Digital Communication I: Modulation and Coding Course

Term 3 - 2008

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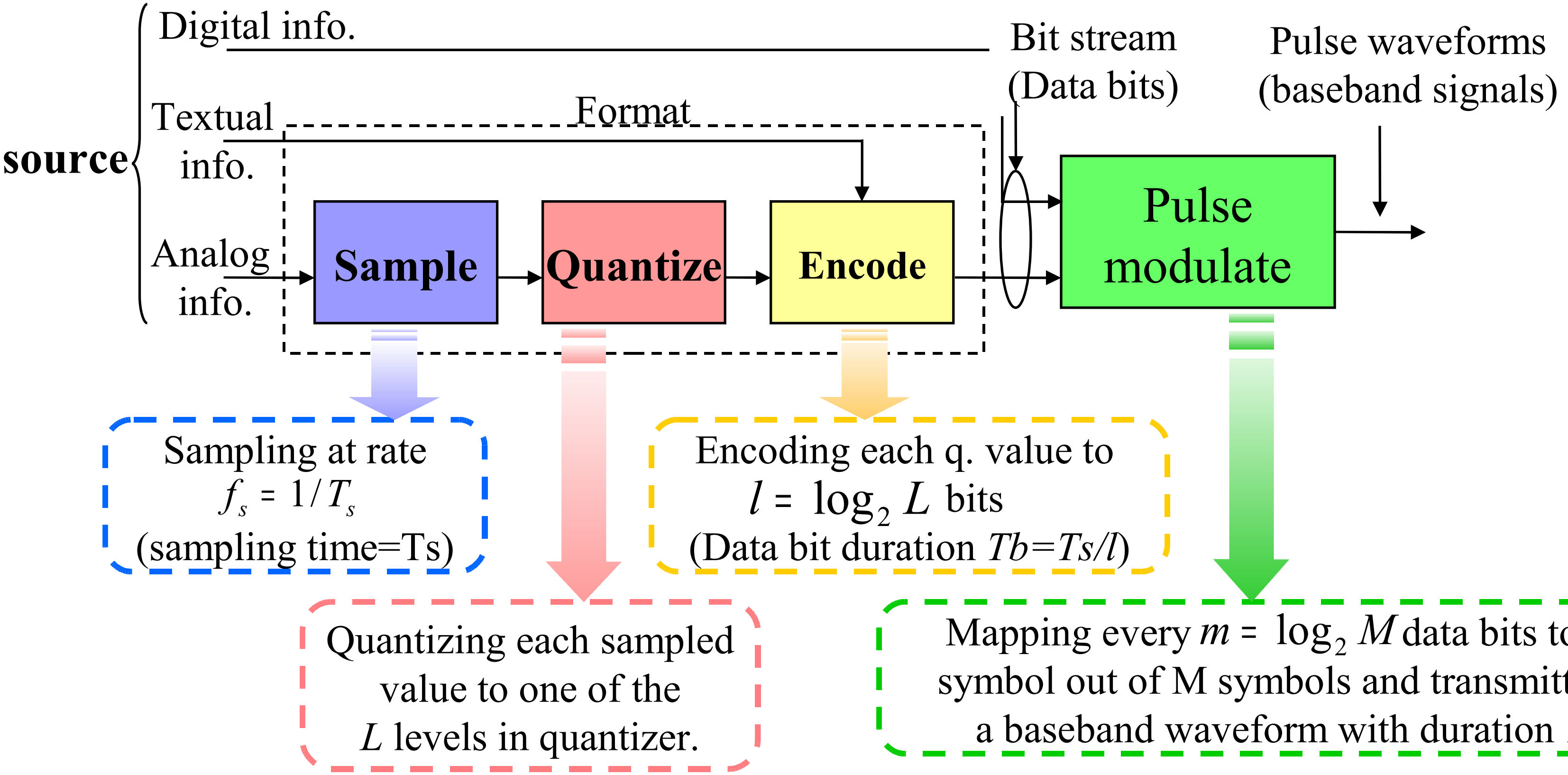
Lecture 3

Last time we talked about:

- Transforming the information source to a form compatible with a digital system
  - Sampling
    - Aliasing
  - Quantization
    - Uniform and non-uniform
  - Baseband modulation
    - Binary pulse modulation
    - M-ary pulse modulation
      - M-PAM (M-ary Pulse amplitude modulation)

Formatting and transmission of baseband signal



Sampling at rate $f_s = 1/T_s$ (sampling time=Ts)

