Tutorial No.3
Period 3 - 2006

Topic: Receiver structure, signal space, detection

Exercise 1
Bipolar pulse signals, \( s_i(t), (i = 1, 2) \), of amplitude ±1 V and duration of 1 sec are received in the presence of AWGN that has a variance 0.1 V\(^2\). Determine the optimum (minimum probability of error) detection threshold \( \gamma_0 \), for matched filter detection if the a priori probabilities are

1. \( Pr(s_1) = 0.5; \)
2. \( Pr(s_1) = 0.7; \)
3. \( Pr(s_1) = 0.2. \)
4. Explain the effect of a priori probabilities on the value of \( \gamma_0 \).

Exercise 2
A binary communication system transmits signals \( s_i(t), (i = 1, 2) \). The receiver test statistic \( z(T) = a_i + n_0 \), where the signal component \( a_i \) is either \( a_1 = +0.8 \) or \( a_2 = -0.8 \) and the noise component \( n_0 \) is uniformly distributed, yielding the conditional density functions \( p(z|s_i) \) given by

\[
\begin{align*}
  p(z|s_1) & = \begin{cases} 
    \frac{1}{2}, & \text{for } -0.2 \leq z \leq 1.8; \\
    0, & \text{otherwise.}
  \end{cases} \\
  p(z|s_2) & = \begin{cases} 
    \frac{1}{2}, & \text{for } -1.8 \leq z \leq 0.2; \\
    0, & \text{otherwise.}
  \end{cases}
\end{align*}
\]

Find the probability of a bit error, \( P_B \), for the case of equally likely signalling and the use of an optimum decision threshold.

Exercise 3
Consider the four waveform shown in Figure 1.
1. Determine the dimensionality of the waveforms and a set of basis functions.

2. Find the basis functions to represent the four waveforms. Determine the vectors $s_1, s_2, s_3, s_4$.

3. Determine the distance between any pair of vectors. What is the minimum distance in the signal space?

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**Exercise 4**

Three equiprobable messages $m_1, m_2$ and $m_3$, are to be transmitted over an AWGN channel with noise power spectral density $\frac{N_0}{2}$. The messages are

$$s_1(t) = \begin{cases} 
1, & 0 \leq t \leq T; \\
0, & \text{otherwise.} 
\end{cases}$$

$$s_2(t) = -s_3(t) = \begin{cases} 
1, & 0 \leq t \leq T/2; \\
-1, & T/2 \leq t \leq T; \\
0, & \text{otherwise} 
\end{cases}$$
1. What is the dimensionality of the signal space?

2. Find an appropriate basis for the signal space (Hint: You can find the basis without using the Gram-Schmidt procedure).

3. Draw the signal constellation (signal space) for this problem.

4. Derive and sketch the optimal decision regions $Z_1$, $Z_2$ and $Z_3$.

5. Which of the three messages is more vulnerable to errors and why? In other words, which of $\Pr(\text{Error}|m_i \text{ transmitted})$, for $i = 1, 2, 3$, is larger?

**Exercise 5**

In a binary antipodal signalling scheme, the signals are given by

$$ s_1(t) = -s_2(t) = \begin{cases} \frac{2A}{T}, & 0 \leq t \leq T/2; \\ 2A(1 - \frac{t}{T}), & T/2 \leq t \leq T; \\ 0, & \text{otherwise} \end{cases} $$

The channel is AWGN and where the noise has the power spectral density $G_n(f) = \frac{N_0}{2}$. The two signals have prior probabilities $p_1$ and $p_2 = 1 - p_1$.

1. Determine the structure of the optimal receiver.

2. Determine an expression for the bit error probability

3. Plot the bit error probability as a function of $0 \leq p_1 \leq 1$. 