In addition, two adaptive feedforward regulators have been developed, based on the off-line optimal design. These regualtors use main output-feedback as well. The combination of feedback and feedforward is superior to the use of feedback or feedforward alone. This is the case also when adaption is utilized. The adaptive regulators have been tested in an extensive simulation study, and found to behave well.

A wide variety of control and signal processing problems are closely related to the linear quadratic feedforward design problem. A number of them are listed below. In some of them, the control/filtering error can be directly measured. In others, it is unmeasureable. This is indicated by M and U, respectively.

o Control strategies utilizing auxiliary outputs

- Disturbance decoupling (M or U)
- Inferential control (mostly U, sometimes M)

o Servofilter optimization (cf Section 3.5) (M)

o Noise cancelling (cf the example in Chapter 1) (M)

o Echo cancelling in telecommunication (M) o Input estimation problems (U)

- Differentiation of noisy signals
- Equalizing (Reconstructing the input to a telecommunication channel)
- Reconstruction of sensor inputs from output data.

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In all the problems above, adaptive techniques are of interest. The structure of the adaptive designs will depend on whether the control/filtering error is directly measurable or not. If it is measurable, the algorithms of Sections 5.2 and 5.3 can be used for scalar signals. In input estimation problems, the filtering error $z=u-\hat{u}$ is unknown, since u is unknown. Cf Section 3.6. For such problems, Ahlén (1986) has suggested an adaptive algorithm based on the polynomial methods utilized in this work. Further development and modification of these algorithms for handling the different problems listed above is an exciting topic for further research. The requirement for fast sampling in some problems (for example equalizing) has already been commented upon. Cases with large and time-varying delays are another challenge. (In echo cancelling problems, delays of up to 200 samples between signal and echo must be handled. The polynomial equations of Chapter 3 become impractical in such situations.)

Other problems for future research are the restatement of the results of Chapter 3 in continuous time, and a generalization to multivariable systems. Finally, it would be of interest to complement the simulation-based comparison of the robustness of the self-tuning LQG and explicit criterion minimization algorithms with a theoretical study.

APPENDIX A1: THE SENSITIVITY FUNCTIONS

From (3.10) and (3.11), it is evident that y(t) and u(t) for the controlled system are given by white noise filtered through asymptotically stable linear filters. Thus, the signals are ergodic, (cf Söderström, 1984), and the criterion (3.6) may be expressed as

$$2J(\theta) = Ey(t,\theta)^2+\rho E(\tilde{\Delta}(q^{-1})u(t-k,\theta))^2 = \int_{-\pi}^{\pi}\Phi(\omega,\theta)d\omega \qquad (A1.1)$$

The expectation is taken with respect to the noise sequences v(t) and e(t). For all θ stabilizing the system, the "equivalent spectral density" $\Phi(\omega,\theta)$ and its derivative $\partial\Phi(\omega,\theta)/\partial\theta$ exist and are continuous functions of ω and θ. (Note that the controlled system is not allowed to have poles on the stability limit.) Thus, we may differentiate with respect to θ, and reverse the order of differentiation and expectation.

$$dJ/d\theta = E\left(\frac{\partial y(t,\theta)}{\partial\theta}\, y(t)+\rho\, \frac{\partial\tilde{\Delta}u(t-k,\theta)}{\partial\theta}\, \tilde{\Delta}u(t-k)\right) =$$

$$= \Big(\underbrace{E(\frac{\partial y}{\partial p_i}\, y+\rho\, \frac{\partial\tilde{\Delta}u}{\partial p_i}\, \tilde{\Delta}u)}_{\substack{i=1,\dots,np \\ \text{(PART 1)}}},\ \underbrace{E(\frac{\partial y}{\partial Q_j}\, y+\rho\, \frac{\partial\tilde{\Delta}u}{\partial Q_j}\, \tilde{\Delta}u)}_{\substack{j=0,\dots,nQ \\ \text{(PART 2)}}},\ \underbrace{E(\frac{\partial y}{\partial r_\ell}\, y+\rho\, \frac{\partial\tilde{\Delta}u}{\partial r_\ell}\, \tilde{\Delta}u)}_{\substack{\ell=1,\dots,nr \\ \text{(PART 3)}}},$$

$$,\underbrace{E(\frac{\partial y}{\partial s_m}\, y+\rho\, \frac{\partial\tilde{\Delta}u}{\partial s_m}\, \tilde{\Delta}u)}_{\substack{m=0,\dots,ns \\ \text{(PART 4)}}}\Big) \qquad (A1.2)$$

The different parts of (A1.2) are expressed in path integral form, using Parsevals relation

$$E[F(q^{-1})e(t)G(q^{-1})e(t)] = \frac{\Lambda}{2\pi i}\oint F(z)G_*(z^{-1})\,\frac{dz}{z} \qquad (A1.3)$$

where F, G are stable transfer operators, e(t) is white noise with variance Λ and the integration path is counter-clockwise around the unit circle. Since covariances do not depend on the signal order, the integrand could just as well be written $G(z)F_*(z^{-1})$. For the optimization method in Appendix A2 to work, it is, however, important to *select the transfer function of lowest degree as the function in* z^{-1} $(G_*(z^{-1})$ in A1.3). Zeros can then cancel poles of G_*.

PART 1. DERIVATIVES WITH RESPECT TO p_i

The use of (3.10) and (3.13) gives

$$\frac{\partial}{\partial p_i}[\alpha HPy(t,\theta)] = \frac{\partial}{\partial p_i}[(q^{-d}DRP-q^{-k}BQ)Gv(t)+RCPHe(t)]$$

$$\alpha H\frac{\partial}{\partial p_i}[y(t,\theta)+p_1y(t-1,\theta)+\ldots+p_{np}y(t-np,\theta)] =$$

$$= \alpha Hy(t-i)+P\alpha H\frac{\partial y(t,\theta)}{\partial p_i} = q^{-d}DRGv(t-i)+HCRe(t-i)$$

The use of (3.10) again to eliminate y(t-i) gives

$$\frac{\partial y(t,\theta)}{\partial p_i} = -\frac{1}{P}y(t-i)+q^{-d}\frac{DRG}{P\alpha H}v(t-i)+\frac{RC}{P\alpha}e(t-i) = q^{-k}\frac{BQG}{P^2\alpha H}v(t-i) \qquad (A1.4)$$

In the same way, using (3.11) and (3.14), we get

$$\frac{\partial}{\partial p_i}[P\alpha H\tilde{\Delta}u(t-k,\theta)] = \frac{\partial q^{-k}}{\partial p_i}[-\tilde{\Delta}(AQ+q^{-d}DSP)Gv(t)-\tilde{\Delta}PHSCe(t)]$$