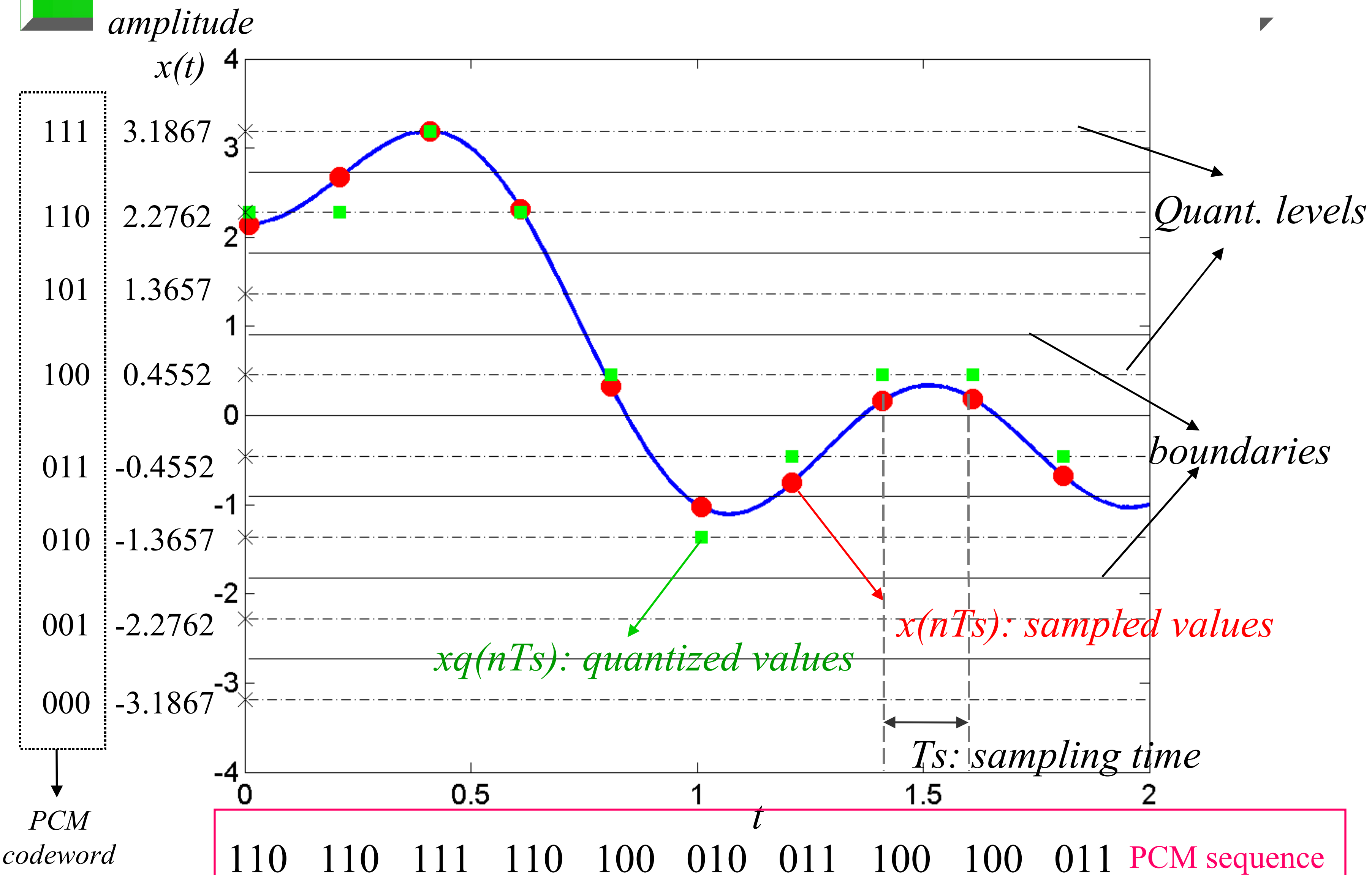
Quantizing each sampled value to one of the L levels in quantizer.

Encoding each q. value to $l = \log_2 L$ bits (Data bit duration $Tb=Ts/l$)

Mapping every $m = \log_2 M$ data bits to a symbol out of M symbols and transmitting a baseband waveform with duration T

- Information (data) rate: $R_b = 1/T_b$ [bits/sec]
