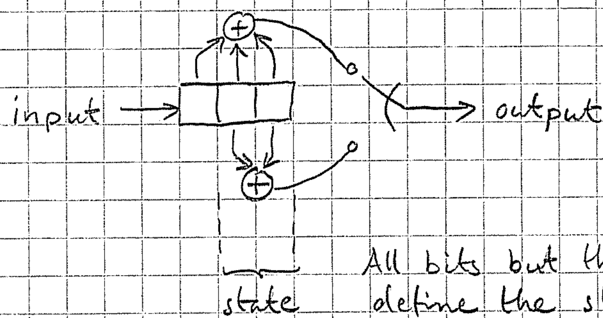
Tut 7, Ex 1

$g_1 = (1, 1, 1)$

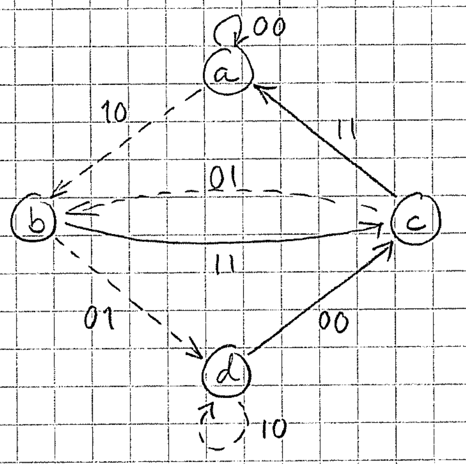
$g_2 = (0, 1, 1)$



All bits but the leftmost define the state

states : $a = \{0, 0\}$ $b = \{1\ 0\}$ $c = \{0\ 1\}$ $d = \{1\ 1\}$

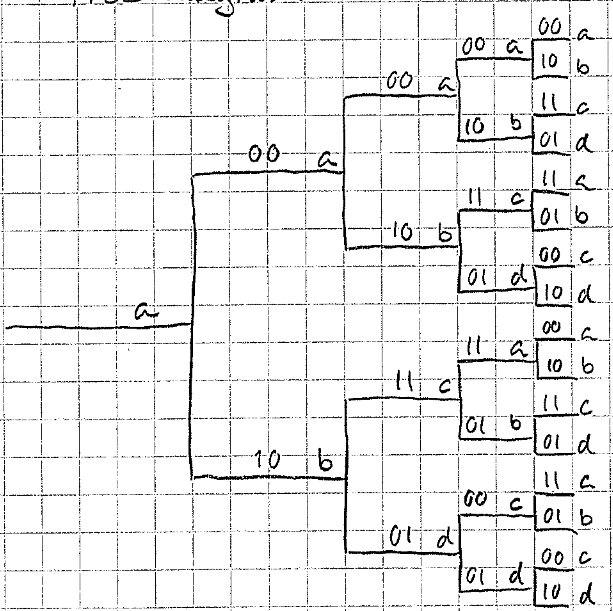
state diagram :



— info bit is 0

- - - info bit is 1

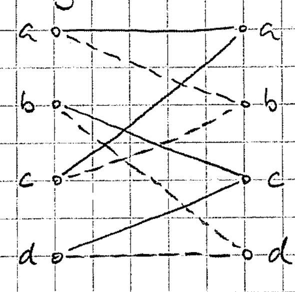
Tree diagram



↑ info bit is 0

↓ info bit is 1

Trellis diagram



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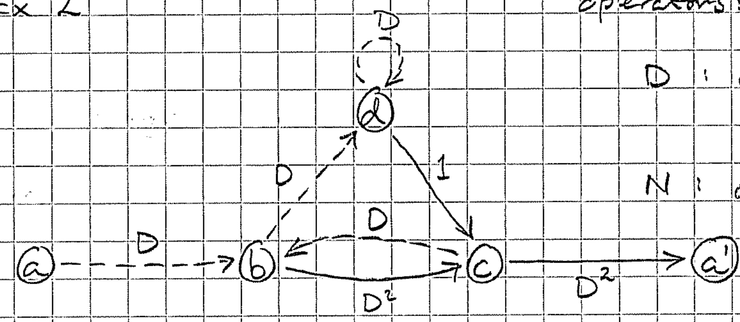
Tut 7, Ex 2



operators:

D : one biterror on the transmission

N : one decoded error (not needed here, therefore ignored)

Take the state diagram and split the all-zero state in two.

Assume the all-zero sequence is the correct one.