The use of (3.11) and (3.14) again to eliminate u(t-i) gives

$$\frac{\partial\tilde{\Delta}u(t-k,\theta)}{\partial p_i} = q^{-k}(-\frac{\tilde{\Delta}}{P}u(t-i)-q^{-d}\frac{\tilde{\Delta}DSG}{P\alpha H}v(t-i)-\frac{\tilde{\Delta}SC}{P\alpha}e(t-i)) =$$

$$= q^{-k}\frac{\tilde{\Delta}AQG}{P^2\alpha H}v(t-i) \qquad (A1.5)$$

Using (3.10), (3.11), (A1.4), (A1.5) and the assumption that v(t), e(t) are mutually independent, part 1 in (A1.2) becomes

$$\frac{\partial J}{\partial p_i} = E\left(q^{-k}\frac{BQG}{P^2\alpha H}v(t-i)\,\frac{MG}{P\alpha H}v(t)\right)-\rho E\left(q^{-k}\frac{\tilde{\Delta}AQG}{P^2\alpha H}v(t-i)\;q^{-k}\frac{\tilde{\Delta}FG}{P\alpha H}v(t)\right)$$

$$= \frac{\Lambda_v}{2\pi i}\oint z^i\frac{QG(z^kBM_*-\rho A\tilde{\Delta}\tilde{\Delta}_*F_*)G_*}{P^2\alpha HP_*\alpha_*H_*}\frac{dz}{z}\quad, i=1,\ldots,np$$

which is (3.15).

PART 2. DERIVATIVES WITH RESPECT TO Q_j

The use of the output expressions (3.10), (3.13) give, in the same way as in the derivation of (A1.4):

$$\frac{\partial y(t,\theta)}{\partial Q_j} = q^{-k}\frac{BG}{P\alpha H}v(t-j) = -q^{-k}\frac{B}{P\alpha}w(t-j) \qquad (A1.6)$$

As in the derivation of (A1.5), the use of the input expressions (3.11),(3.14) give

$$\frac{\partial\tilde{\Delta}u(t-k,\theta)}{\partial Q_j} = -q^{-k}\frac{\tilde{\Delta}AG}{P\alpha H}v(t-j) = -q^{-k}\frac{A\tilde{\Delta}}{P\alpha}w(t-j) \qquad (A1.7)$$

Using (3.10), (3.11), (A1.6), (A1.7) and the assumption that v(t), e(t) are mutually independent, part 2 in (A1.2) becomes

$$\frac{\partial J}{\partial Q_j} = -E\left(q^{-k}\frac{BG}{P\alpha H}v(t-j)\,\frac{MG}{P\alpha H}v(t)\right)-\rho E\left(\frac{\tilde{\Delta}AG}{P\alpha H}v(t-j)\,\frac{\tilde{\Delta}FG}{P\alpha H}v(t)\right) =$$

$$= -\frac{\Lambda_v}{2\pi i}\oint z^j\frac{G(z^kBM_*-\rho A\tilde{\Delta}\tilde{\Delta}_*F_*)G_*}{P\alpha HP_*\alpha_*H_*}\frac{dz}{z}\quad, j=0,\ldots,nQ$$

which is (3.16).

PART 3. DERIVATIVES WITH RESPECT TO r_ℓ

Differentiation of the expression (3.10) for the output leads to

$$\frac{\partial y(t,\theta)}{\partial r_\ell} = \frac{1}{\alpha}\,[-Ay(t-\ell)+q^{-d}D\,\frac{G}{H}\,v(t-\ell)+Ce(t-\ell)] = -q^{-k}\,\frac{B}{\alpha}\,u(t-\ell) \qquad \text{(A1.8)}$$

where the system equation (3.7) was used in the last step. The use of (3.11) to eliminate $u(t-\ell)$ then gives

$$\frac{\partial y(t,\theta)}{\partial r_\ell} = q^{-k}\,\frac{BFG}{P\alpha^2H}\,v(t-\ell)+q^{-k}\,\frac{BSC}{\alpha^2}\,e(t-\ell) \qquad \text{(A1.9)}$$

Differentiation of the input expression (3.11) leads to

$$\frac{\partial\tilde{\Delta}u(t-k,\theta)}{\partial r_\ell} = -q^{-k}\,\frac{\tilde{\Delta}A}{\alpha}\,u(t-\ell) \qquad \text{(A1.10)}$$

$$= q^{-k}\,\frac{\tilde{\Delta}AFG}{P\alpha^2H}\,v(t-\ell)+q^{-k}\,\frac{\tilde{\Delta}ASC}{\alpha^2}\,e(t-\ell) \qquad \text{(A1.11)}$$

Using (3.10), (3.11), (A1.9), (A1.11) and the assumption that e(t) and v(t) are mutually independent, part 3 in (A1.2) becomes

$$\frac{\partial J}{\partial r_\ell} = E[q^{-k}\,\frac{BFG}{P\alpha^2H}\,v(t-\ell)+q^{-k}\,\frac{BSC}{\alpha^2}\,e(t-\ell)][\frac{MG}{P\alpha H}\,v(t)+\frac{RC}{\alpha}\,e(t)]+$$

$$+\rho E[q^{-k}\,\frac{\tilde{\Delta}AFG}{P\alpha^2H}\,v(t-\ell)+q^{-k}\,\frac{\tilde{\Delta}ASC}{\alpha^2}\,e(t-\ell)][-q^{-k}\,\frac{\tilde{\Delta}FG}{P\alpha H}\,v(t)-q^{-k}\,\frac{\tilde{\Delta}SC}{\alpha}\,e(t)] =$$

$$= E\Big(q^{-k}\,\frac{BFG}{P\alpha^2H}\,v(t-\ell)\,\frac{MG}{P\alpha H}\,v(t)-\rho q^{-k}\,\frac{\tilde{\Delta}AFG}{P\alpha^2H}\,v(t-\ell)q^{-k}\,\frac{\tilde{\Delta}FG}{P\alpha H}\,v(t)\,+$$

$$+\,q^{-k}\,\frac{BSC}{\alpha^2}\,e(t-\ell)\,\frac{RC}{\alpha}\,e(t)-\rho q^{-k}\,\frac{\tilde{\Delta}ASC}{\alpha^2}\,e(t-\ell)q^{-k}\,\frac{\tilde{\Delta}SC}{\alpha}\,e(t)\Big) =$$

$$= \frac{\Lambda_v}{2\pi i}\oint z^\ell\,\frac{FG\,(z^kBM_*-\rho A\tilde{\Delta}\tilde{\Delta}_*F_*)G_*}{P\alpha^2HP_*\alpha_*H_*}\,\frac{dz}{z}\,+$$

$$+\,\frac{\Lambda_e}{2\pi i}\oint z^\ell\,\frac{SC(z^kBR_*-\rho A\tilde{\Delta}\tilde{\Delta}_*S_*)C_*}{\alpha^2\alpha_*}\,\frac{dz}{z} \qquad \ell=1,\ldots,nr \qquad \text{(A1.12)}$$

This is (3.17).

