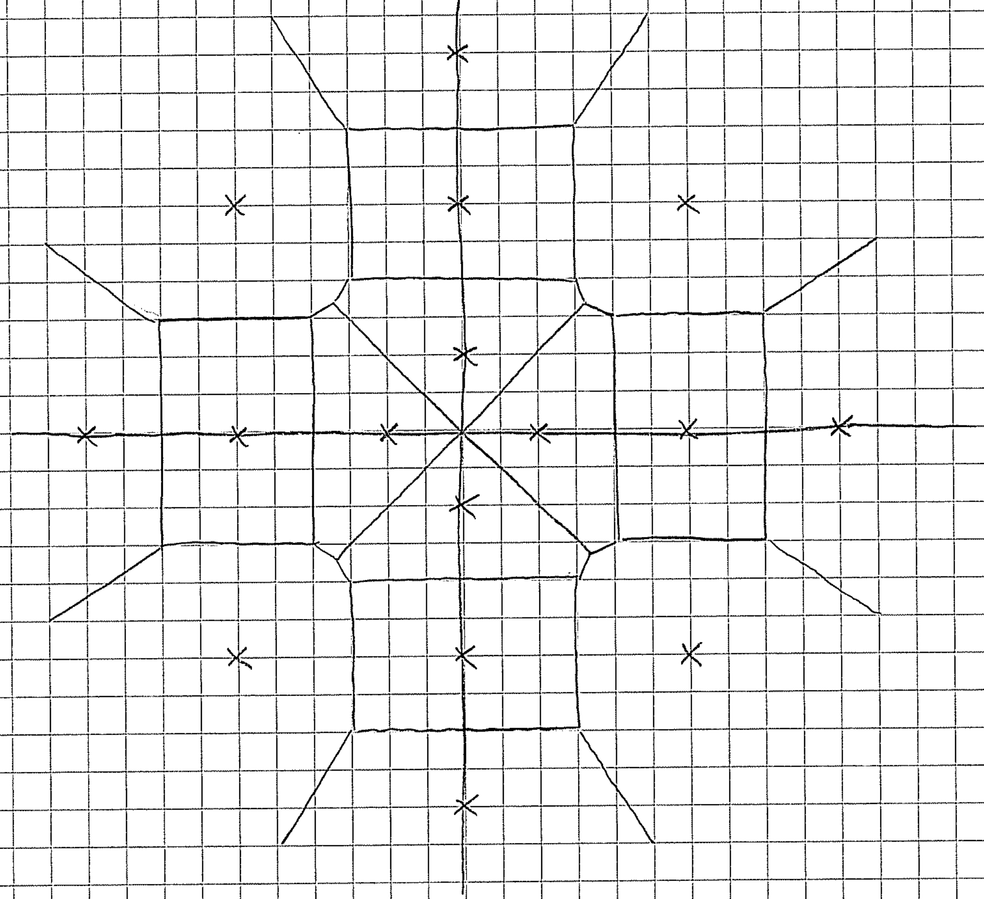
Tut 4, Ex 1

We have seen that when two symbols are equiprobable (and we assume equiprobable symbols here), then the decision boundary of a point is determined by the perpendicular bisectors of each line segment connecting the point with its neighbours.



This is easy to prove. Assume AWGN and equiprobable symbols.

Maximise $p(s_i|z) \Rightarrow$ maximise $p(s_i)\,p(z|s_i)/p(z) \Rightarrow$ maximise $p(z|s_i)$

$\Rightarrow$ maximise $\frac{1}{\pi N_0} \exp\left\{-\frac{(z_x - s_{i,x})^2 + (z_y - s_{i,y})^2}{N_0}\right\} \Rightarrow$ minimise $(z_x - s_{i,x})^2 + (z_y - s_{i,y})^2$

But this is the squared euclidean distance $\Rightarrow$ choose the point that is nearest to the received z!

