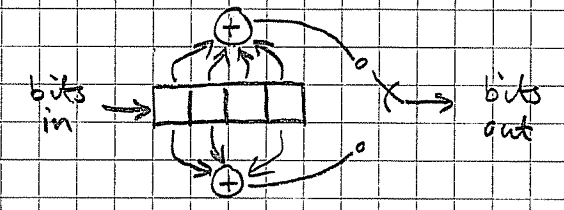
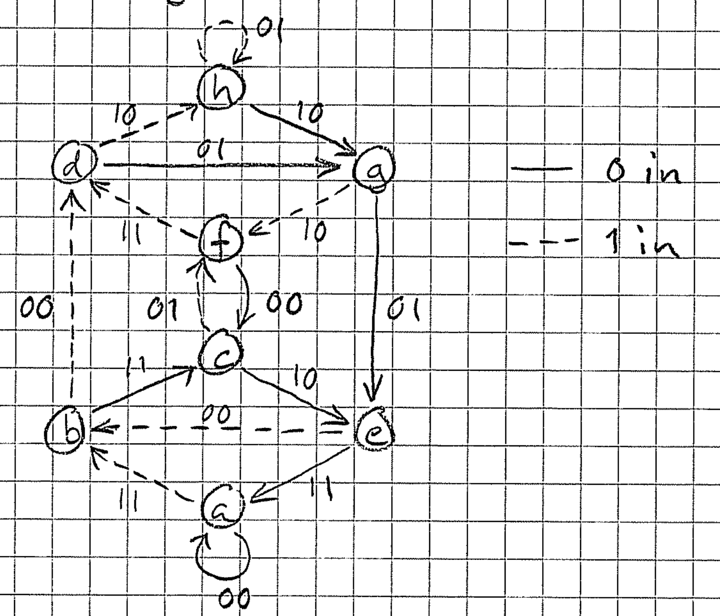
Tut 8, Ex 1

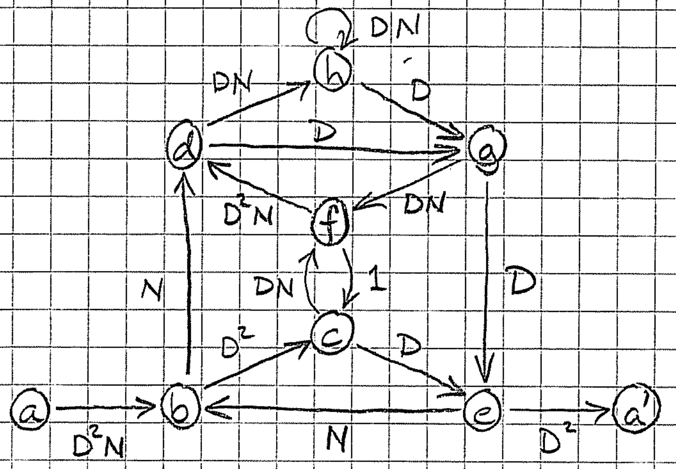


State diagram



States:

$a = \{0\ 0\ 0\}$
$b = \{1\ 0\ 0\}$
$c = \{0\ 1\ 0\}$
$d = \{1\ 1\ 0\}$
$e = \{0\ 0\ 1\}$
$f = \{1\ 0\ 1\}$
$g = \{0\ 1\ 1\}$
$h = \{1\ 1\ 1\}$



D: number of bit errors required in order for the receiver to think that a transition was the correct one.

N: number of generated bit errors

To calculate the decoded bit error probability we need the transfer function $T(D,N) = X_{a'}/X_a$. We saw in a previous exercise that this can be rather cumbersome to calculate when we have four states. Now we have eight states. However, the shortest path will dominate the behaviour so we only need to find that. Note that we needed only D when we looked for the free distance d_{free}. To find the bit error performance we need also N.

Shortest path: $a \to b \to d \to g \to e \to a' \Rightarrow T(D,N) \approx D^6N^2$

$$P_B \leq dT(D,N)/dN\Big|_{\substack{N=1\\ D=2\sqrt{p(1-p)}}} \approx 2D^6N\Big|_{\substack{N=1\\ D=2\sqrt{p(1-p)}}} = 127\,p^3(1-p)^3$$

$p = 10^{-3} \Rightarrow P_B \leq 1{,}3 \cdot 10^{-7}$

$d_{free} = 6$. Input: 10111 $\Rightarrow$ output: 11 11 01 11 10 (assume starting in state a and follow the state diagram)

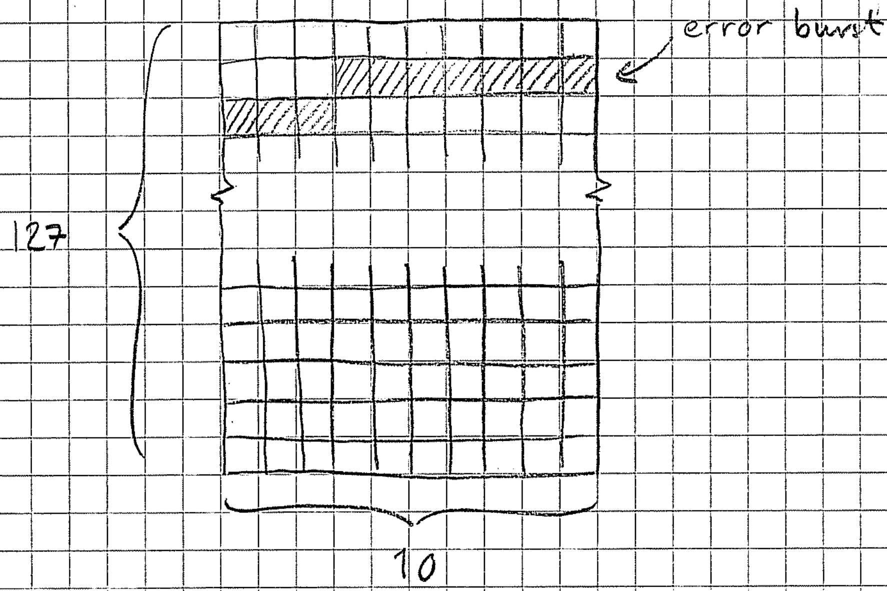
Tut 8. Ex 3.