- Symbol rate : $R = 1/T$ [symbols/sec]
  - For real time transmission: $R_b = mR$

Quantization example



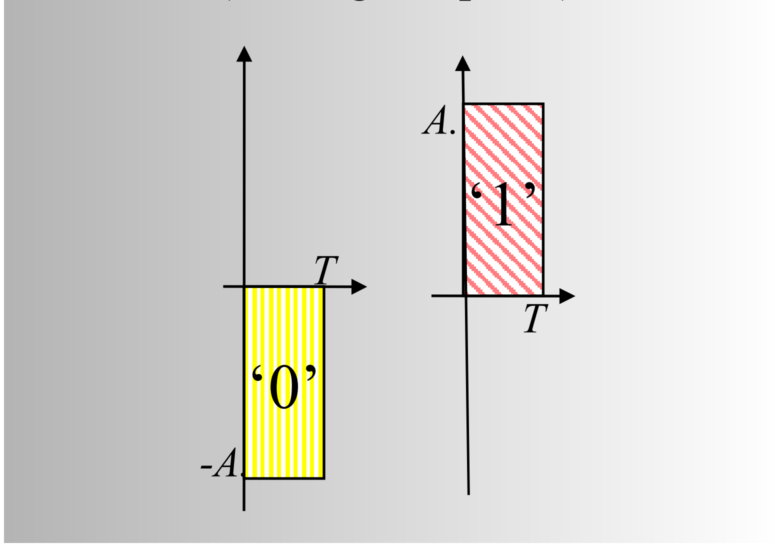
110 110 111 110 100 010 011 100 100 011 PCM sequence

Example of M-ary PAM

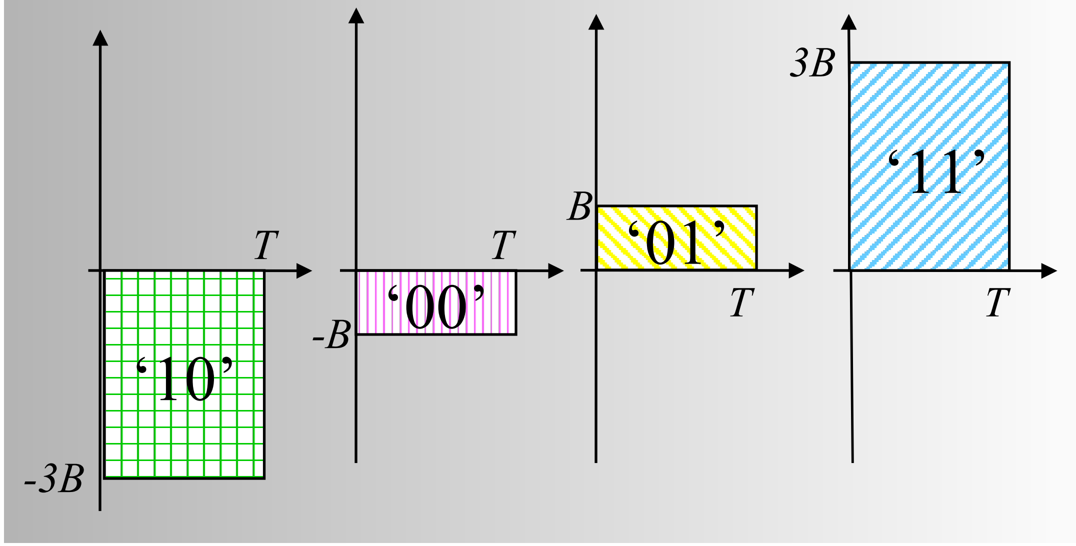
Assuming real time transmission and equal energy per transmission data bit for binary-PAM and 4-ary PAM:

- 4-ary: $T=2T_b$ and Binary: $T=T_b$
- $A^2 = 10B^2$

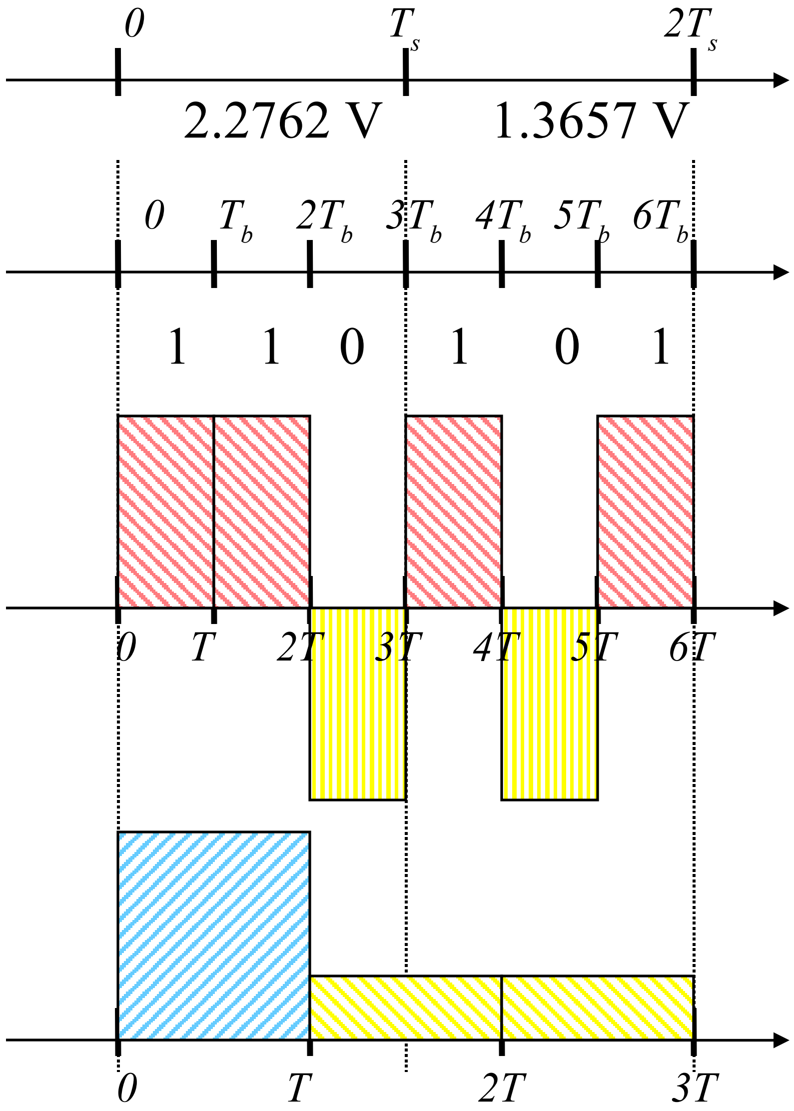
Binary PAM
(rectangular pulse)



4-ary PAM
(rectangular pulse)



Example of M-ary PAM …



$R_b=1/T_b=3/T_s$

$R=1/T=1/T_b=3/T_s$

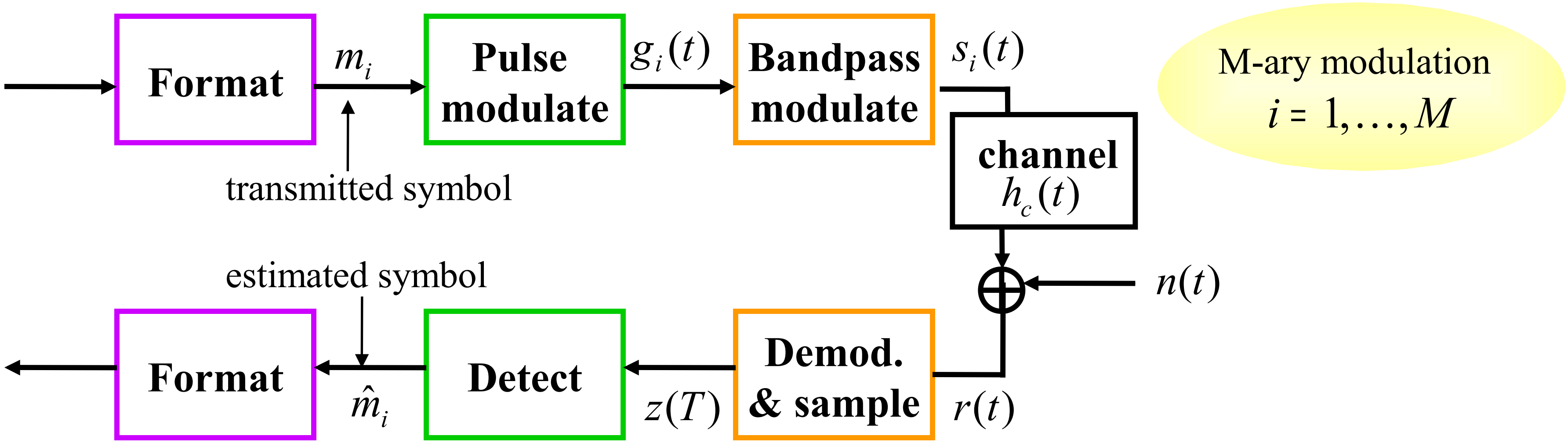
$R_b=1/T_b=3/T_s$

$R=1/T=1/2T_b=3/2T_s=1.5/T_s$

Today we are going to talk about:

- Receiver structure
  - Demodulation (and sampling)
  - Detection
- First step for designing the receiver
  - Matched filter receiver
    - Correlator receiver

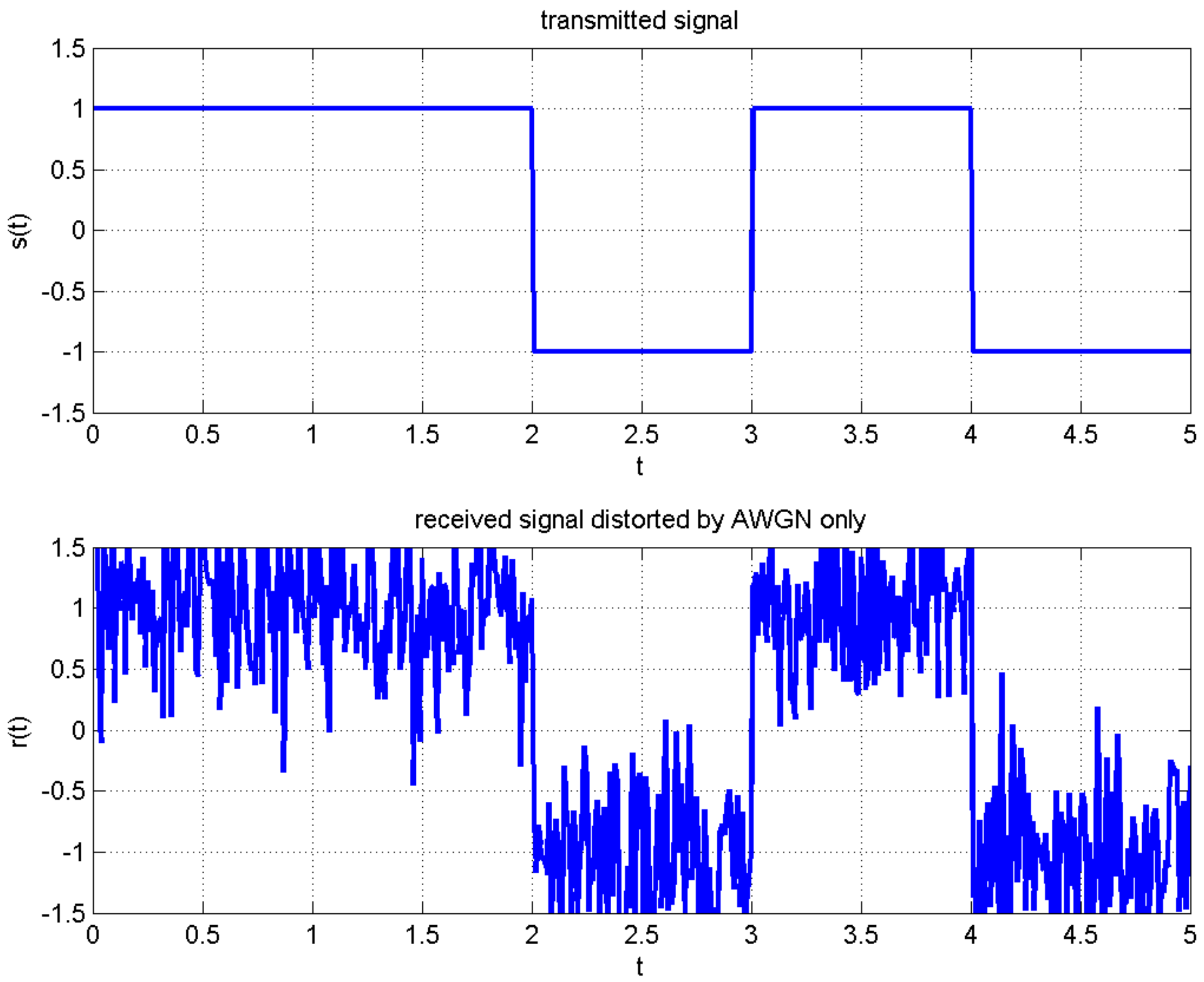
Demodulation and detection



M-ary modulation
$i = 1, \dots, M$

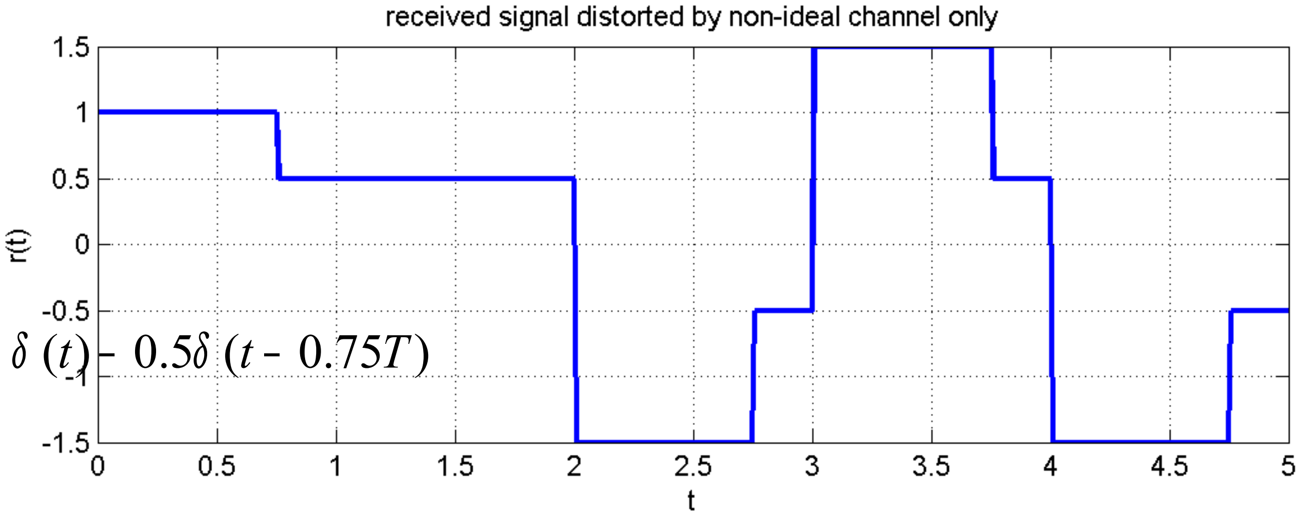
- Major sources of errors:
  - Thermal noise (AWGN)
    - disturbs the signal in an additive fashion (Additive)
    - has flat spectral density for all frequencies of interest (White)
    - is modeled by Gaussian random process (Gaussian Noise)
  - Inter-Symbol Interference (ISI)
    - Due to the filtering effect of transmitter, channel and receiver, symbols are “smeared”.

