Last time we talked about:

- Another source of error due to filtering effect of the system:
  - Inter-symbol interference (ISI)
- The techniques to reduce ISI
  - Pulse shaping to achieve zero ISI at the sampling time
  - Equalization to combat the filtering effect of the channel
Today, we are going to talk about:

- Some bandpass modulation schemes used in DCS for transmitting information over channel
  - M-PAM, M-PSK, M-FSK, M-QAM
- How to detect the transmitted information at the receiver
  - Coherent detection
  - Non-coherent detection
Block diagram of a DCS

Format → Source encode → Channel encode → Pulse modulate → Bandpass modulate

Digital modulation

Source decode → Channel decode → Detect → Demod. Sample

Digital demodulation
Bandpass modulation

■ **Bandpass modulation:** The process of converting a data signal to a sinusoidal waveform where its amplitude, phase or frequency, or a combination of them, are varied in accordance with the transmitting data.

■ **Bandpass signal:**

\[
s_i(t) = g_T(t) \sqrt{\frac{2E_i}{T}} \cos(\omega c t + (i - 1)\Delta \omega t + \phi_i(t)) \quad 0 \leq t \leq T
\]

where \( g_T(t) \) is the baseband pulse shape with energy \( E_g \).

■ We assume here (otherwise will be stated):
  ■ \( g_T(t) \) is a rectangular pulse shape with unit energy.
  ■ Gray coding is used for mapping bits to symbols.
  ■ \( E_s \) denotes average symbol energy given by \( E_s = \frac{1}{M} \sum_{i=1}^{M} E_{s_i} \).
Demodulation and detection

- **Demodulation**: The receiver signal is converted to baseband, filtered and sampled.
- **Detection**: Sampled values are used for detection using a decision rule such as the ML detection rule.

\[
\int_0^T r(t) \psi_1(t) \, dt = z_1
\]

\[
\int_0^T r(t) \psi_N(t) \, dt = z_N = [z_1, \ldots, z_N]
\]

Decision circuits (ML detector)

\[\hat{m}\]
Coherent detection

- Coherent detection
  - requires carrier phase recovery at the receiver and hence, circuits to perform phase estimation.
  - Sources of carrier-phase mismatch at the receiver:
    - Propagation delay causes carrier-phase offset in the received signal.
    - The oscillators at the receiver which generate the carrier signal, are not usually phased locked to the transmitted carrier.
Coherent detection ..

- Circuits such as Phase-Locked-Loop (PLL) are implemented at the receiver for carrier phase estimation ($\alpha \approx \hat{\alpha}$).

$$\begin{align*}
    r(t) &= g_T(t) \sqrt{\frac{2E_i}{T}} \cos(\omega_c t + \phi_i(t) + \alpha) + n(t) \\

    \text{Oscillator} &\rightarrow \text{PLL} \\
    \sqrt{\frac{2}{T}} \cos(\omega_c t + \hat{\alpha}) &\rightarrow \text{I branch} \\
    \sqrt{\frac{2}{T}} \sin(\omega_c t + \hat{\alpha}) &\rightarrow \text{Q branch}
\end{align*}$$

Used by correlators
Bandpass Modulation Schemes

- One dimensional waveforms
  - Amplitude Shift Keying (ASK)
  - M-ary Pulse Amplitude Modulation (M-PAM)

- Two dimensional waveforms
  - M-ary Phase Shift Keying (M-PSK)
  - M-ary Quadrature Amplitude Modulation (M-QAM)

- Multidimensional waveforms
  - M-ary Frequency Shift Keying (M-FSK)
Amplitude Shift Keying (ASK) modulation:

\[ s_i(t) = \sqrt{\frac{2E_i}{T}} \cos(\omega_c t + \phi) \]

\[ s_i(t) = a_i \psi_1(t) \quad i = 1, \ldots, M \]

\[ \psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t + \phi) \]

\[ a_i = \sqrt{E_i} \]

On-off keying (M=2):