Hence,

$$\left.\begin{aligned} X_b &= DX_a + DX_c \\ X_c &= D^2X_b + X_d \\ X_d &= DX_b + DX_d \\ X_{a'} &= D^2X_c \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} X_b - X_c &= DX_a \\ -D^2X_b + X_c - X_d &= 0 \\ -DX_b + (1-D)X_d &= 0 \\ -D^2X_c + X_{a'} &= 0 \end{aligned}\right.$$

Solve for $X_{a'}$:

$$\begin{bmatrix} 1 & -D & 0 & 0 \\ -D^2 & 1 & -1 & 0 \\ -D & 0 & 1-D & 0 \\ 0 & -D^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_b \\ X_c \\ X_d \\ X_{a'} \end{bmatrix} = X_a \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad \left[\begin{array}{cccc|c} 1 & -D & 0 & 0 & D \\ -D^2 & 1 & -1 & 0 & 0 \\ -D & 0 & 1-D & 0 & 0 \\ 0 & -D^2 & 0 & 1 & 0 \end{array}\right] \begin{matrix} D^2 \; D \\ \\ \\ \\ \end{matrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -D & 0 & 0 & D \\ 0 & 1-D^3 & -1 & 0 & D^3 \\ 0 & -D^2 & 1-D & 0 & D^2 \\ 0 & -D^2 & 0 & 1 & 0 \end{array}\right] \begin{matrix} \\ \frac{D^2}{1-D^3} \\ \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -D & 0 & 0 & D \\ 0 & 1-D^3 & -1 & 0 & D^3 \\ 0 & 0 & 1-D-\frac{D^2}{1-D^3} & 0 & D^2+\frac{D^5}{1-D^3} \\ 0 & 0 & -\frac{D^2}{1-D^3} & 1 & \frac{D^5}{1-D^3} \end{array}\right]$$

$$1-D-\frac{D^2}{1-D^3} = \frac{1}{1-D^3}\left((1-D)(1-D^3)-D^2\right) = \frac{1}{1-D^3}\left(1-D^3-D+D^4-D^2\right) = \frac{1-D-D^2-D^3+D^4}{1-D^3}$$

$$D^2-\frac{D^5}{1-D^3} = \frac{D^2-D^5+D^5}{1-D^3} = \frac{D^2}{1-D^3}$$

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$$\begin{bmatrix} 1 & -D & 0 & 0 & 0 \\ 0 & 1-D^3 & -1 & 0 & D^3 \\ 0 & 0 & \frac{1-D-D^2-D^3+D^4}{1-D^3} & 0 & \frac{D^2}{1-D^3} \\ 0 & 0 & \frac{-D^2}{1-D^3} & 1 & \frac{D^5}{1-D^3} \end{bmatrix} \quad \frac{\cancel{1-D^3}}{1-D-D^2-D^3+D^4} \cdot \frac{D^2}{\cancel{1-D^3}} = \frac{D^2}{1-D-D^2-D^3+D^4}$$

$$\Rightarrow T(D) = X_{a'}/X_a = \frac{D^5}{1-D^3} + \frac{D^2}{1-D^3} \cdot \frac{D^2}{1-D-D^2-D^3+D^4}$$

$$= \frac{D^5(1-D-D^2-D^3+D^4)+D^4}{(1-D^3)(1-D-D^2-D^3+D^4)}$$

But the nominator is $D^5(1-D-D^2-D^3+D^4)+D^4 = D^4+D^5-D^6-D^7-D^8+D^9$

$$= (1-D^3)(D^4+D^5-D^6)$$

$$\Rightarrow T(D) = \frac{D^4+D^5-D^6}{1-(D+D^2+D^3-D^4)} = (D^4+D^5-D^6)\left[1+(D+D^2+D^3-D^4)+(D+D^2+D^3-D^4)^2+\dots\right]$$

$$= D^4 + O(D^5) \quad \Rightarrow \quad d_{free} = 4$$

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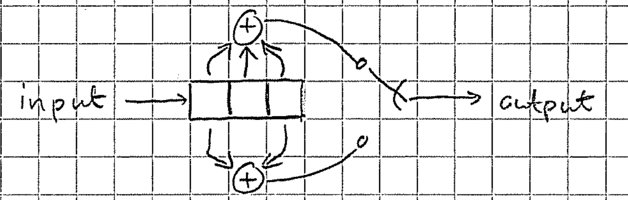
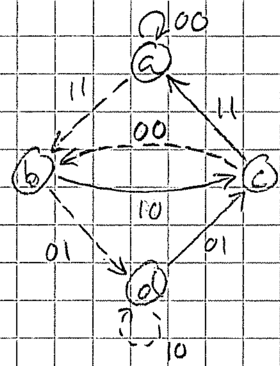
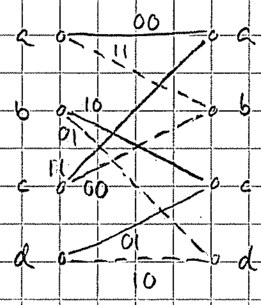
Tut 7, Ex 3.