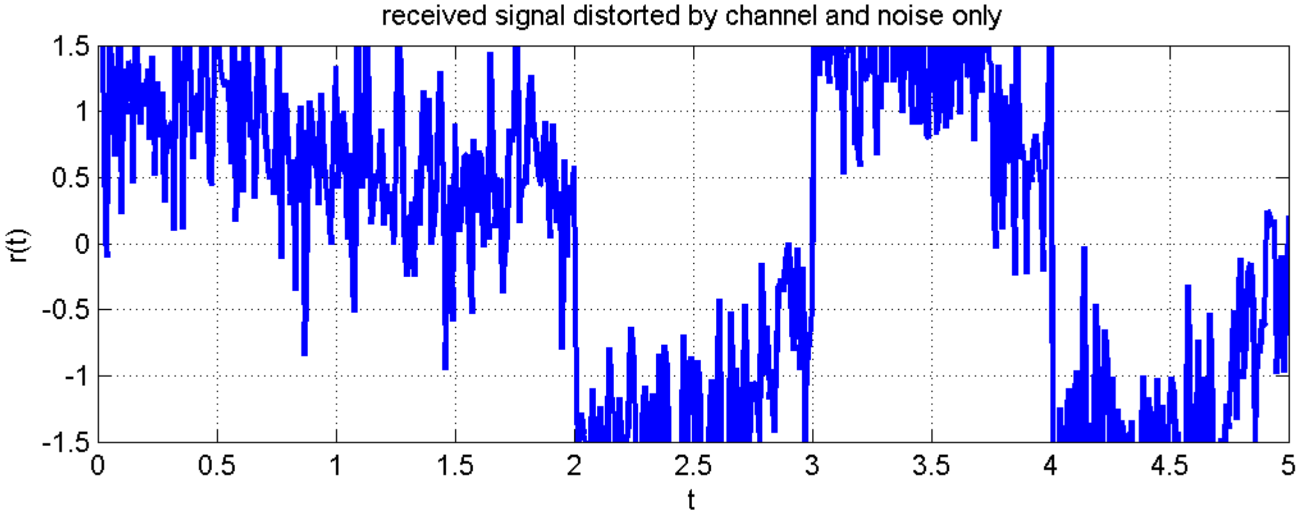
Example: Impact of the channel



Example: Channel impact …

$$h_c(t) = \delta(t) - 0.5\delta(t - 0.75T)$$

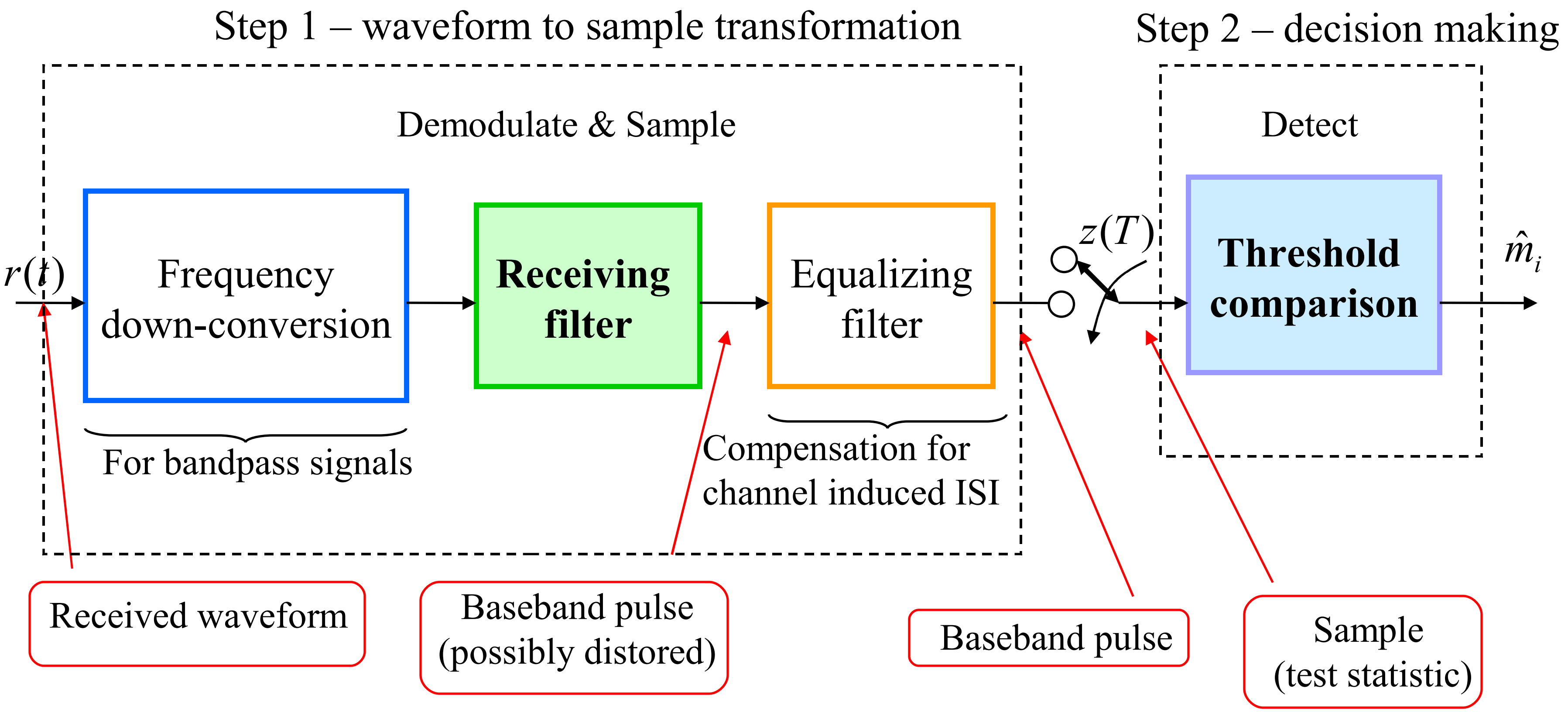

received signal distorted by non-ideal channel only


received signal distorted by channel and noise only

Receiver tasks

- Demodulation and sampling:
  - Waveform recovery and preparing the received signal for detection:
    - Improving the signal power to the noise power (SNR) using matched filter
    - Reducing ISI using equalizer
    - Sampling the recovered waveform
- Detection:
  - Estimate the transmitted symbol based on the received sample

Receiver structure



Baseband and bandpass

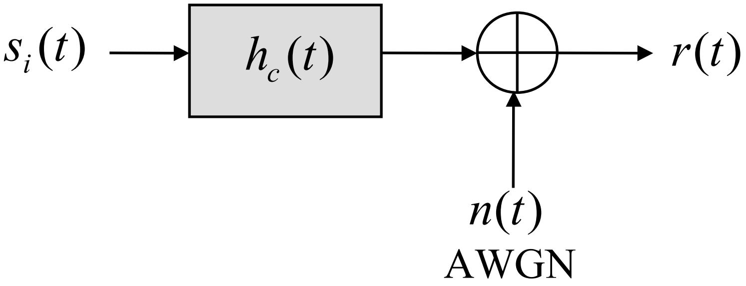
- Bandpass model of detection process is equivalent to baseband model because:
    - The received bandpass waveform is first transformed to a baseband waveform.
    - Equivalence theorem:
        - Performing bandpass linear signal processing followed by heterodyning the signal to the baseband, yields the same results as heterodyning the bandpass signal to the baseband , followed by a baseband linear signal processing.

Steps in designing the receiver

- Find optimum solution for receiver design with the following goals:
  1. Maximize SNR
  2. Minimize ISI
- Steps in design:
  - Model the received signal
  - Find separate solutions for each of the goals.
- First, we focus on designing a receiver which maximizes the SNR.