PART 4. DERIVATIVES WITH RESPECT TO s_m

Differentiation of the expression (3.10) for the output leads to

$$\frac{\partial y(t,\theta)}{\partial s_m} = -q^{-k}\,\frac{B}{\alpha}\,y(t-m) = \qquad \text{(A1.13)}$$

$$= -q^{-k}\,\frac{BMG}{P\alpha^2H}\,v(t-m)-q^{-k}\,\frac{BRC}{\alpha^2}\,e(t-m) \qquad \text{(A1.14)}$$

Differentiation of the expression (3.11) leads to

$$\frac{\partial\tilde{\Delta}u(t-k,\theta)}{\partial s_m} = q^{-k}\,\frac{\tilde{\Delta}}{\alpha}\,(-q^{-k}Bu(t-m)-q^{-d}\,\frac{DG}{H}\,v(t-m)-Ce(t-m)) =$$

$$= -q^{-k}\,\frac{\tilde{\Delta}A}{\alpha}\,y(t-m) \qquad \text{(A1.15)}$$

where the system equation (3.7) was used in the last step, Thus, (3.10) gives

$$\frac{\partial\tilde{\Delta}u(t-k,\theta)}{\partial s_m} = -q^{-k}\,\frac{\tilde{\Delta}AMG}{P\alpha^2H}\,v(t-m)-q^{-k}\,\frac{\tilde{\Delta}ARC}{\alpha^2}\,e(t-m) \qquad \text{(A1.16)}$$

Compare (A1.14),(A1.16) with (A1.9),(A1,11) in Part 3. The only difference is that

1) F-polynomials have been replaced by -M

2) S-polynomials have been replaced by -R

When these replacements are used in the derivation of (A1.12), part 4 in (A1.2) is seen to be given by (3.18).

APPENDIX A2: PROOF OF THEOREM 3.1

In the parameter space spanned by the polynomial coefficients of $P(q^{-1})$ and $Q(q^{-1})$, stationary points are defined by (cf (3.15), (3.16))

$$\frac{\partial J}{\partial p_i} = \frac{\Lambda_v}{2\pi i} \oint z^i \frac{QGW(z,z^{-1})}{P^2 \alpha H} dz = 0 \quad , i=1,\dots,np \qquad (A2.1)$$

$$-\frac{\partial J}{\partial Q_j} = \frac{\Lambda_v}{2\pi i} \oint z^j \frac{PGW(z,z^{-1})}{P^2 \alpha H} dz = 0 \quad , j=0,\dots,nQ \qquad (A2.2)$$

with $W(z,z^{-1})$ given by (3.19).

The proof is divided into four sections:

1) First, (3.27) and (3.28) are shown to be necessary and sufficient conditions for the attainment of stationarity because the integrands of (A2.1) and (A2.2) are analytical in the unit circle. (The path integrals will then vanish because no poles exist inside the integration path.)

2) Stationary points might concievably be attained for another reason: The integrands (written as rational functions of z only) might have poles inside the unit circle, but the sum of all residues cancel. In section 2, it is shown that this cannot be the case if P and Q have optimal (or higher) degree. The integrands of (A2.1) and (A2.2) must then indeed be analytical at stationary points. This part of the proof uses a method suggested by Åström and Söderström (1974). It has also been used by Trulsson (1985).

3) Section 3 explains why attainment of a stationary point implies that J attains its minimal value.

4) The expression (3.30) for the minimal criterion value is finally derived in section 4.

SECTION 1

To allow for the possibility that P and Q may have stable common factors, introduce *coprime factors* $\tilde{P},\tilde{Q}$ of the feedforward filter as:

$$P(q^{-1}) = \tilde{P}(q^{-1})T(q^{-1}) \quad ; \quad Q(q^{-1}) = \tilde{Q}(q^{-1})T(q^{-1}) \qquad (A2.3)$$

where T is the greatest common factor of P and Q.

Necessary and sufficient conditions for the integrands of (A2.1) and (A2.2) to be analytic inside the unit circle are sought. The characteristic polynomial $P\alpha H$ of the closed loop system is required to be stable. $P(z)$, $\alpha(z)$ and $H(z)$ thus have all zeros outside the unit circle. (Recall that q^{-1}, and not q, was exchanged for z.) Since

$$z^{np+n\alpha+nh}P(z^{-1})\alpha(z^{-1})H(z^{-1}) = \bar{P}(z)\bar{\alpha}(z)\bar{H}(z)$$

is unstable, W will have all its poles *inside* the unit circle (with some possibly at the origin). The integrands are analytic inside the unit circle only if all poles of W are cancelled by zeros of W. (The stable numerator polynomial G(z) in (A2.1),(A2.2) cannot cancel any unstable zeros. The numerator factors z^i and z^j are of no help either, because j=0 in one integral.) *Thus, W must be a polynomial in z.* Let us call this polynomial L(z).

$$W(z,z^{-1}) = \frac{(z^k BM_* - \rho \tilde{A}\tilde{\Delta}\tilde{\Delta}_* F_*)G_*}{P_*\alpha_* H_* z} = L(z) \qquad (A2.4)$$

The use of the expressions (3.13),(3.14) for M and F gives

$$(z^k Bz^{-d}D_*R_*P_* - BB_*Q_* - \rho \tilde{A}\tilde{\Delta}\tilde{\Delta}_* A_*Q_* - \rho \tilde{A}\tilde{\Delta}\tilde{\Delta}_* z^{-d}D_*S_*P_*)G_* = zLP_*\alpha_*H_*$$

The use of the spectral factorization (3.24) and elimination of a possible common regulator factor T_* gives

$$(z^{-d+k}BD_*R_*G_*-\rho z^{-d}A\tilde{\Delta}\tilde{\Delta}_*D_*S_*G_*-zL\alpha_*H_*)\tilde{P}_* = r\beta\beta_*\tilde{Q}_*G_* \quad (A2.5)$$