“0”  “1”

\[ s_2 \quad 0 \quad \sqrt{E_1} \quad s_1 \]

\[ \psi_1(t) \]
M-ary Pulse Amplitude modulation (M-PAM)

\[ s_i(t) = a_i \sqrt{\frac{2}{T}} \cos(\omega_c t) \]

\[ s_i(t) = a_i \psi_1(t) \quad i = 1, \ldots, M \]

\[ \psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \]

\[ a_i = (2i - 1 - M) \sqrt{E_g} \]

\[ E_i = \|s_i\|^2 = E_g (2i - 1 - M)^2 \]

\[ E_s = \frac{(M^2 - 1)}{3} E_g \]

4-PAM:

- “00”
- “01”
- “11”
- “10”

Values:
- -3\(\sqrt{E_g}\)
- -\(\sqrt{E_g}\)
- 0
- \(\sqrt{E_g}\)
- 3\(\sqrt{E_g}\)
Example of bandpass modulation: Binary PAM
Coherent detection of M-PAM

One dimensional mod.,...–cont’d
Two dimensional modulation, demodulation and detection (M-PSK)

- **M-ary Phase Shift Keying (M-PSK)**

Given a signal $s_i(t) = \sqrt{2E_s/T} \cos(\omega_c t + \frac{2\pi i}{M})$

where

$$s_i(t) = a_{i1} \psi_1(t) + a_{i2} \psi_2(t) \quad i = 1, \ldots, M$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \quad \psi_2(t) = -\sqrt{\frac{2}{T}} \sin(\omega_c t)$$

$$a_{i1} = \sqrt{E_s} \cos\left(\frac{2\pi i}{M}\right) \quad a_{i2} = \sqrt{E_s} \sin\left(\frac{2\pi i}{M}\right)$$

$$E_s = E_i = \|s_i\|^2$$
Two dimensional mod.,... (MPSK)

BPSK (M=2)

8PSK (M=8)

QPSK (M=4)
Two dimensional mod.,... (MPSK)

- Coherent detection of MPSK

\[
\begin{align*}
\psi_1(t) & \quad \int_0^T z_1 \\
\psi_2(t) & \quad \int_0^T z_2 \\
\text{arctan} \frac{z_1}{z_2} & \quad \hat{\phi} \\
\text{Compute} \ |\phi_i - \hat{\phi}| & \\
\text{Choose smallest} & \quad \hat{m}
\end{align*}
\]
Two dimensional mod.,... (M-QAM)

- M-ary Quadrature Amplitude Mod. (M-QAM)

\[ s_i(t) = \sqrt{\frac{2E_i}{T}} \cos(\omega_c t + \varphi_i) \]

\[ s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \ldots, M \]

\[ \psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \quad \psi_2(t) = \sqrt{\frac{2}{T}} \sin(\omega_c t) \]

where \( a_{i1} \) and \( a_{i2} \) are PAM symbols and \( E_s = \frac{2(M - 1)}{3} \)

\[
(a_{i1}, a_{i2}) = \begin{bmatrix}
(-\sqrt{M} + 1, \sqrt{M} - 1) & (-\sqrt{M} + 3, \sqrt{M} - 1) & \cdots & (\sqrt{M} - 1, \sqrt{M} - 1) \\
(-\sqrt{M} + 1, \sqrt{M} - 3) & (-\sqrt{M} + 3, \sqrt{M} - 3) & \cdots & (\sqrt{M} - 1, \sqrt{M} - 3) \\
\vdots & \vdots & \ddots & \vdots \\
(-\sqrt{M} + 1, -\sqrt{M} + 1) & (-\sqrt{M} + 3, -\sqrt{M} + 1) & \cdots & (\sqrt{M} - 1, -\sqrt{M} + 1)
\end{bmatrix}
\]
Two dimensional mod.,... (M-QAM)

16-QAM

- Points represent different combinations of 4-bit binary sequences.
- Each point corresponds to a unique 4-bit sequence.
Two dimensional mod.,... (M-QAM)

- Coherent detection of M-QAM

\[
\int_{0}^{T} (1) t \psi
\]

\[
\int_{0}^{T} (2) t \psi
\]

ML detector

(Compare with \(\sqrt{M} - 1\) thresholds)