Design the receiver filter to maximize the SNR

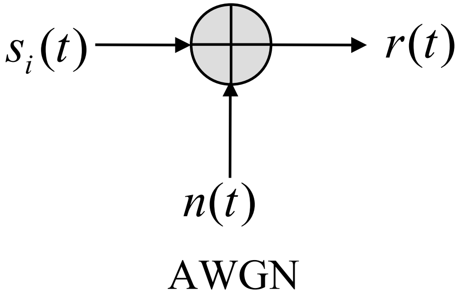
- Model the received signal



$$r(t) = s_i(t) * h_c(t) + n(t)$$

- Simplify the model:
  - Received signal in AWGN

Ideal channels
$h_c(t) = \delta(t)$



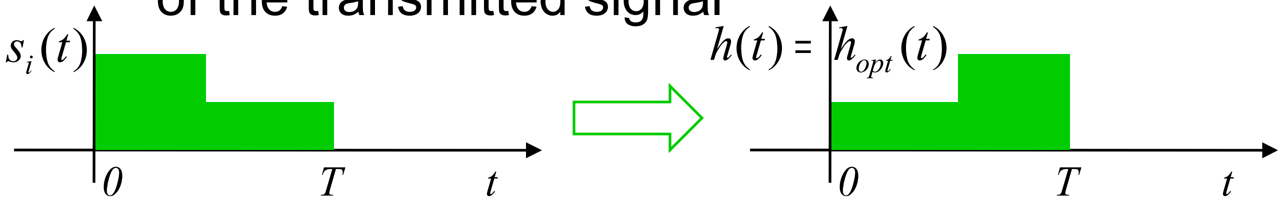
$$r(t) = s_i(t) + n(t)$$

Matched filter receiver

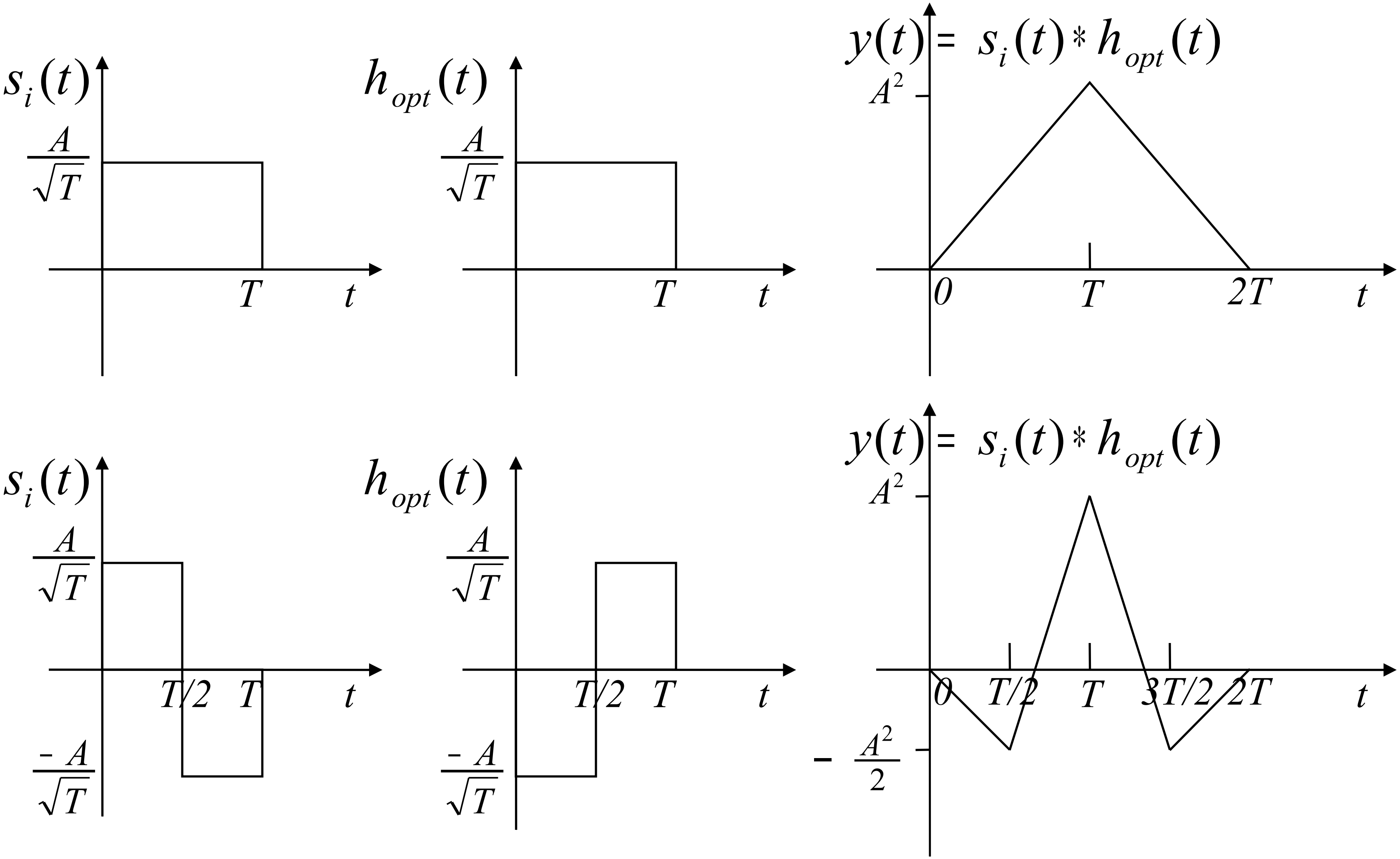
- Problem:
  - Design the receiver filter $h(t)$ such that the SNR is maximized at the sampling time when $s_i(t), i = 1,\ldots,M$ is transmitted.
- Solution:
  - The optimum filter, is the Matched filter, given by

$$\boxed{\begin{gathered} h(t) = h_{opt}(t) = s_i^*(T - t) \\ H(f) = H_{opt}(f) = S_i^*(f)\exp(-j2\pi fT) \end{gathered}}$$

which is the time-reversed and delayed version of the conjugate of the transmitted signal



Example of matched filter



Properties of the matched filter

The Fourier transform of a matched filter output with the matched signal as input is, except for a time delay factor, proportional to the ESD of the input signal.

$$\boxed{Z(f) = |S(f)|^2 \exp(-j2\pi f T)}$$

The output signal of a matched filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.

$$\boxed{z(t) = R_s(t-T) \Rightarrow z(T) = R_s(0) = E_s}$$

The output SNR of a matched filter depends only on the ratio of the signal energy to the PSD of the white noise at the filter input.

$$\boxed{\max\left(\frac{S}{N}\right)_T = \frac{E_s}{N_0/2}}$$

Two matching conditions in the matched-filtering operation:

- spectral phase matching that gives the desired output peak at time T.
- spectral amplitude matching that gives optimum SNR to the peak value.

