Tutorial No.1

Period 3 - 2006

Topic: Signals and spectra

Exercise 1

Determine which, if any, of the following functions have the properties of auto-correlation functions. Justify your answer.

1. $x(\tau) = \begin{cases} 1, & \text{for } -1 \leq \tau \leq 1; \\ 0, & \text{otherwise.} \end{cases}$
2. $x(\tau) = \delta(\tau) + \sin(2\pi f_0 \tau)$
3. $x(\tau) = \exp(|\tau|)$
4. $x(\tau) = 1 - |\tau|$ for $-1 \leq \tau \leq 1, 0$ elsewhere

Exercise 2

Determine whether these signals are energy-type or power-type. In each case, find the energy or power-spectral density and also the energy or power content of the signal.

1. $x(t) = \exp(-\alpha t)u(t) \qquad \alpha > 0$
2. $x(t) = sinc(t)$
3. $x(t) = \sum_{n=-\infty}^{+\infty} \Lambda(t - 2n)$
4. $x(t) = u(t)$

Exercise 3

The signal $Y(t)$ is generated by filtering the output of an Gaussian source with zero mean and unit variance through a filter with the impulse response given by
$h(t) = rect\left(\frac{t-10^{-3}}{10-4}\right)$.

Find the value of the bandwidth of the output signal $Y(t)$ using the following bandwidth definitions:

1. Half-power bandwidth.

2. Noise equivalent bandwidth.
3. Null-to-null bandwidth.
4. 99% of power bandwidth.
5. Bandwidth beyond which the attenuation is 35 dB.
6. Absolute bandwidth.

Exercise 4

For each of the following processes, find the spectral density.

1. $X(t) = A\cos(2\pi f_0 t+\Theta)$, where A is a constant and Θ is a random variable uniformly distributed on $[0, \frac{\pi}{4}]$.
2. $X(t) = X + Y$, where X and Y are independent, X is uniform on $[-1, 1]$ and Y is uniform on $[0, 1]$.

Exercise 5

Let $\{A_k\}_{k=-\infty}^{+\infty}$ be a sequence of random variables with $E[A_k] = m$ and $E[A_k A_j] = R_A(k-j)$. We further assume that $R_A(k-j) = R_A(j-k)$. Let $p(t)$ be any deterministic signal whose Fourier transform is $P(f)$, and define the random process
$X(t) = \sum_{k=-\infty}^{+\infty} A_k p(t - kT)$
where T is a constant.

1. Find $m_X(t)$.
2. Find $R_X(t + \tau, t)$.
3. Show that the process is cyclostationary with period T.
4. Show that
   $\bar{R}_X(\tau) = \frac{1}{T}\int_0^T R_X(t+\tau, t)dt = \frac{1}{T}\sum_{-\infty}^{+\infty} R_A(n)R_p(\tau - nT)$
   where $R_p(\tau) = p(\tau)\star p(-\tau)$ is the (deterministic) autocorrelation function of $p(t)$.
5. Show that the power spectral density of $X(t)$ is given by
   $G_X(f) = \frac{|P(f)|^2}{T}\left[R_A(0) + 2\sum_{k=1}^{\infty} R_A(k)\cos(2\pi k f T)\right]$.