Exam in

1TT850, 1E275

Modulation, Demodulation and Coding course

EI, TF, IT programs

16th of August 2004, 14:00-19:00

Signals and systems, Uppsala university

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Material allowed is

- Any calculator (no computer)
- Mathematics handbook
- Swedish-English dictionary

Grading There are 5 sets of questions, each with 12 points. Total number of points is 60. The grade of the exam is according to the table below.

Total score	0-29	30-39	40-49	50-60
Grade	Fail	3	4	5

Results will be posted on the course homepage on Monday 23rd of August 2004. The exam review is on Wednesday, 30th of August, 2004, at 13:00-14:00, at the library on 2nd floor of Magistern.

Good luck!

Question 1

Assume the low-pass wide-sense stationary signal $X(t)$ which has the probability density function (pdf) given by

$$f_X(x) = \begin{cases} \frac{1}{4}(x+2), & -2 \leq x < 0; \\ -\frac{1}{4}(x-2), & 0 < x \leq 2; \\ 0, & \text{otherwise.} \end{cases}$$

and a non-zero power spectral density limited on the range of [-2500, 2500] Hertz.

The signal is sampled at the Nyquist rate and quantized by a uniform linear quantizer with 32 levels which has a dynamic amplitude range of ± 1.5 [Volts]. The bits are then modulated by an 8-ary PAM (8-ary pulse amplitude modulation) scheme using square-root raised cosine (SRRC) pulse shaping with roll-off factor $\alpha = 0.2$. Assume real-time and base-band transmission. The transmitted waveforms experience a base-band channel with a bandwidth of 40 [kHz].

Answer the following questions:

A. Calculate the signal-to-quantization noise ratio, denoted by $\left(\frac{S}{N}\right)_q$, in [dB]. (6p)

B. Calculate information rate and transmission symbol rate.(4p)

C. Calculate the bandwidth efficiency, given by $\eta = R_b/W_t$ [bps/Hz], where R_b is the information bit rate and W_t is the transmission bandwidth. (2p)

Question 2

In a binary communication system, information is transmitted using antipodal signaling. The prior probability for the bits are 1/3 and 2/3. The received signal is given by

$$r(t) = s(t) + n(t)$$

where $s(t)$ is the transmitted signal given by

$$s(t) = \pm A\psi(t)$$

where

$$\psi(t) = \begin{cases} \frac{1}{\sqrt{T}}, & 0 \leq t < T/2; \\ -\frac{1}{\sqrt{T}}, & T/2 \leq t \leq T; \\ 0, & \text{otherwise.} \end{cases}$$

where T is the duration of the transmitted symbol in time. Moreover, A is the signal amplitude in [Volt] and $n(t)$ is the AWGN with zero-mean and a double-sided power spectral density of $N_0/2$ [W/Hz].
Answer the following questions:

A. Plot the impulse response of the matched filter. Label and determine the values on all the axes. (2p)

B. Plot the output of the matched filter where its input is $s(t)$. Label and determine the values on all the axes.(2p)

C. Determine the threshold of the maximum-likelihood detector and calculate the bit error probability. (4p)

D. Determine the threshold of the optimum detector and calculate the bit error probability. (4p)

Question 3

In a real-time communication system, a linear block code with the generator matrix

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

and 4-ary pulse amplitude modulation (PAM) with square root raised cosine (SRRC) pulse shaping and the roll-off factor $\alpha = 0.2$ are implemented. Answer the following questions:

A. Assuming base-band transmission, what is the value of the bandwidth efficiency that this system provides? (3p)

B. What is the minimum distance of the code and the asymptotic coding gain? What are the error correction and error detection capabilities of the code? (4p)

C. For a given transmitted message, the received sequence at the detector output is found to be $\mathbf{r} = (0, 0, 1, 1, 1, 0)$. What is the estimated transmitted message? (5p)

Question 4

You are required to design a real-time communication system operating over an AWGN channel with an available bandwidth of 2400 [Hz], by choosing a modulation scheme and possibly an error-correction coding scheme. The available E_b/N_0 is 14 [dB]. The required data rate and the bit error probability are 9600 [bits/s] and 10^{-5}, respectively. For design, it is allowed to choose of one the two modulation types - either non-coherent orthogonal 8-FSK or 16-QAM with matched filter detection. It is also allowed to choose one of the two codes - either the (127,92) BCH code or a rate $\frac{1}{2}$ convolutional code that provides 5 [dB] of coding gain at a bit error probability of 10^{-5}. Assuming ideal filtering, verify that your choices achieve the desired band-width and error performance requirements. (12p)

Question 5

A 2-tap linear predictor is used in a DPCM system. The predicted signal sample is given by

$$\hat{x}(n) = a_1 x(n-1) + a_2(n-2)$$

where a_1 and a_2 are the predictor filter taps and $x(n)$ is the sample of the signal at the time index n. Assume that the sequence

$$\{x(n)\}_{n=0}^{7} = \{-3, -2, 0, -1, 2, 2, 1, 2\}$$

is used to determine the filter taps.
Answer the following questions:

A. Find the optimum filter coefficients such that the prediction mean square error variance is minimized. (4p)

B. Using the predictor in part [A], predict the signal sample at at time indices 8 and 9. (4p)

C. What is the SNR, the signal power to prediction error variance ratio in [dB] for the predictor in part [A]? (4p)

Hint: The auto-correlation sequence of a random process from a finite number of samples, for example $x(n)$ for $n = 0, \cdots, N-1$, can be estimated using the sample auto-correlation

$$\hat{R}_x(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) x^*(n-k)$$

where * is the conjugate operator.