Tutorial No.3

Period 3 - 2006

Topic: Receiver structure, signal space, detection

Exercise 1

Bipolar pulse signals, $s_i(t), (i = 1, 2)$, of amplitude ± 1 V and duration of 1 sec are received in the presence of AWGN that has a variance 0.1 V^2. Determine the optimum (minimum probability of error) detection threshold γ_0, for matched filter detection if the a priori probabilities are

1. $Pr(s_1) = 0.5$;
2. $Pr(s_1) = 0.7$;
3. $Pr(s_1) = 0.2$.
4. Explain the effect of a priori probabilities on the value of γ_0.

Exercise 2

A binary communication system transmits signals $s_i(t), (i = 1, 2)$. The receiver test statistic $z(T) = a_i + n_0$, where the signal component a_i is either $a_1 = +0.8$ or $a_2 = -0.8$ and the noise component n_0 is uniformly distributed, yielding the conditional density functions $p(z|s_i)$ given by

$$p(z|s_1) = \begin{cases} \frac{1}{2}, & \text{for } -0.2 \leq z \leq 1.8; \\ 0, & \text{otherwise.} \end{cases}$$

$$p(z|s_2) = \begin{cases} \frac{1}{2}, & \text{for } -1.8 \leq z \leq 0.2; \\ 0, & \text{otherwise.} \end{cases}$$

Find the probability of a bit error, P_B, for the case of equally likely signalling and the use of an optimum decision threshold.

Exercise 3

Consider the four waveform shown in Figure 1.

1. Determine the dimensionality of the waveforms and a set of basis functions.
2. Find the basis functions to represent the four waveforms. Determine the vectors $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4$.
3. Determine the distance between any pair of vectors. What is the minimum distance in the signal space?

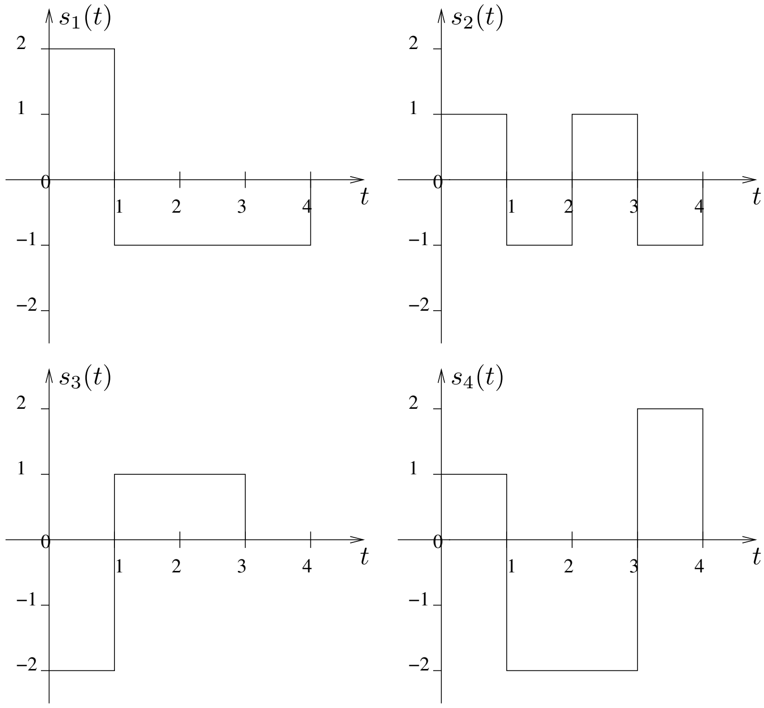

Figure 1: Baseband signals in Exercise 1

Exercise 4

Three equiprobable messages m_1, m_2 and m_3, are to be transmitted over an AWGN channel with noise power spectral density $\frac{N_0}{2}$. The messages are

$$s_1(t) = \begin{cases} 1, & 0 \leq t \leq T; \\ 0, & \text{otherwise.} \end{cases}$$

$$s_2(t) = -s_3(t) = \begin{cases} 1, & 0 \leq t \leq T/2; \\ -1, & T/2 \leq t \leq T; \\ 0, & \text{otherwise} \end{cases}$$

1. What is the dimensionality of the signal space?
2. Find an appropriate basis for the signal space (Hint: You can find the basis without using the Gram-Schmidt procedure).
3. draw the signal constellation (signal space) for this problem.
4. Derive and sketch the optimal decision regions Z_1, Z_2 and Z_3.
5. Which of the three messages is more vulnerable to errors and why? In other words, which of $\Pr(Error|m_i\ transmitted)$, for $i = 1, 2, 3$, is larger?

Exercise 5

In a binary antipodal signalling scheme, the signals are given by

$$s_1(t) = -s_2(t) = \begin{cases} \frac{2At}{T}, & 0 \le t \le T/2; \\ 2A(1 - \frac{t}{T}), & T/2 \le t \le T; \\ 0, & \text{otherwise} \end{cases}$$

The channel is AWGN and where the noise has the power spectral density $G_n(f) = \frac{N_0}{2}$. The two signals have prior probabilities p_1 and $p_2 = 1 - p_1$.

1. Determine the structure of the optimal receiver.
2. Determine an expression for the bit error probability
3. Plot the bit error probability as a function of $0 \le p_1 \le 1$.