Since $\tilde{P}_*$ is a factor of the left side, it must be a factor of the right. Consider the right-hand side of (A2.5). $\tilde{Q}_*$ and $\tilde{P}_*$ are defined as coprime. $\beta(z)$ is stable, while $z^{n\tilde{p}}\tilde{P}_*=\bar{\tilde{P}}$ is unstable, since P must be stable. Thus, no factors of $\beta\tilde{Q}_*$ can be factors of $\tilde{P}_*$. As a consequence, $\tilde{P}_*$ must divide the polynomial $G_*\beta_*$. Therefore, there exists a polynomial T_{1*} such that

$$\beta_*G_* = \tilde{P}_*T_{1*} \quad (A2.6)$$

Equation (A2.5) is then reduced to the polynomial equation (3.28) in $\tilde{Q}_*T_{1*}$ and L:

$$(BR_*-\rho z^{-k}A\tilde{\Delta}\tilde{\Delta}_*S_*)z^{-d+k}D_*G_* = r\beta(\tilde{Q}_*T_{1*})+\alpha_*H_*zL \quad (A2.7)$$

The equations (A2.6) and (A2.7) define a feedforward filter $Q/P=\tilde{Q}T_1/\tilde{P}T_1$ with a possible stable common factor T_1. They are necessary for (A2.4) to hold, and thus necessary for (A2.1),(A2.2) to have analytic integrands inside the integration path. Sufficiency is simple to prove by inserting (A2.6) and (A2.7) into the integrands and by verifying the pole cancellation.

Q_* is a polynomial in z^{-1} while L is a polynomial in z. They must have degrees large enough to cover the maximal powers in z^{-1} and z, respectively in (3.28). Writing (3.28) as

$$r\beta Q_* = -Lz\alpha_*H_*+z^{-d+k}BR_*D_*G_*-\rho z^{-d}A\tilde{\Delta}\tilde{\Delta}_*S_*D_*G_* \quad (A2.8a)$$

or

$$L\alpha_*H_* = -z^{-1}r\beta Q_*+z^{-d+k-1}BR_*D_*G_*-\rho z^{-d-1}A\tilde{\Delta}\tilde{\Delta}_*S_*D_*G_* \quad (A2.8b)$$

and taking (3.25) into account, it is evident that degree conditions for the unique solution are given by (3.29).

If a higher degree is assigned to $Q_*(z^{-1})$ the superfluous coefficients will equal zero, since no corresponding powers of z^{-1} exist on the right hand side of (A2.8a). The same is true of L(z) in (A2.8b).

SECTION 2

By expanding (A2.1) and (A2.2) as sums of path integrals, they are rewritten as

$$\begin{pmatrix} 0 & \tilde{Q}_o & \cdots & \tilde{Q}_{nQ} & & 0 \\ & \ddots & & & \ddots & \\ 0 & & \tilde{Q}_o & \cdots & & \tilde{Q}_{n\tilde{Q}} \\ 1 & \tilde{p}_1 & \cdots & \tilde{p}_{np} & & 0 \\ & \ddots & & & \ddots & \\ 0 & & 1 & \tilde{p}_1 & \cdots & \tilde{p}_{n\tilde{p}} \end{pmatrix} \begin{pmatrix} h_1 \\ \vdots \\ h_m \end{pmatrix} = 0 \quad (A2.9)$$

where $m=n\tilde{p}+nQ+1=\deg\tilde{P}\tilde{Q}T+1$, where (A2.3) has been used and where

$$h_\ell = \frac{\Lambda_v}{2\pi i}\oint z^\ell \underbrace{\frac{TG(z^kBM_*-\rho A\tilde{\Delta}\tilde{\Delta}_*F_*)G_*}{P^2\alpha H(P_*\alpha_*H_*z)}}_{I(z)}\frac{dz}{z} \quad ; \ell=1,\ldots,m \quad (A2.10)$$

Note that since T is a factor in both P and Q, its degree will only be counted once in m, Note also that numerator and denominator of(A2.10) have been multiplied with z. (ℓ starts with 1, not 0.)

Since $(\tilde{P},\tilde{Q})$ are coprime by definition, it follows from the theory of resultants, see for example van der Waerden (1970), that the matrix in (A2.9) with $np+nQ+1$ rows and $n\tilde{p}+nQ+1$ columns has full rank $m=np+n\tilde{Q}+1$. Thus, the only solution to the system (A2.9) is the trivial one. Consequently, at all stationary points

$$h_\ell \triangleq \frac{\Lambda_e}{2\pi i} \oint z^\ell I(z) \frac{dz}{z} = 0 \ , \quad \ell=1,\dots,m \qquad \text{(A2.11)}$$

where $I(z)$ has ν poles inside the unit circle.

Lemma 1 in Aström and Söderström (1974) guarantees that if

$$m \geq \nu \qquad \text{(A2.12)}$$

then $I(z)$ will in fact be analytic inside the unit circle at all stationary points. The poles inside the integration path will be cancelled by zeros. Fulfillment of (3.27) and (3.28) (which are necessary and sufficient for having no poles inside the unit circle) would then be necessary and sufficient for attainment of a stationary point.

The use of the expressions (3.13), (3.14), (3.24) and (A2.3) gives

$$I = \frac{TG(z^{-d+k}BD_*R_*\tilde{P}_*-r\beta\beta_*\tilde{Q}_*-\rho A\tilde{\Delta}\tilde{\Delta}_*z^{-d}D_*S_*\tilde{P}_*)G_*}{P^2\alpha H\tilde{P}_*\alpha_*H_*z} \qquad \text{(A2.13)}$$

The integrand (A2.13) will now be converted into a rational function in z, and the number of poles inside the unit circle (unstable poles) is investigated.

If $\max\{d-k+\deg DR\tilde{P}G, \deg\beta\tilde{Q}G, d+\deg\tilde{\Delta}DS\tilde{P}G\} \leq \deg\tilde{P}\alpha H-1$ we can multiply with $z^{\deg\tilde{P}\alpha H-1}$ to obtain

$$I(z) = \frac{TGX_1}{P^2\alpha H\bar{\tilde{P}}\bar{\alpha}\bar{H}}$$

where $X_1(z)=z^{\deg\tilde{P}\alpha H-1}(z^{-d+k}BD_*R_*\tilde{P}_*-r\beta\beta_*\tilde{Q}_*-\rho A\tilde{\Delta}\tilde{\Delta}_*z^{-d}D_*S_*\tilde{P}_*)G_*$ is a polynomial in z and $\bar{\tilde{P}}\bar{\alpha}\bar{H}$ has all zeros inside the unit circle.