Daniel Aronsson

Tut 4. Ex 2

The most we could send over a 4 kHz bandpass channel is 4000 symbols/s.
Here however, the rolloff will steal some bandwidth. See Figure 3.17 in Sklar. Sklar 3.81:

$$W_{DSB} = (1+r) R_s \quad , \quad r = 0.5 , \; W_{DSB} = 4000 \Rightarrow R_s = 2667 \text{ symbols/s}.$$

This gives

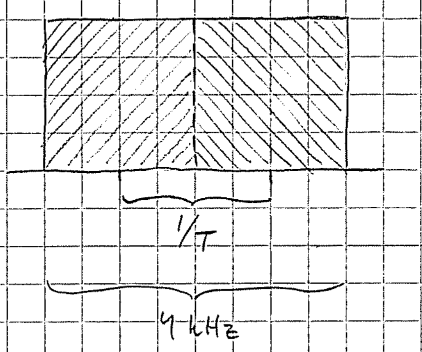
1. $R_B = R_s$

2. $R_B = 2R_s$

3. $R_B = 3R_s$

What about noncoherent orthogonal FSK signalling? We know that the tone spacing need to be at least $\frac{1}{T} = R_s$ (if you've forgot why, see Sklar 4.5.4.)

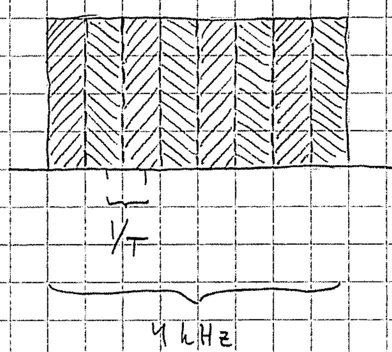
4. BFSK



$\Rightarrow \frac{1}{T} = 2000$ symbols/s

$\Rightarrow R_B = 2$ kbps

5. 8FSK



$\Rightarrow \frac{1}{T} = 500$ symbols/s

$\Rightarrow R_B = 1,5$ kbps

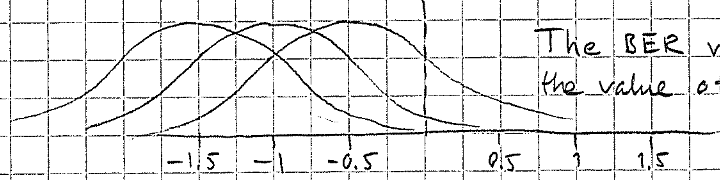
Observe that in FSK we only send one frequency at a time. How are these symbols located in the signal space?
Did you notice that the bit rate <u>slowed down</u> when we went from BFSK to 8FSK. That's the other way around from QAM signalling.

Daniel Aronsson

Tut 4. Ex 3.

Correlation metric: $y_m = \underbrace{a_m + i_m}_{x_m} + n_m$

Up to this point we've only had AWGN, but now we also have a noise term caused by ISI. What is the probability of a symbol error (= BER in this case)?



The BER varies depending on the value of i_m.

Skipping the index m and assuming that $a = -1$ (which we can do by symmetry),

$$p(y|i) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{(y-x)^2}{2\sigma^2}\right\} \Rightarrow p(\text{error}|i) = \int_0^\infty p(y|i)\,dy$$

$$= Q\left(\frac{0-x}{\sigma}\right) = Q\left(-\frac{x}{\sigma}\right) \quad \left(\text{Remember } \int_{\text{limit}}^{\infty} \text{gaussian pdf } dz = Q\left(\frac{\text{limit} - \text{mean value}}{\text{standard dev}}\right)\right)$$

$$p(\text{error}) = p(i=0.5)\,p(\text{error}|i=0.5) + p(i=0)\,p(\text{error}|i=0) + p(i=-0.5)\,p(\text{error}|i=-0.5)$$

$$= 0.25\,Q\left(\frac{0.5}{\sigma}\right) + 0.5\,Q\left(\frac{1}{\sigma}\right) + 0.25\,Q\left(\frac{1.5}{\sigma}\right)$$

Daniel Aronsson

Tut 4. Ex 4.