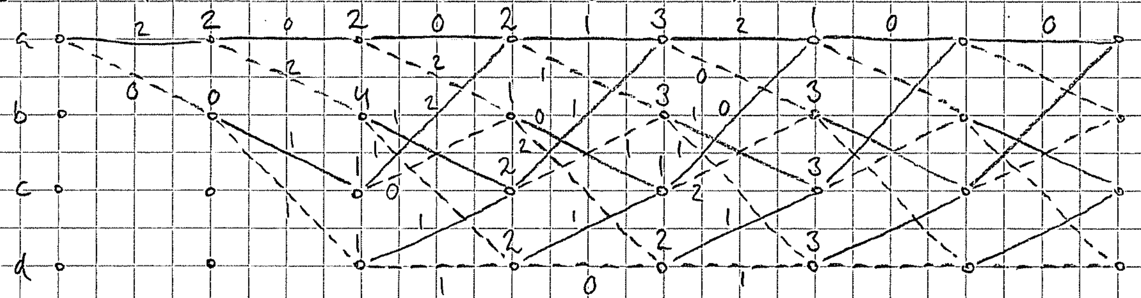


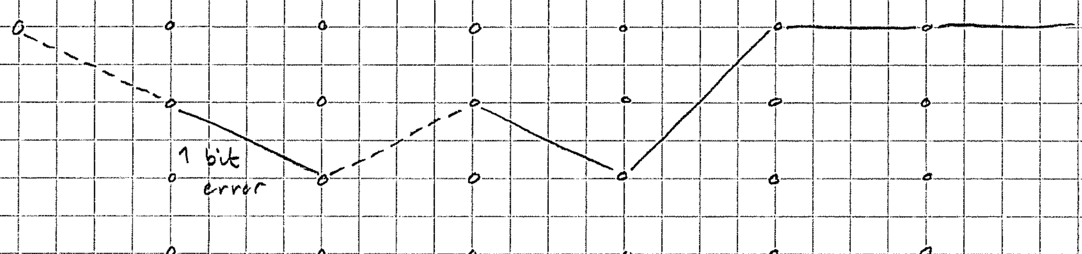
states: a = {0 0} b = {1 0} c = {0 1} d = {1 1}





Z = 11 00 00 10 11 00 00





m = 1 0 1 0 0 0 0 ...

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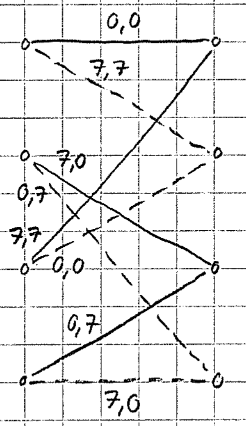
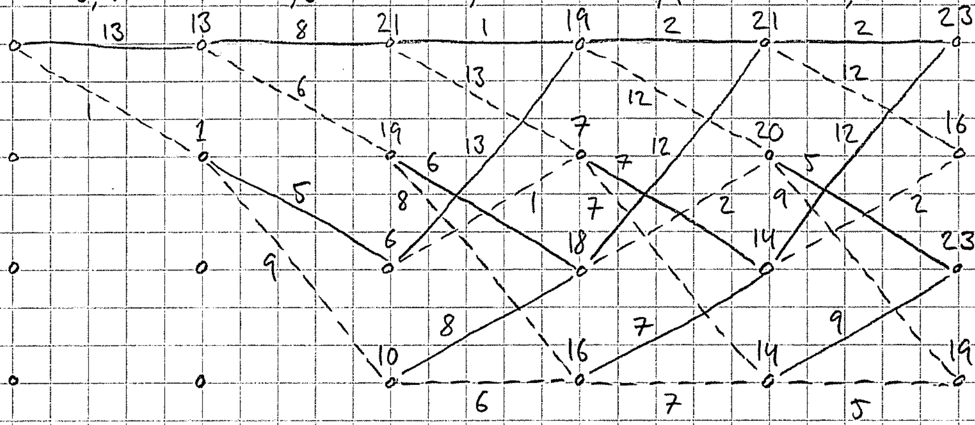
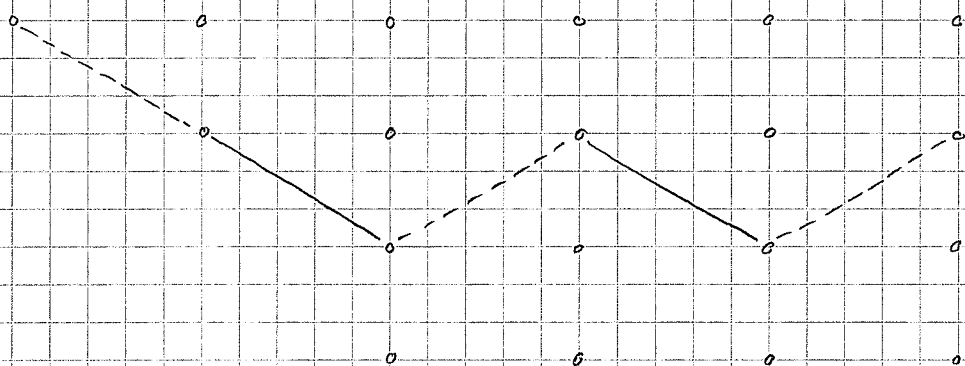
Tut 7. Ex 4.



z = 6,7 5,3 1,0 1,1 2,0





m =	1	0	1	0	1
#error-levels	1	5	1	7	2

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