Otherwise, multiply with z^f, where

$$f \triangleq \max\{d-k+\deg DR\tilde{P}G,\ \deg\beta\tilde{Q}G,\ d+\deg\tilde{\Delta}DS\tilde{P}G\} \qquad \text{(A2.14)}$$

Then $I(z)$ becomes

$$I(z) = \frac{TGX_2}{P^2\alpha H\bar{\tilde{P}}\bar{\alpha}\bar{H}z^e}$$

where z^e are "surplus poles at the origin".

$$e = f-\deg\tilde{P}\alpha H+1 \qquad \text{(A2.15)}$$

To summarize, the number of poles inside the unit circle of I is

$$\nu = \deg\tilde{P}\alpha H+\max(e,0) = \deg\tilde{P}\alpha H+\max\{f-\deg\tilde{P}\alpha H+1,0\} =$$

$$= 1+\max\{f,\deg\tilde{P}\alpha H-1\}$$

Using f from (A2.14) and $m=\deg\tilde{P}\tilde{Q}T+1$, the condition (A2.12) may then be expressed as

$$\deg\tilde{P}\tilde{Q}T \geq \max\{d-k+\deg DR\tilde{P}G,\ \deg\beta\tilde{Q}G,\ d+\deg\tilde{\Delta}DS\tilde{P}G,\ \deg\tilde{P}\alpha H-1\}$$

Considering the different parts of the right-hand side separately, the following conditions should all be fulfilled:

1) $\deg \tilde{Q}T \geq d-k+\deg DRG$

2) $\deg \tilde{P}T \geq \deg \beta G$

3) $\deg \tilde{Q}T \geq d+\deg \tilde{\Delta} DSG$

4) $\deg \tilde{Q}T \geq \deg \alpha H-1$

Conditions 1)-4) are together equivalent with

$$np = \deg \tilde{P}T \geq \deg \beta G \qquad \text{(A2.16)}$$

$$nQ = \deg \tilde{Q}T \geq \max\{\deg \alpha H-1,\ d-k+\deg DRG,\ d+\deg \tilde{\Delta} DSG\} \qquad \text{(A2.17)}$$

Whenever we choose polynomials with degrees according to (3.27) and (3.29), these conditions are indeed fulfilled. Fulfillment of (3.27) and (3.28) are thus necessary and sufficient for stationarity for (P,Q) with $nP=\deg \beta G$ and nQ given by (3.29a). Cf (A2.12).

If regulator polynomial degrees are restricted below (A2.16),(A2.17), the integrands cannot be analytic at stationary points: Equation (3.28) has no solution. Such *restricted complexity optimal control problems* may have several local minima, where $dJ/d\theta=0$ because residues corresponding to poles inside the unit circle cancel.

SECTION 3

In general, the global extremum of a function might be attained at a point where the gradient does not exist: Consider for example a cone. This cannot be the case in our problem. J and $dJ/d\theta$ exist and are well-defined for all stabilizing regulators. Thus, the minimal value of J must be attained at a stationary point or, possibly, at the boundary of admissible (stable) filter parameters. But since $J\to\infty$ whenever the stability boundary is approached, this possibility is excluded.

The stationary points constitute a manifold with dimension equal to the number of common factors to P and Q. Since stable common factors do not influence the stationary behaviour of a regulator, all regulators covered by the theorem attain the same criterion value.

Consider the subspace spanned by the coefficients of the coprime factors $\tilde{P}$, $\tilde{Q}$, i e by $\tilde{p}_1,\ldots,\tilde{p}_{n\tilde{p}}$, $\tilde{q}_0,\ldots,\tilde{q}_{n\tilde{Q}}$. $dJ/d\theta=0$ is attained at a unique point in this subspace, since the coprime factors of P, Q calculated from (3.27),(3.28) with degree condition (3.29) are unique. The value of J attained at this point is the global minimum. It cannot be a maximum because

1) $J\to\infty$ whenever $\|\tilde{\theta}\|_2 = \|\tilde{p}_1,\ldots,\tilde{p}_{n\tilde{p}}, \tilde{q}_0,\ldots,\tilde{q}_{n\tilde{Q}}\|_2 \to \infty$

and

2) no other stationary points exist.

For the same reason, it cannot be a saddle point.

($\|\tilde{\theta}\|_2\to\infty$ means that $\tilde{P}$ crosses the stability boundary and/or that the $\tilde{Q}$-polynomial coefficients go to infinity. $Ey(t)^2$ and $E\tilde{\Delta}u(t)^2$ then increase without bounds.)

SECTION 4

Using (3.10)-(3.14) with e(t)=0 and P=βG, the minimal criterion value becomes

$$2J_{min} = Ey(t)^2+\rho E(\widetilde{\Delta}(q^{-1})u(t-k))^2 =$$

$$= \frac{\Lambda_v}{2\pi i}\oint \frac{(z^d DR\beta G - z^{-k}BQ)(z^{-d}D_*R_*\beta_*G_* - z^{-k}B_*Q_*) + \rho\widetilde{\Delta\Delta}_*(AQ+z^d DS\beta G)(A_*Q_*+z^{-d}D_*S_*\beta_*G_*)}{\beta\alpha H\beta_*\alpha_*H_*}\frac{dz}{z} \tag{A2.18}$$

Using (3.24)

$$\beta\beta_* = \frac{BB_*}{r} + \frac{\rho}{r}A\widetilde{\Delta\Delta}_*A_*$$

the numerator of (A2.18) may be expressed term by term in the following way:

$$\overset{(1)}{\frac{1}{r}DRGBD_*R_*G_*B_*} + \overset{(2)}{\frac{\rho}{r}DRGA\widetilde{\Delta\Delta}_*A_*D_*R_*G_*} \overset{-(3)}{-z^{d-k}DR\beta GB_*Q_*}$$

$$\overset{-(4)}{-z^{-d+k}D_*R_*\beta_*G_*BQ} + \overset{(5)}{r\beta\beta_*QQ_*} + \overset{(6)}{z^{-d}\rho AQD_*S_*\beta_*G_*\widetilde{\Delta\Delta}_*}$$

$$\overset{(7)}{+z^d\rho DS\beta GA_*Q_*\widetilde{\Delta\Delta}_*} + \overset{(8)}{\frac{\rho}{r}DD_*SS_*GG_*BB_*\widetilde{\Delta\Delta}_*} + \overset{(9)}{\frac{\rho^2}{r}DD_*SS_*GG_*\widetilde{\Delta\Delta}_*AA_*\widetilde{\Delta\Delta}_*}$$

$$\overset{(a)}{+\frac{\rho}{r}z^kBR_*D_*G_*A_*\widetilde{\Delta\Delta}_*SDG} + \overset{(b)}{\frac{\rho}{r}z^{-k}A\widetilde{\Delta\Delta}_*S_*D_*G_*B_*RDG}$$

$$\overset{-(a)}{-\frac{\rho}{r}z^kBR_*D_*G_*A_*\widetilde{\Delta\Delta}_*SDG} \overset{-(b)}{-\frac{\rho}{r}z^{-k}A\widetilde{\Delta\Delta}_*S_*D_*G_*B_*RDG} \tag{A2.19}$$

(The last four terms in (A2.19): (a)+(b)-(a)-(b)=0 have been added to clarify the following steps.)