The bursts last 2 ms each, corresponding to 10 symbols / coded bits.

Interleaving: Fill the columns, send the rows (do the opposite at the receiver).

Assume that the bursts occur just once per interleaver block.

We want to spread the bits so that we at most get one error in a column ⇒ use 10 columns.



We need to fill the entire block (practically) before we can start sending, and then we need to do the same at the receiver.

$$\text{Delay} = 2 \cdot 127 \cdot 10 \cdot 0.2 \text{ ms} = 0.508 \text{ s}$$

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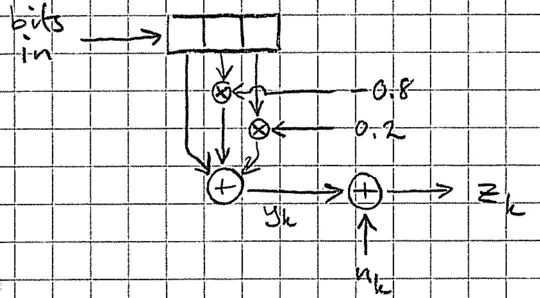
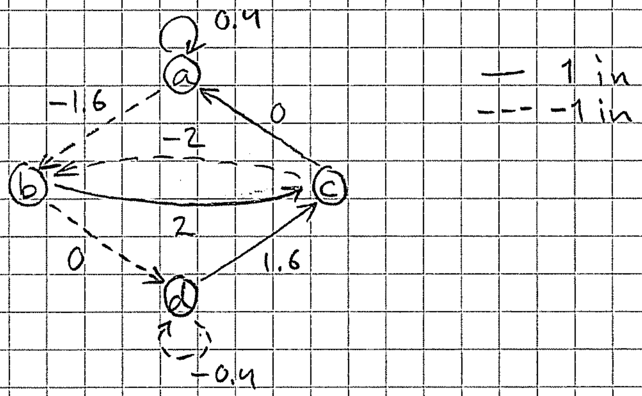
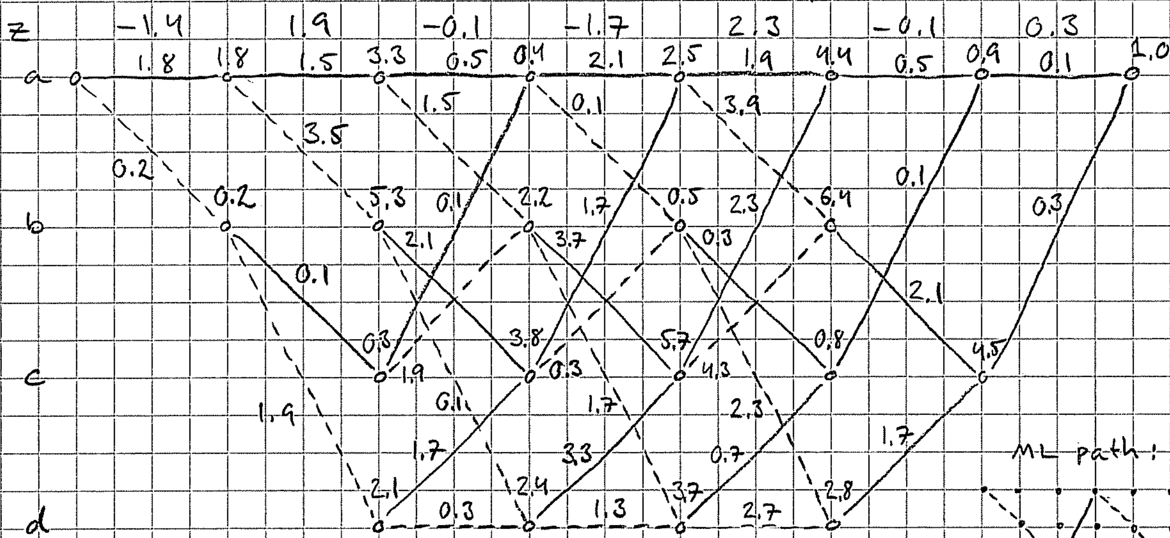
Tut 8 Ex 2

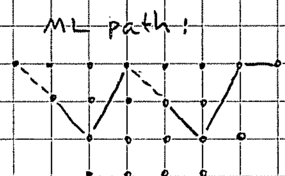
Here we'll use a state machine and the Viterbi algorithm to combat ISI instead of decoding a convolutional code.



states: $a = \{1\ 1\}$
$b = \{-1\ 1\}$
$c = \{1\ -1\}$
$d = \{-1\ -1\}$







$\hat{m}_{ML} =$ -1 1 1 -1 1 1 1

$\hat{m}_{\text{symbol by symbol}} =$ -1 1 $\underline{\underline{-1}}$ -1 1 $\underline{\underline{-1}}$ 1

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