$H(f) = 1 + \alpha \cos(2\pi f t_0)$, $|\alpha| < 1$, $|f| \leq W$, $H(f) = 0$ otherwise

1 We know that the signal has bandwidth W so we may simply say that the channel frequency response is

$$H(f) = 1 + \alpha \cos(2\pi f t_0) \quad |\alpha| < 1$$

The impulse response is simply the inverse Fourier transform.
If you don't know it already you can look it up in e.g Sklar:

$$\int_{-\infty}^{\infty} \cos 2\pi f_0 t \, e^{-2\pi j f t} dt = \frac{1}{2}[\delta(f-f_0) + \delta(f+f_0)]$$

Let $f \to t$, $t \to f$, $f_0 \to t_0$:

$$\int_{-\infty}^{\infty} \cos 2\pi t_0 f \, e^{-2\pi t f} df \underset{\text{since cos is even}}{=} \underbrace{\int_{-\infty}^{\infty} \cos 2\pi f t_0 \, e^{2\pi f t} df}_{\text{inv Fourier transform}} = \frac{1}{2}[\delta(t-t_0) + \delta(t+t_0)]$$

$$\Rightarrow h(t) = \delta(t) + \frac{\alpha}{2}[\delta(t-t_0) + \delta(t+t_0)]$$

$y(t) = s * h(t)$ gives the result we're looking for.
(It is worth pointing out that one of the 'echoes' comes before the signal, i.e the channel is noncausal. What adjustment to $H(f)$ is required to make it causal?)

2. Let's say that the sent signal is $s(t) = \sum_{n=-\infty}^{\infty} a_n p(t-nT)$, where $p(t)$ is the pulse shape. Then $y(t) = \sum_{n=-\infty}^{\infty} [a_n p(t-nT) + \frac{\alpha}{2} a_n p(t-nT-t_0) + \frac{\alpha}{2} a_n p(t-nT+t_0)]$

$y(t)$ is sent through the receiving filter, i.e convoluted with $p(T-t)$.

Let $\int_{-\infty}^{\infty} p(u) p(T-(t-u)) du = \int_{-\infty}^{\infty} p(u) p(u-T-t) du = x(t)$

Then $\int_{-\infty}^{\infty} p(u-nT) p(u-T-t) du = x(t-nT)$, $\int_{-\infty}^{\infty} p(u-nT-t_0) p(u-T-t) dt = x(t-nT-t_0)$

and $\int_{-\infty}^{\infty} p(u-nT+t_0) p(u-T-t) du = x(t-nT+t_0)$

So, if $f(t) = p(T-t)$, the signal after the matched filter is

$$w(t) = y * f(t) = \sum_{n=-\infty}^{\infty} [a_n x(t-nT) + \frac{\alpha}{2} a_n x(t-nT-t_0) + \frac{\alpha}{2} a_n x(t-nT+t_0)]$$

and insert $t = kT$

Daniel Aronsson

3. $t_0 = T \Rightarrow w(kT) = \sum_{n=-\infty}^{\infty} \left[a_n x((k-n)T) + \frac{\alpha}{2} a_n x((k-n-1)T) + \frac{\alpha}{2} a_n x((k-n+1)T) \right]$

$$= a_k \left[x(0) + \frac{\alpha}{2} x(-T) + \frac{\alpha}{2} x(T) \right] + \underbrace{\sum_{n \neq k} \left[a_n x((k-n)T) + \frac{\alpha}{2} a_n x((k-n-1)T) + \frac{\alpha}{2} a_n x((k-n+1)T) \right]}_{\text{ISI}}$$

Pretend that the channel doesn't produce any echoes, i.e let $\alpha = 0$.

Then how do we wish the autocorrelation of the pulse shape, $x(t)$, to behave?

Daniel Aronsson