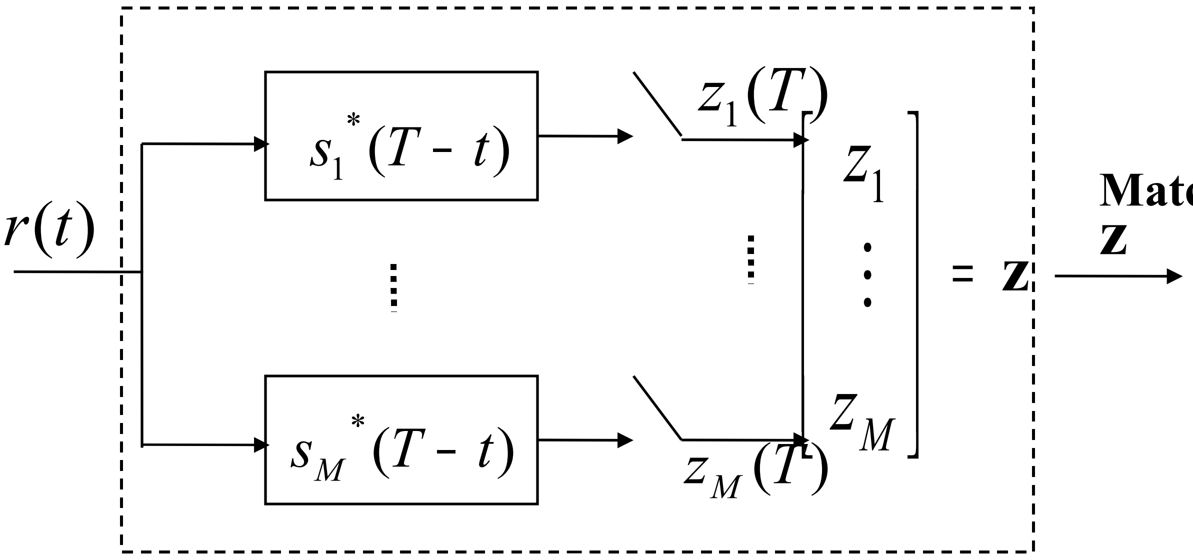
Correlator receiver

- The matched filter output at the sampling time, can be realized as the correlator output.

$$
\begin{aligned}
z(T) &= h_{opt}(T) * r(T) \\
&= \int_0^T r(\tau)s_i^*(\tau)\,d\tau = < r(t), s(t) >
\end{aligned}
$$

Implementation of matched filter receiver

Bank of M matched filters



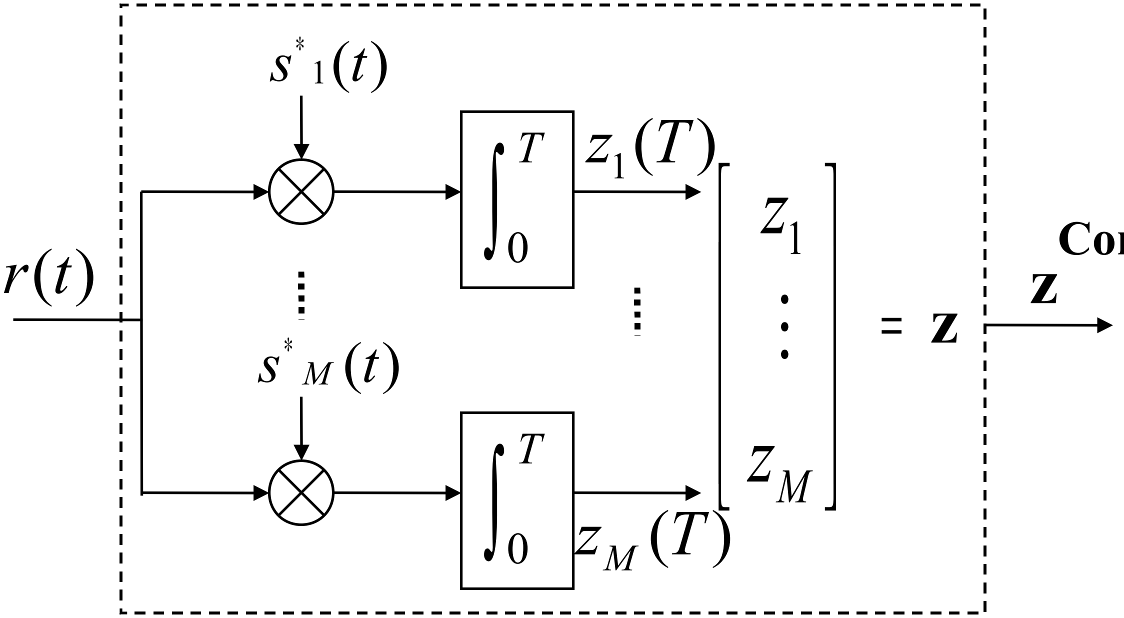
Matched filter output:
z **Observation vector**

$$z_i = r(t) * s^*_i(T-t) \quad i = 1,\ldots,M$$

$$\mathbf{z} = (z_1(T), z_2(T),\ldots,z_M(T)) = (z_1, z_2,\ldots,z_M)$$

Implementation of correlator receiver

Bank of M correlators

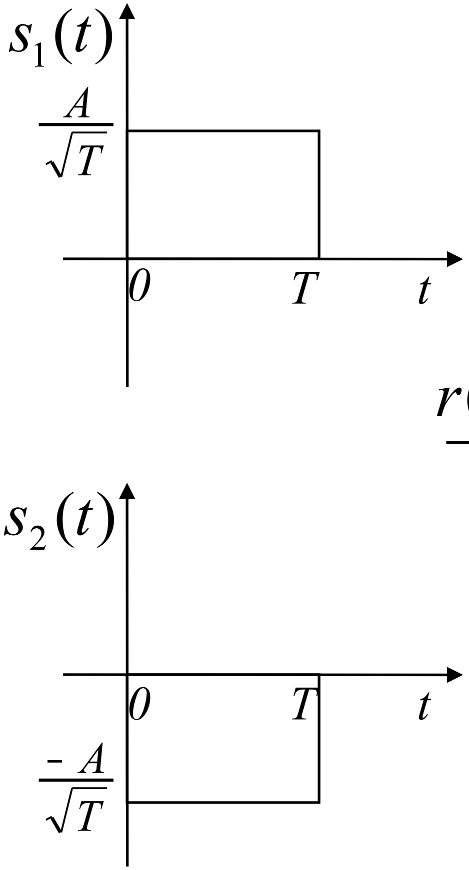


Correlators output:
Observation vector

$$\mathbf{z} = (z_1(T), z_2(T), \dots, z_M(T)) = (z_1, z_2, \dots, z_M)$$

$$z_i = \int_0^T r(t) s_i(t)\,dt \qquad i = 1,\dots,M$$

Implementation example of matched filter receivers



Bank of 2 matched filters