Using the fact that the optimal Q_* will saitsfy (3.28), a number of terms in the numerator (A2.19) may be simplified in the following way:

$$(1)-(a)-(4)-(b)+(9)+(6)-(3)+(7)+(5) =$$

$$= \left(\frac{1}{\sqrt{r}}BR_*z^{-d+k}D_*G_* - \frac{\rho}{\sqrt{r}}z^{-d}A\widetilde{\Delta\Delta}_*S_*D_*G_* - \sqrt{r}\beta Q_*\right)\left(\frac{1}{\sqrt{r}}B_*Rz^{d-k}DG - \frac{\rho}{\sqrt{r}}z^dA_*\widetilde{\Delta}_*\widetilde{\Delta}SDG - \sqrt{r}\beta_*Q\right) =$$

$$= \frac{1}{r}\alpha_*\alpha H_*HL_*L$$

The rest of the terms may be written as

$$(2)+(b)+(a)+(8) = \frac{\rho}{r}GG_*DD_*\widetilde{\Delta\Delta}_*(AR+z^kBS)(A_*R_*+z^{-k}B_*S_*)$$

With these simplifications of the numerator, the minimal criterion value (A2.18) is given by (3.30)

$$2J_{min} = \frac{\Lambda_v}{2\pi i}\oint\frac{\left(\frac{1}{r}\alpha\alpha_*H_*HL_*L + \frac{\rho}{r}GG_*DD_*\widetilde{\Delta\Delta}_*\alpha\alpha_*\right)}{\beta\alpha H\beta_*\alpha_*H_*}\frac{dz}{z},$$

$$= \frac{\Lambda_v}{2\pi i}\oint\frac{LL_*}{r\beta\beta_*}\frac{dz}{z} + \frac{\rho\Lambda_v}{2\pi i}\oint\frac{GG_*DD_*\widetilde{\Delta\Delta}_*}{r\beta\beta_*HH_*}\frac{dz}{z}$$

APPENDIX A3: PROOF OF THEOREM 3.6

From Theorem 3.1 and Appendix A2, we know that $P=\beta G$ and a Q solving (3.28) are necessary and sufficient for $\partial J/\partial p_i=0$ and $\partial J/\partial Q_j=0$ if $np=n\beta+ng$ and nQ is given by (3.29). The integrands of (3.15) and (3.16) are then analytic inside the unit circle, because of (A2.4):

$$W(z,z^{-1}) = L(z)$$

But this implies that the first parts of $\partial J/\partial r_\ell$, $\partial J/\partial s_m$ in (3.17), (3.18), namely

$$\frac{\Lambda_v}{2\pi i}\oint z^\ell \frac{FGW(z,z^{-1})}{P\alpha^2 H}dz \quad \text{and} \quad \frac{\Lambda_v}{2\pi i}\oint z^m \frac{MGW(z,z^{-1})}{P\alpha^2 H}dz$$

will also have analytic integrands inside the integration path, and thus be zero. (P, α and H are stable.)

Given that P and Q are optimal, we thus have a stationary point if and only if the second parts of (3.17) and (3.18) are zero. From Theorem 3.4, part 2, we recognize that this is achieved by designing (R,S) to be optimal with respect to the unmeasurable disturbance e(t).

Since the system is linear, and v(t), e(t) are mutually independent, we may simply add J_{FF} (which gives the minimal criterion value if e(t)=0) with J_{FB} (which gives the minimal value if v(t)=0) to get the total global minimal value $J_{min}=J_{FF}+J_{FB}$.

The discussion of Appendix A2, section 3 may be repeated with $\tilde{\theta}=\{\tilde{P}, \tilde{Q}, \tilde{R}, \tilde{S}\}$ to show that stationary points really correspond to the global minimal value of J.

Equations (3.43) are used to optimize R and S, as in Theorem 3.4. They imply pole placement in

$$\beta C = AR+q^{-k}BS \triangleq \alpha \tag{A3.1}$$

Let us exchange z^{-1} for q^{-1} in (A3.1) and multiply it with $\rho A\tilde{\Delta}\tilde{\Delta}_*$. This gives

$$-\rho A\tilde{\Delta}\tilde{\Delta}_* z^{-k}B_*S_* = \rho A\tilde{\Delta}\tilde{\Delta}_* A_*R_* - \rho A\tilde{\Delta}\tilde{\Delta}_* \beta_* C_*$$

Addition of BB_*R_* and multiplication with z^{k-1} gives

$$z^{k-1}(BB_*R_* - \rho A\tilde{\Delta}\tilde{\Delta}_* z^{-k}B_*S_*) = z^{k-1}(r\beta\beta_*R_* - \rho A\tilde{\Delta}\tilde{\Delta}_*\beta_*C_*)$$

By dividing with β_*B_* and comparing with (3.43a) it is found that

$$\frac{z^{-1}(z^kBR_* - \rho A\tilde{\Delta}\tilde{\Delta}_*S_*)}{\beta_*} = \frac{z^{k-1}(r\beta R_* - \rho\tilde{\Delta}\tilde{\Delta}_*AC_*)}{B_*} = X(z) \tag{A3.2}$$

The optimal feedforward polynomial Q satisfies (3.28). The use of (A3.1) and (A3.2) in (3.28) gives

$$(z\beta_*X)z^{-d}D_*G_* = r\beta Q_* + \beta_*C_*H_*zL \tag{A3.3}$$

Since β_* is a factor of the other two terms, it must be a factor of Q_*. (It cannot have common factors with β, since $z^{-n\beta}\beta=\bar{\beta}_*$ will be strictly unstable, while β_* is stable.) Let

$$Q_* = Q_*'\beta_*$$

Equation (A3.3) then reduces to (3.52):

$$z^{-d+1}D_*G_*X = r\beta Q_*' + C_*H_*zL$$

If the stable common factor β in

$$\frac{Q}{P} = \frac{Q'\beta}{G\beta}$$

is cancelled and Q' is named Q, we have the regulator (3.43), (3.50),(3.52),(3.55).

The degree of L is found by inspecting the maximal power in z in (3.52), and using (3.44a):

$$nL+1 = \max\{n\beta,\ nx-d+1\} = \max\{n\beta,\ n\beta+k-d\}$$

$$nL = n\beta-1+\max\{0,\ k-d\}$$

The degree of Q is given directly by the maximal power in z^{-1} of (3.52).

