Tut 9, Ex 1

We require sending 9600 bps at $P_b = 10^{-5}$. Depending on what modulation format and coding scheme we choose to use, we will require a certain bandwidth and achieve a certain BER.

Generally, we may say that we use an (n,k) code. The information bit rate R_b relates to the coded bit rate R_c as

$$n R_b = k R_c$$

If we assume Nyquist signalling, then we can transmit 1 symbol/s/Hz. The required bandwidth for 16QAM is then $R_c/4$.

For noncoherent 8FSK, each band is $R_c/3$ wide, but we need 8 bands (and the separation is $1/T$) so the total bandwidth is $\frac{8}{3}R_c$. The required bandwidths for the different modulator/code combinations are therefore

	uncoded	(127,92) BCH	rate $\frac{1}{2}$ conv. code
16QAM	2400 Hz	3313 Hz	4800 Hz
8FSK	25600 Hz	35339 Hz	51200 Hz

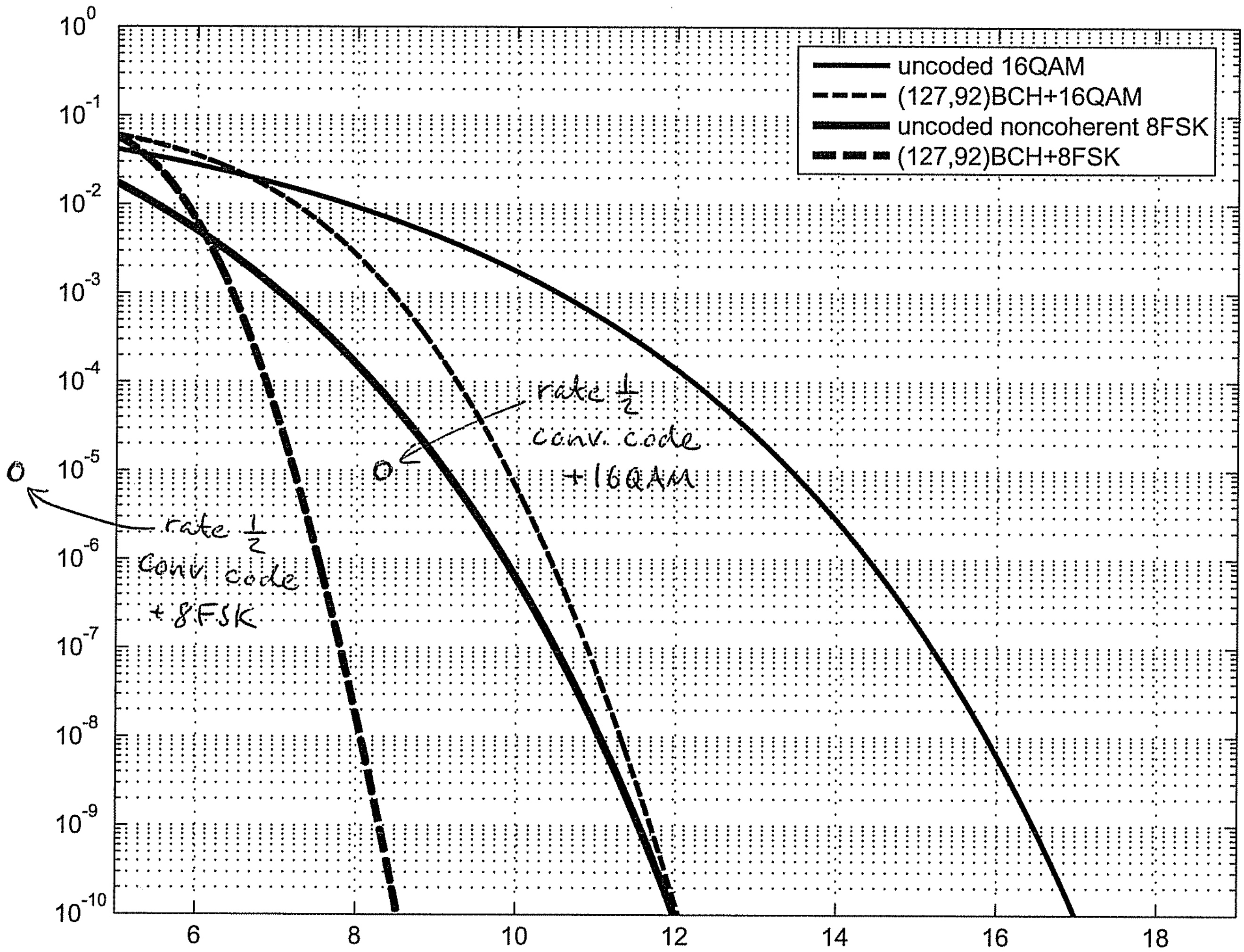
What about P_B? For the coded bits we have

