Tut 2. Ex 1

700 information bits and 800 coded bits are transmitted in 2 seconds.

Uncoded bit rate : 350 bps , Coded bit rate : 400 bps.

Modulation type	Bits per symbol ($\log_2 M$)	# symbols	Symbol rate [baud]
32PAM	5	160	80
16 PAM	4	200	100
8 PAM	3	266	133
4 PAM	2	400	200
PCM	1	800	400

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Tut 2. Ex 2

Our signal has a PSD that looks like the spectrum of white noise that has passed through a Butterworth filter. From eq 1.65:

$$|H_n(f)| = \frac{1}{\sqrt{1+(f/f_u)^{2n}}}$$

1. We have $f_u = 1000$ Hz and $n=6$ $\Rightarrow$ $G_x(f) = \frac{1}{(1+f/1000)^{12}}$

   This signal is not bandlimited, so obviously there will always be some aliasing involved. Here we seek the frequency where the spectral density has dropped -50 dB below its maximum.

   $Max = G_x(0) = 1$. $\quad 10^{-5} = \frac{1}{(1+f/1000)^{12}} \Rightarrow f = 1000 \cdot (10^{5/12} - 1) = 2,6$ kHz

   $\Rightarrow$ We must sample at 5,2 kHz.

2. Using $n=12$ we get $f = 0.6$ kHz $\Rightarrow$ We must sample at 1.2 kHz

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Tut 2, Ex 3.

The highest frequency is 4 kHz so clearly we have to sample at 8 kHz.
The quantisation distortion mustn't exceed $\pm 1\%$ of the peak-to-peak value,
so we have to divide the amplitude span into at least 50 levels.

1. We need at least 50 levels. Let's make this easy and say that we'll use 64 levels. Then we need 6 bits per sample.

2. Analog sampling rate: 8 kHz
   Bit rate: $6 \cdot 8$ kHz = 48 kHz

3. We use 16PAM to transmit our bits $\Rightarrow$ 4 bits per symbol
   Symbol rate: 12 kHz (This is what determines the bandwidth of our transmitted signal)

So we turned our original 8 kHz signal (−4 kHz to 4 kHz) into a 12 kHz digital signal. What was the point? Couldn't we just transmit the original one?

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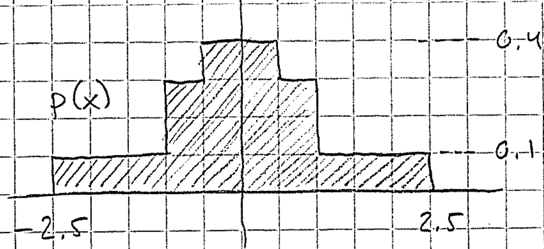
Tut 2. Ex 4.

Different values have different probabilities of being output from the source as described by the pdf

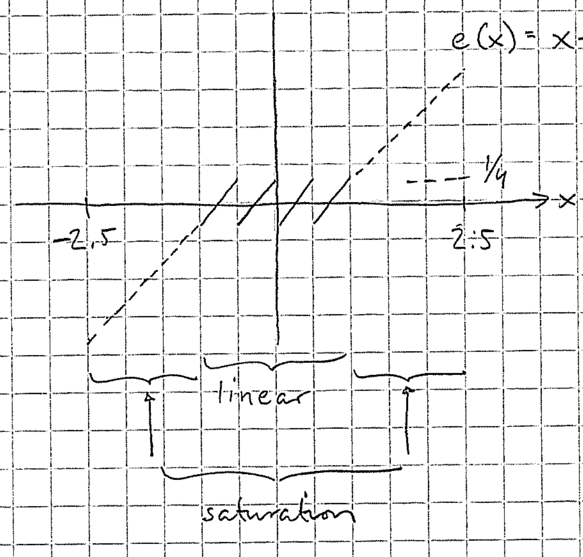


The ADC can only output one of the values

$-3/4$ V, $-1/4$ V, $1/4$ V, and $3/4$ V.

The actual value is hence rounded to one of these values.

The quantisation noise is therefore given by



The mean value of e(x) is clearly zero, so the quantisation noise variance is given by the expected value of $e^2(x)$:

$$\sigma^2_{lin} = 4(0.4+0.3)\int_0^{1/4} x^2\,dx = \frac{28}{10}\left[\frac{1}{3}x^3\right]_0^{1/4} = \frac{7}{480}$$

$$\sigma^2_{sat} = 2\cdot 0.1\int_0^{1.5}(x+1/4)^2\,dx = \frac{2}{10}\left[\frac{1}{3}x^3+\frac{1}{4}x^2+\frac{1}{16}x\right]_0^{1.5} = \frac{57}{160} = \frac{171}{480}$$

$$\sigma^2 = \sigma^2_{lin}+\sigma^2_{sat} = \frac{178}{480} = \frac{89}{240}$$

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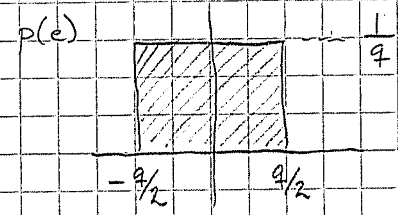
Tut 2. Ex 5

1. What is the signal power?

$$S = \frac{1}{T}\int_0^T |A\cos(2\pi f_0 t)|^2\, dt = \frac{A^2}{T}\underbrace{\int_0^T \frac{1+\cos 4\pi f_0 t}{2}\, dt}_{=T/2} = \frac{A^2}{2}, \quad 2A = \text{peak-to-peak value.}$$

What about the quantisation noise? We usually say that the quantisation noise is uniformly distributed over the quantisation interval (Why?) :



$$N_q = \frac{1}{q}\int_{-q/2}^{q/2} e^2\, de = \frac{1}{q}\left[\frac{1}{3}e^3\right]_{-q/2}^{q/2} = \frac{2q^2}{3\cdot 8} = \frac{q^2}{12}$$

The well-known variance for a uniform distribution

But $2A = q(L-1) \approx qL$ where L is the number of levels, so

$S = A^2/2 \approx \frac{1}{8}q^2L^2$

$\Rightarrow S/N_q \approx \frac{12}{8}L^2 = \frac{3}{2}L^2$ and $L = 65536$ (16-bit ADC)

$\Rightarrow S/N_q \approx 98$ dB

2. The peak-to-rms ratio is 20 $\Rightarrow$ The average signal power is a factor $\frac{1}{400}$ of the maximum power A^2.

$\Rightarrow$ The average signal power is a factor $\frac{1}{200}$ of the power of the signal in 1). $200 \approx 23$ dB

$\Rightarrow S/N_q \approx 98$ dB $- 23$ dB $= 75$ dB

3. $16 \cdot 44100 \cdot 2 \cdot 2 = 2.8$ Mbit/s

4. $2.8 \cdot 10^6 \cdot 3600 / 8 = 1.27$ GB (but half of it is overhead)

$\Rightarrow$ 635 MB can be stored.

5. $1500 \cdot 2 \cdot 100 \cdot 7 \cdot 6 \cdot 6 / 8 = 9.45$ MB $\Rightarrow$ 67 dictionaries can be stored.

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