APPENDIX A4: DERIVATION OF MINIMUM VARIANCE REGULATORS

PART 1: PROOF OF COROLLARY (3.7)

With $\rho=0$ and $B=cB_sB_u$, the spectral factorization (3.24) reduces to

$$r\beta\beta_* = c^2B_sB_uB_{s*}B_{u*} = c^2(B_sz^uB_{u*})(B_{s*}z^{-u}B_u) = c^2(B_s\bar{B}_u)(B_{s*}\bar{B}_{u*})$$

Thus, the scalar r equals c^2. Consider the optimal feedforward regulator of Corollary 3.3. The use of $\beta=B_s\bar{B}_u$ in (3.38) gives (3.61). Multiplication of (3.39) with z^{-nb-g} transforms it into an equation in z^{-1} only. $g=\max(0,k-d)$.

$$z^{-d+k-g}z^{-nb}BD_*G_* = z^{-nb-g}c^2\bar{B}_uB_sQ_*+A_*H_*z^{-nb-g+1}L$$

From (3.40), we know that $z^{-nb-g+1}L \triangleq \bar{L}_*$ will be a polynomial in z^{-1}. Use $z^{-nb}B=\bar{B}_*$ and $z^{-nb}\bar{B}_uB_s=B_{u*}\bar{B}_{s*}$. Substitute $q^{-1}=z$ for z^{-1}. This gives

$$q^{-d+k-g}c\bar{B}_u\bar{B}_sDG = q^{-g}c^2B_u\bar{B}_sQ+AH\bar{L} \qquad (A4.1)$$

Since $c\bar{B}_s$ is a factor of the two first terms, it must be a factor of $\bar{L}$. (No factor of $\bar{B}_s$ may be part of AH, since A and H are assumed stable.)

Let

$$\bar{L} = c\bar{B}_sL_1$$

This reduces (A4.1) to (3.62). The degree of L_1 is given by

$$nL_1 = nL-\deg B_s = \max\{nb-1,nb-d+k-1\}-\deg B_s = u-1+g$$

PART 2: PROOF OF COROLLARY 3.8

Consider the optimal combined feedback-feedforward regulator of Theorem 3.6. When A and B have no common factors, R and S are determined uniquely by (3.45). (This equation then corresponds to a square system of simultaneous equations with full rank if nr and ns are given by (3.44b,c).)

The use of $B=cB_sB_u$ and $\beta=B_s\bar{B}_u$ in (3.45) gives

$$B_s\bar{B}_uC = AR+q^{-k}cB_sB_uS \qquad \text{(A4.2)}$$

Since B_s is a factor of the other terms, and A,B are assumed coprime, B_s must be a factor in R.

Let

$$R = B_sR_1 \qquad \text{(A4.3)}$$

(Since R and B_s are monic, R_1 will be monic.) This reduces (A4.2) to (3.68). The polynomial degree expressions (3.70) follow from (3.44b,c).

The use of $R=B_sR_1$ and $\rho=0$ in the second part of (A3.2) gives

$$X = \frac{z^{k-1}c^2B_s\bar{B}_uB_{s*}R_{1*}}{cB_{s*}B_{u*}} = z^{+nb}z^{k-1}\frac{c\bar{B}_{s*}B_{u*}R_{1*}}{B_{u*}} = z^{nb+k-1}c\bar{B}_{s*}R_{1*}$$

Equation (3.52) then becomes

$$z^{nb+k-d}c\bar{B}_{s*}R_{1*}D_*G_* = c^2\bar{B}_uB_sQ_*+C_*H_*zL$$

Multiplication with z^{-nb-g} transforms this equation into an equation in z^{-1} only.

$$z^{-d+k-g}c\bar{B}_{s*}R_{1*}D_*G_* = c^2z^{-g}B_{u*}\bar{B}_{s*}Q_*+C_*H_*z^{-nb-g+1}L$$

From (3.53) we know that $z^{-nb-g+1}L=\bar{L}_*$ will be a polynomial in z^{-1}. If $q^{-1}=z$ is substituted for z^{-1} we get

$$q^{-d+k-g}c\bar{B}_sR_1DG = c^2q^{-g}B_u\bar{B}_sQ+CH\bar{L} \qquad \text{(A4.4)}$$

Since $c\bar{B}_s$ is a factor of the two first terms, it must be a factor of $\bar{L}$. (No factor of $\bar{B}_s$ may be part of CH, since C and H are assumed stable.)

Let

$$\bar{L} = c\bar{B}_sL_1$$

This reduces (A4.4) to (3.69). The polynomial degrees (3.71) follow from (3.53).

APPENDIX A5: PROOF OF THEOREM 3.10

The proof follows a technique of proving the optimality of the LQG feedback regulator used in Åström and Wittenmark (1984). The method is easy to use when one already knows the answer.

Since no unmeasurable disturbances e(t) are present, the system is described by

$$Ay(t) = q^{-k}Bu(t)+q^{-d}Dw(t)$$
$$w(t) = \frac{G}{H}v(t) \quad ; \quad H = H_s\Delta' \qquad \text{(A5.1)}$$

The criterion

$$2J = Ey(t)^2+\rho E(\tilde{\Delta}u(t))^2 \qquad \text{(A5.2)}$$

is to be minimized. The factor Δ' with zeros on the unit circle is assumed to be a factor of either $\tilde{\Delta}$ or D. Let us write an arbitrary input sequence u(t) as

$$u(t) = -\frac{Q}{P}w(t)+m(t) \qquad \text{(A5.3)}$$

where $Q(q^{-1})$ and $P(q^{-1})$ are given by Corollary 3.3. Any additional control action is represented by m(t). Our goal is to demonstrate that it is optimal to choose m(t)=0.

Thus, P and Q are given by

$$P = \beta G$$

and, exchanging z for z^{-1} in (3.39):

$$z^{d-k}B_*DG = r\beta_*Q+AHz^{-1}L_* \qquad \text{(A5.4)}$$

When the system (A5.1) is controlled with (A5.3), the result is

$$y(t) = \frac{MG}{PAH}v(t)+q^{-k}\frac{B}{A}m(t)$$

$$u(t) = -\frac{QG}{PH}v(t)+m(t)$$

where $M=q^{-d}DP-q^{-k}BQ$. Thus, the criterion (A5.2) may be written as

$$2J = E\left(\frac{MG}{PAH}v(t)+q^{-k}\frac{B}{A}m(t)\right)^2+\rho E\left(-\frac{\tilde{\Delta}QG}{PH}v(t)+\tilde{\Delta}m(t)\right)^2 = J_1+2J_2+J_3$$

where

$$J_1 = E\left(\frac{MG}{PAH}v(t)\right)^2+\rho E\left(\frac{\tilde{\Delta}QG}{PH}v(t)\right)^2 \geq 0$$

$$J_2 = E\left(\frac{MG}{PAH}v(t)\right)\left(q^{-k}\frac{B}{A}m(t)\right)-\rho E\left(\frac{\tilde{\Delta}QG}{PH}v(t)\right)\tilde{\Delta}m(t)$$

$$J_3 = E\left(\frac{B}{A}m(t)\right)^2+\rho(\tilde{\Delta}m(t))^2 \geq 0$$

We know that since P,Q satisfy Corollary 3.3, J_1, which is the criterion value when m(t)=0, will be given by (3.30):

$$J_1 = \frac{\Lambda_v}{2\pi i}\oint\frac{LL_*}{r\beta\beta_*}\frac{dz}{z}+\frac{\rho\Lambda_v}{2\pi i}\oint\frac{GG_*DD_*\tilde{\Delta}\tilde{\Delta}_*}{r\beta\beta_*HH_*}\frac{dz}{z}$$