16QAM : $$P_c \approx \frac{3}{4} Q\left(\sqrt{\frac{4}{5}\frac{\mathcal{E}_c}{N_0}}\right) = \frac{3}{4} Q\left(\sqrt{\frac{4}{5}\frac{k}{n}\frac{\mathcal{E}_b}{N_0}}\right)$$ [Sklar 9.54]

8FSK : $$P_c < \frac{M/2}{M-1} \cdot \frac{M-1}{2} \exp\left(-\frac{\mathcal{E}_s}{2N_0}\right) = 2\exp\left(-\frac{3}{2}\frac{\mathcal{E}_c}{N_0}\right) = 2\exp\left(-\frac{3}{2}\frac{k}{n}\frac{\mathcal{E}_b}{N_0}\right)$$ [Sklar 4.111/112]

Now we must find out how the codes improve the BER. From Sklar Table 6.4 we see that (127,92) BCH has $t = 5$. Now we can calculate the approximate P_B: $$P_B \approx \frac{1}{n}\sum_{j=t+1}^{n} j\binom{n}{j} P_c^j (1-P_c)^{n-j}$$ [Sklar 6.46]

For the convolutional code, we know that it achieves $P_B = 10^{-5}$ 5 dB below the uncoded cases. See the plot.



uncoded 16QAM
(127,92)BCH+16QAM
uncoded noncoherent 8FSK
(127,92)BCH+8FSK
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
6
8
10
12
14
16
18
rate $\frac{1}{2}$ conv. code +16QAM
O
rate $\frac{1}{2}$ conv. code +8FSK
O

1.

Only uncoded 16QAM meets the requirements.

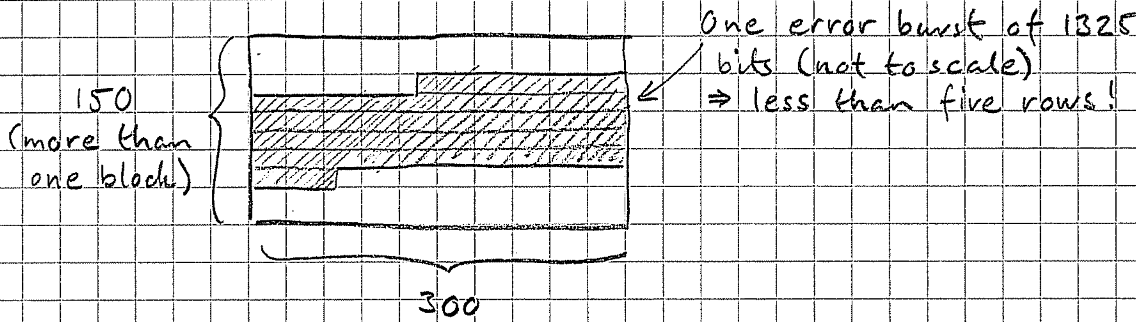
2.

Only (127,92) BCH + 8FSK meets the requirements.

3.

Only (127,92) BCH + 16QAM meets the requirements.

But what about the interleaving? We send $\frac{127}{92} \cdot 9600 = 13252$ coded bits per second. This means that 1325 bits are affected by a 100 ms burst. A 16x32 interleaver won't even hold one such burst so that won't do. We test the 150x300 interleaver:



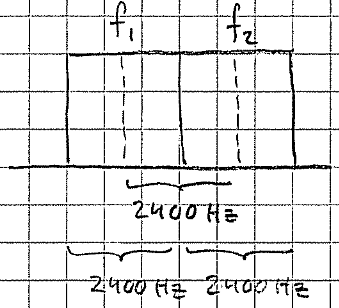
Since the columns can hold a block and the bursts cover less than five rows, the 150x300 will suffice (the code can correct five errors and we assume that we get at most one burst per interleaver block)

Tut 9 Ex 2

We need to transmit 2400 coded bits per second.

Noncoherent FSK $\Rightarrow$ frequency separation needs to be $1/T$

If we assume ideal Nyquist sampling, then each band is $1/T$



$\Rightarrow$ Total BW is 4800 Hz

$$\left.\begin{aligned} \mathcal{E}_b &= 2 \cdot 10^{-6}\ \text{W} / 1200\ \text{bps} = 1.67 \cdot 10^{-9}\ \text{J} \\ \tfrac{1}{2} N_0 &= 10^{-10}\ \text{W/Hz} = 10^{-10}\ \text{J} \end{aligned}\right\} \frac{\mathcal{E}_b}{N_0} = 8.33 \quad (= 9.2\ \text{dB})$$

Error probability for the coded bits:

$$P_c < \frac{M/2}{M-1} \cdot \frac{M-1}{2} \exp\left(-\frac{\mathcal{E}_s}{2N_0}\right) = \frac{1}{2} \exp\left(-\frac{1}{2} \frac{k}{n} \frac{\mathcal{E}_b}{N_0}\right)$$

