Tutorial No.6

Period 3 - 2006

Topic: Linear block codes

Exercise 1

Calculate the improvement in probability of message error relative to an uncoded transmission for a (24,12) double-error-correcting linear block code. Assume that coherent BPSK modulation is used and that the received $E_b/N_0 = 10$ dB (i.e here we will compare the two systems *at the same* E_b/N_0).

Exercise 2

The telephone company uses a "best-of-five" encoder for some of its digital data channels. In this system every data bit is repeated five times, and at the receiver, a majority vote decides the value of each data bit. If the uncoded probability of bit error is 10^{-3}, calculate the decoded bit-error probability when using such a best-of-five code. Here the uncoded system has excessive bit rate capacity but the error performance is to poor. We're looking for a simple technical solution to simply increase the bit error probability. Hence the E_c/N_0 in the coded case will be the same as E_b/N_0 in the uncoded case.

Exercise 3

Consider a (7,4) code whose generator matrix is

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

1. Find all the codewords of the code.
2. What is the error-correcting capability of the code?
3. What is the error-detecting capability of the code?
4. Find **H**, the parity-check matrix of the code.
5. Construct the syndrome table for the code.
6. Compute the syndrome for the received vector 1 1 0 1 1 0 1. Is this a valid vector? If not, what was the most probable sent message?

Exercise 4

A (15,5) cyclic code has a generator polynomial as follows:
$\mathbf{g}(X) = 1 + X + X^2 + X^5 + X^8 + X^{10}$.

1. Find the code polynomial (in systematic form) for the message $\mathbf{m}(X) = 1 + X^2 + X^4$.
2. Is $\mathbf{V}(X) = 1 + X^4 + X^6 + X^8 + X^{14}$ a code polynomial in this system? Justify your answer.

Exercise 5

BPSK is used to transmit data in an AWGN channel. To obtain error free transmission we use a (15,7) block code in combination with ARQ (automatic repeat request, which means retransmission until a code word is received correctly). The block code is only used for error detection and you can assume that it detects all errors. The data rate is 10 [kbps] and the average $E_b/N_0 = 5$ [dB].

1. What is the word error probability without block code and ARQ (block length = 7 bits)?
2. What is the throughput of information bits if we use the block code and ARQ?