This expression will be finite. The integrands do not have any poles on the unit circle, since such factors of H are assumed to be factors also of $\tilde{\Delta}D$. L(z) has finite coefficients since (A5.4) will be solvable for all $\rho<\infty$.

If m(t) is nonstationary, J_3 is infinite, and we cannot have a minimum. Assume m(t) to be stationary. The cross spectrum between the white noise v(t) and m(t) is $\Phi_{vm}(e^{i\omega})$. Using $P=\beta G$, J_2 may be written as

$$J_2 = \frac{1}{2\pi i} \oint \frac{Mz^{-k}B_*}{\beta AHA_*} \Phi_{vm*} \frac{dz}{z} - \frac{\rho}{2\pi i} \oint \frac{\tilde{\Delta}Q}{\beta H} \tilde{\Delta}_* \Phi_{vm*} \frac{dz}{z} =$$

$$= \frac{1}{2\pi i} \oint \frac{[(z^d D\beta G - z^k BQ)z^{-k}B_* - \rho AA_* \tilde{\Delta}\tilde{\Delta}_* Q]}{\beta AHA_*} \Phi_{vm*} \frac{dz}{z} =$$

$$= \frac{1}{2\pi i} \oint \frac{\beta AHz^{-1}L_*}{\beta AHA_*} \Phi_{vm*} \frac{dz}{z} = \frac{1}{2\pi i} \oint \frac{z^{-1}L_*}{A_*} \Phi_{vm*} \frac{dz}{z} \qquad \text{(A5.5)}$$

The polynomial equation (A5.4) was used in the second equality. From (A5.5), it is seen that J_2 will be finite and well defined even when H is unstable. J_2 may be written as

$$J_2 = E[v(t)(q^{-1} \frac{L(q^{-1})}{A(q^{-1})} m(t)]$$

The second term is a function of m(t-1),m(t-2),... These terms are all independent of v(t). Thus, it is evident that J_2 will vanish.

The loss function is

$$2J = J_1 + J_3(m(t))$$

where J_1 is independent of m(t) and J_3 is nonnegative. It is minimized by choosing m(t)=0. Thus, the control law u=-(Q/P)w is optimal also for systems where H has zeros on the unit circle.

APPENDIX A6: SYSTEM IDENTIFICATION

In both the LQG regulator and the explicit criterion minimization algorithm, system identification is performed as follows.

The main output and auxiliary output model (3.4),(3.5) are estimated by two separate single output recursive prediction error algorithms. As prediction errors

$$\varepsilon_y(t) = y(t) - \varphi_1(t)^T \theta_y(t) \quad ; \quad \varepsilon_w(t) = w(t) - \varphi_2(t)^T \theta_w(t) \qquad \text{(A6.1)}$$

are used, with regressor vectors

$$\varphi_1(t)^T = (-y(t-1),\ldots,-y(t-na),\ u(t-1),\ldots,u(t-nb),\ w(t),\ldots,w(t-nd),\ \varepsilon_y(t-1),\ldots,\varepsilon_y(t-nc))$$

$$\varphi_2(t)^T = (-w(t-1),\ldots,-w(t-nh),\ u(t-1),\ldots,u(t-nn),\ \varepsilon_w(t-1),\ldots,\varepsilon_w(t-ng))$$

and parameter vectors

$$\theta_y^T = (a_1,\ldots,a_{na},\ b_1,\ldots,b_{nb},\ d_0,\ldots,d_{nd},\ c_1,\ldots,c_{nc})$$

$$\theta_w^T = (h_1,\ldots,h_{nh},\ n_1,\ldots,n_{nn},\ g_1,\ldots,g_{ng})$$

respectively. The complete model parameter vector is $\theta_m^T \triangleq (\theta_y^T, \theta_w^T)$. The model polynomials A, H, C and G have fixed leading coefficients 1. If w(t) is measured before y(t) at the sampling instant, w(t) may affect y(t). (This time delay d=0.) This is why w(t) is included in the regressor vector $\varphi_1(t)$.

By using two separate algorithms, instead of a multiple output identification method with parameter vector θ_m, the computational load is reduced significantly.

To prevent biased estimation because of non-zero mean disturbances, two options for filtering the regressor signals y, u and w in φ_1 and φ_2 have been included: The signals can be differentiated. Alternatively, they can be high-pass filtered with a filter $(1-q^{-1})/(1-0.98q^{-1})$. Compared with differentiation, this filtering does not emphasize the high frequency properties of the models.

Stability of C, H and G is monitored. Unknown and time-varying delays are handled by selecting degrees of B, D and N which cover the maximal expected delays. The terms $b_1,..,b_{k-1}$, $d_o,...,d_{d-1}$ and $n_1,...,n_{n-1}$ will then converge to zero. (The algorithm would, however, have to be modified to cope with time delays above 10-15.) Residuals (a posteriori prediction errors, using θ(t) instead of θ(t-1)), defined by (A6.1), are used in the regressor vectors to speed up convergence (Ljung and Söderström, 1983).

In the adaptive control simulation experiments, identification has been performed in the following way: For the first 20 samples, the system is identified in open loop. The forgetting factor is 0.95 initially and goes to a final value between 0.98 and 1 exponentially. Updating of the model is discontinued if the standard deviation of the residuals decrease below a preset bound, or if the trace of the covariance matrix increases above another bound. Model updating is resumed (with a covariance matrix reset to unity) if the standard deviation of the residuals increase a factor of 2, compared with the value when updating was discontinued. These modifications guard against bad excitation and estimator wind-up.

A variant of the algorithm where identification initially is performed in an ELS mode, as suggested by Friedlander (1982), has also been tested. Up to time 250, filtering with the (still rather uncertain) C and G-estimates is not performed in this variant. This modification was found to give only small improvements in the speed of convergence, and an insignificant improvement in the control performance.

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