Parallel-to-serial converter

\(\hat{m}\)
Multi-dimensional modulation, demodulation & detection

- M-ary Frequency Shift keying (M-FSK)

\[ s_i(t) = \sqrt{\frac{2E_s}{T}} \cos(\omega_i t) = \sqrt{\frac{2E_s}{T}} \cos(\omega_c t + (i - 1)\Delta \omega t) \]

\[ \Delta f = \frac{\Delta \omega}{2\pi} = \frac{1}{2T} \]

\[ s_i(t) = \sum_{j=1}^{M} a_{ij} \psi_j(t) \quad i = 1, \ldots, M \]

\[ \psi_i(t) = \sqrt{\frac{2}{T}} \cos(\omega_i t) \quad a_{ij} = \begin{cases} \frac{\sqrt{E_s}}{M} & i = j \\ 0 & i \neq j \end{cases} \]

\[ E_s = E_i = \| s_i \|^2 \]
Multi-dimensional mod.,…(M-FSK)

$$\int_{0}^{T} r(t) \psi_{M}(t) = z_{M}$$

ML detector:
Choose the largest element in the observed vector

$$\hat{m}$$
Non-coherent detection

Non-coherent detection:

- *No need for a reference in phase* with the received carrier

- *Less complexity* compared to coherent detection at the price of *higher error rate*.
Non-coherent detection ...

- **Differential coherent detection**
  - **Differential encoding of the message**
    - The symbol phase changes if the current bit is different from the previous bit.

\[
s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \theta_i(t)), \quad 0 \leq t \leq T, \quad i = 1, \ldots, M
\]

\[
\theta_k(nT) = \theta_k((n-1)T) + \phi_i(nT)
\]

Symbol index: \( k \)

<table>
<thead>
<tr>
<th>Symbol index: ( k )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data bits: ( m_k )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Diff. encoded bits</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Symbol phase: ( \theta_k )</td>
<td>( \pi )</td>
<td>( \pi )</td>
<td>( \pi )</td>
<td>0</td>
<td>0</td>
<td>( \pi )</td>
<td>( \pi )</td>
<td>( \pi )</td>
</tr>
</tbody>
</table>
Non-coherent detection ...

- Coherent detection for diff encoded mod.
  - assumes slow variation in carrier-phase mismatch during two symbol intervals.
  - correlates the received signal with basis functions
  - uses the phase difference between the current received vector and previously estimated symbol

\[
r(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \theta_i(t) + \alpha) + n(t), \quad 0 \leq t \leq T
\]

\[
(\theta_i(nT) + \alpha) - (\theta_j((n-1)T) + \alpha) = \theta_i(nT) - \theta_j((n-1)T) = \phi_i(nT)
\]
Non-coherent detection ...

- **Optimum differentially coherent detector**
  \[ r(t) \rightarrow \psi_1(t) \rightarrow \int_0^T \rightarrow \text{Decision} \]

- **Sub-optimum differentially coherent detector**
  \[ r(t) \rightarrow \rightarrow \int_0^T \rightarrow \text{Decision} \]

- Performance degradation about 3 dB by using sub-optimal detector
Non-coherent detection ...

- **Energy detection**
  - Non-coherent detection for orthogonal signals (e.g. M-FSK)
    - Carrier-phase offset causes partial correlation between I and Q branches for each candidate signal.
    - The received energy corresponding to each candidate signal is used for detection.
Non-coherent detection of BFSK

\[
\frac{\sqrt{2}}{T} \cos(\omega_1 t) \rightarrow \int_0^T \rightarrow Z_{11} \rightarrow (\ )^2 \rightarrow Z_{11}^2 + Z_{12}^2
\]

\[
\frac{\sqrt{2}}{T} \sin(\omega_1 t) \rightarrow \int_0^T \rightarrow Z_{12} \rightarrow (\ )^2 \rightarrow Z_{11}^2 + Z_{12}^2 + Z(T)
\]

\[
\frac{\sqrt{2}}{T} \cos(\omega_2 t) \rightarrow \int_0^T \rightarrow Z_{21} \rightarrow (\ )^2 \rightarrow \hat{m} = 1 \text{ if } z(T) > 0
\]

\[
\frac{\sqrt{2}}{T} \sin(\omega_2 t) \rightarrow \int_0^T \rightarrow Z_{22} \rightarrow (\ )^2 \rightarrow \hat{m} = 0 \text{ if } z(T) < 0
\]

Decision stage:
if \( z(T) > 0 \), \( \hat{m} = 1 \)
if \( z(T) < 0 \), \( \hat{m} = 0 \)