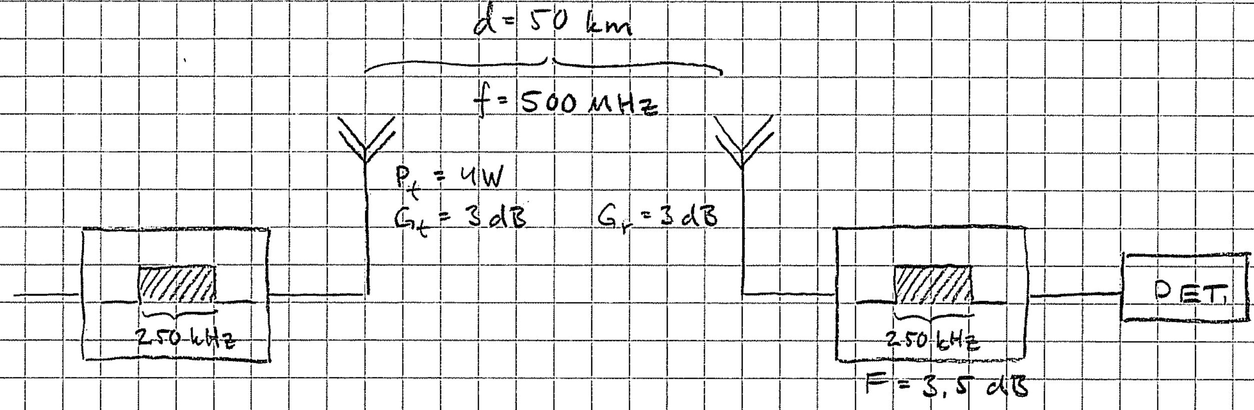
Here $k/n = 1/2 \Rightarrow P_c < \frac{1}{2} \exp\left(-\frac{1}{4} \frac{\mathcal{E}_b}{N_0}\right) = 6.2 \cdot 10^{-2}$

We know that the optimal rate $\frac{1}{2}$ with constraint length 5 has $d_{free} = 7$ giving rise to one decoded error

$$\Rightarrow T(D,N) = X_{a'}/X_a \approx D^7 N$$

$$P_B \leq dT(D,N)/dN \Big|_{\substack{N=1 \\ D = 2\sqrt{P_c(1-P_c)}}} = 128\sqrt{P_c^7(1-P_c)^7} = 6.2 \cdot 10^{-3}$$

Tut 9. Ex 3



Assume ideal Nyquist signalling $\Rightarrow$ we transmit 250 ksymbols/s and the filters are rectangular.

$f = 500$ MHz $\Rightarrow$ the wavelength $\lambda = c_0/f = 0.6$ m

The receiver antenna has the effective area $A_{er} = \frac{G_r \lambda^2}{4\pi}$

The received power is hence

$$P_r = \underbrace{\frac{P_t G_t}{4\pi d^2}}_{\substack{\text{area of a sphere} \\ \text{of radius } d}} A_{er} = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2}$$

This gives us $\mathcal{E}_b$ before the receiver amplifier:

$$\mathcal{E}_b = P_r / 250\,000\ s^{-1} = 0.58 \cdot 10^{-16}\ J$$

N_0 before the receiver amplifier is simple: $N_0 = kT^\circ$

$k = 1.38 \cdot 10^{-23}$ J/K and $T^\circ = 290$ K $\Rightarrow N_0 = 0.40 \cdot 10^{-20}$ W/Hz

$\Rightarrow \mathcal{E}_b/N_0$ (before amplifier) $= 41.6$ dB

$\Rightarrow \mathcal{E}_b/N_0$ (at the detector) $= 41.6$ dB $- F = 38.1$ dB

BPSK is used, so clearly $P_B = 0$ at 38 dB!

The noise factor or the noise figure describes the SNR degradation over an amplifier: $F = \frac{(SNR)_{in}}{(SNR)_{out}}$. Its value depends on how much noise the amplifier adds (how "hot" it is). Read Sklar 5.5.

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Tut 9. Ex 4.

It is convenient to do the calculations in dB.

We saw in Ex 3 that

$$\mathcal{E}_b = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2} T_s$$ except for extra loss (here 50 dB) and the noise figure (here 5 dB)

Counting in dB we then have

$$(\mathcal{E}_b)_{dB} = (P_t)_{dB} + 5 + 2 + 20\log\frac{\lambda}{4\pi d} - 10\log(R_b) - 50 - 5 = (P_t)_{dB} - 210.6 \text{ dBJ}$$

$$(N_0)_{dB} = 10\log(kT^0) = -204 \text{ dBJ}$$

Here, $\lambda = 1/3$ m, $d = 8000$ m, $R_b = 200000$ bits/s (assume ideal Nyquist signalling), $T^0 = 290$ K

In order to have $P_B = 0.02$, we plot $Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$ and find $\mathcal{E}_b/N_0 = 3.25$ dB

$$\Rightarrow \quad (P_t)_{dB} - 210.6 + 204 = 3.25 \quad \Rightarrow \quad (P_t)_{dB} = 9.85 \text{ dBW}$$