Digital Communications I: Modulation and Coding Course

Term 3 – 2008
Catharina Logothetis
Lecture 2
Last time, we talked about:

- Important features of digital communication systems
- Some basic concepts and definitions such as signal classification, spectral density, random process, linear systems and signal bandwidth.
Today, we are going to talk about:

- The first important step in any DCS:
  - Transforming the information source to a form compatible with a digital system
Formatting and transmission of baseband signal

source

Digital info.
Textual info.
Analog info.

Sample → Quantize → Encode

Pulse modulate

Transmit
Channel
Receive

sink

Digital info.
Textual info.
Analog info.

Low-pass filter → Decode

Demodulate/Detect

Bit stream
Pulse waveforms

Format
Format analog signals

To transform an analog waveform into a form that is compatible with a digital communication system, the following steps are taken:

1. Sampling
2. Quantization and encoding
3. Baseband transmission
Sampling

Time domain

\[ x_s(t) = x_\delta(t) \times x(t) \]

Frequency domain

\[ X_s(f) = X_\delta(f) \ast X(f) \]
Aliasing effect

\[ |X_s(f)| \]

LP filter

Nyquist rate

\[ f_s = 2f_m \]

\[ f_s < 2f_m \]

aliasing

\[ f_s > 2f_m \]
Sampling theorem

Analog signal → Sampling process → Pulse amplitude modulated (PAM) signal

- **Sampling theorem**: A bandlimited signal with no spectral components beyond $f_m$, can be uniquely determined by values sampled at uniform intervals of

$$T_s \leq \frac{1}{2f_m}$$

- The sampling rate, $f_s = \frac{1}{T_s} = 2f_m$ is called **Nyquist rate**.
Amplitude quantizing: Mapping samples of a continuous amplitude waveform to a finite set of amplitudes.

- Average quantization noise power
  \[ \sigma^2 = \frac{q^2}{12} \]
- Signal peak power
  \[ V_p^2 = \frac{L^2 q^2}{4} \]
- Signal power to average quantization noise power
  \[ \left( \frac{S}{N} \right)_q = \frac{V_p^2}{\sigma^2} = 3L^2 \]
A uniform linear quantizer is called Pulse Code Modulation (PCM).

Pulse code modulation (PCM): Encoding the quantized signals into a digital word (PCM word or codeword).
- Each quantized sample is digitally encoded into an \( l \) bits codeword where \( L \) in the number of quantization levels and

\[
l = \log_2 L
\]
Lecture 2

Quantization example

amplitude $x(t)$

$\text{quant. levels}$

$\text{boundaries}$

$\text{PCM codeword}$

$\text{PCM sequence}$

$\text{Ts: sampling time}$

$x(q\text{Ts}): \text{quantized values}$

$x(n\text{Ts}): \text{sampled values}$

111 3.1867
110 2.2762
101 1.3657
100 0.4552
011 -0.4552
010 -1.3657
001 -2.2762
000 -3.1867
Quantization error

- **Quantizing error:** The difference between the input and output of a quantizer

\[ e(t) = \hat{x}(t) - x(t) \]

**Process of quantizing noise**

**Model of quantizing noise**
Quantization error …

- **Quantizing error:**
  - **Granular or linear errors** happen for inputs within the dynamic range of quantizer
  - **Saturation errors** happen for inputs outside the dynamic range of quantizer
    - Saturation errors are larger than linear errors
    - Saturation errors can be avoided by proper tuning of AGC

- **Quantization noise variance:**

\[
\sigma_q^2 = E\{[x - q(x)]^2\} = \int_{-\infty}^{\infty} e^2(x)p(x)dx = \sigma_{Lin}^2 + \sigma_{Sat}^2
\]

\[
\sigma_{Lin}^2 = 2\sum_{l=0}^{L/2-1} \frac{q_l^2}{12} p(x_l)q_l \quad \text{Uniform } q. \quad \sigma_{Lin}^2 = \frac{q^2}{12}
\]
Uniform and non-uniform quant.

- Uniform (linear) quantizing:
  - No assumption about amplitude statistics and correlation properties of the input.
  - Not using the user-related specifications
  - Robust to small changes in input statistic by not finely tuned to a specific set of input parameters
  - Simple implementation

- Application of linear quantizer:
  - Signal processing, graphic and display applications, process control applications

- Non-uniform quantizing:
  - Using the input statistics to tune quantizer parameters
  - Larger SNR than uniform quantizing with same number of levels
  - Non-uniform intervals in the dynamic range with same quantization noise variance

- Application of non-uniform quantizer:
  - Commonly used for speech
Non-uniform quantization

- It is achieved by uniformly quantizing the "compressed" signal.
- At the receiver, an inverse compression characteristic, called "expansion" is employed to avoid signal distortion.
Statistics of speech amplitudes

- In speech, weak signals are more frequent than strong ones.

- Using equal step sizes (uniform quantizer) gives low \( \left( \frac{S}{N} \right)_q \) for weak signals and high \( \left( \frac{S}{N} \right)_q \) for strong signals.

  - Adjusting the step size of the quantizer by taking into account the speech statistics improves the SNR for the input range.
To transmit information through physical channels, PCM sequences (codewords) are transformed to pulses (waveforms).

- Each waveform carries a symbol from a set of size $M$.
- Each transmit symbol represents $k = \log_2 M$ bits of the PCM words.
- PCM waveforms (line codes) are used for binary symbols ($M=2$).
- M-ary pulse modulation are used for non-binary symbols ($M>2$).
PCM waveforms

PCM waveforms category:

- Nonreturn-to-zero (NRZ)
- Return-to-zero (RZ)
- Phase encoded
- Multilevel binary

NRZ-L
Unipolar-RZ
Bipolar-RZ
Manchester
Miller
Dicode NRZ
PCM waveforms …

- Criteria for comparing and selecting PCM waveforms:
  - Spectral characteristics (power spectral density and bandwidth efficiency)
  - Bit synchronization capability
  - Error detection capability
  - Interference and noise immunity
  - Implementation cost and complexity
Spectra of PCM waveforms

![Graph showing spectral density vs. normalized bandwidth](image)

- **Duobinary**
- **Delay modulation**
- **NRZ**
- **Dicode NRZ**
- **Bi-phase**

*WT* (normalized bandwidth, where *T* is the signal pulse width)
M-ary pulse modulation

M-ary pulse modulations category:
- M-ary pulse-amplitude modulation (PAM)
- M-ary pulse-position modulation (PPM)
- M-ary pulse-duration modulation (PDM)

- M-ary PAM is a multi-level signaling where each symbol takes one of the $M$ allowable amplitude levels, each representing $k = \log_2 M$ bits of PCM words.
- For a given data rate, M-ary PAM ($M > 2$) requires less bandwidth than binary PCM.
- For a given average pulse power, binary PCM is easier to detect than M-ary PAM ($M > 2$).
PAM example

8-ary PAM

binary PAM

Lecture 2