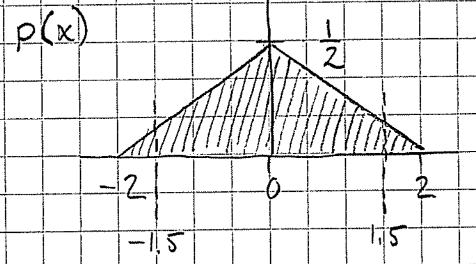
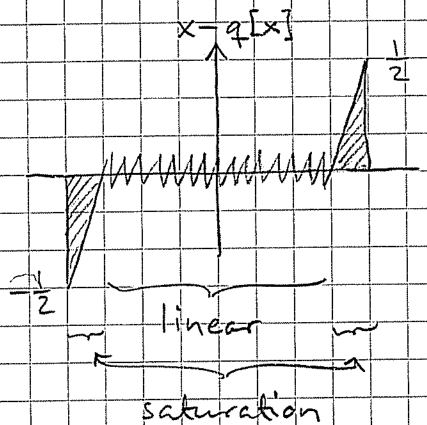
Tut 10, Ex 1.



Signal power: $\sigma_s^2 = E(x^2) = \int_{-2}^{2} x^2 p(x) = 2\int_0^2 x^2 \cdot \frac{1}{4}(2-x)\,dx$

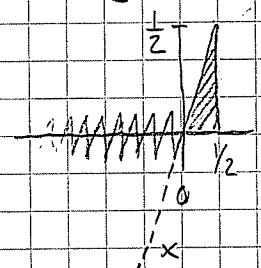
$= \left[\frac{1}{3}x^3 - \frac{1}{8}x^4\right]_0^2 = \frac{2}{3}$

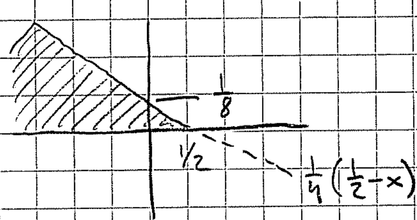
Quantisation noise power:



$\sigma_{sat}^2 = 2\int_0^{0.5} x^2 \frac{1}{4}\left(\frac{1}{2}-x\right)dx = \frac{1}{2}\left[\frac{1}{6}x^3 - \frac{1}{4}x^4\right]_0^{0.5} = \frac{1}{16}\left[\frac{1}{6}-\frac{1}{8}\right] = \frac{1}{384}$

For simplicity, we have moved everything 1.5 steps to the left:





Approximate $p(x)$ to be uniform within each quantisation level.

$\Delta = [1.5-(-1.5)]/32 = 3/32 \Rightarrow \sigma_{lin}^2 = \frac{\Delta^2}{12}$ if we assume that the saturation noise is negligible, so here $\sigma_{lin}^2 = \frac{\Delta^2}{12} \cdot P(-1.5<x<1.5)$

$P(-1.5<x<1.5) = 1 - 2\,\frac{1/2 \cdot 1/8}{2} = 15/16 \Rightarrow \sigma_{lin}^2 = \frac{1}{12}\cdot\frac{3^2}{32^2}\cdot\frac{15}{16} = \frac{45}{65536}$

$\sigma_s^2 / (\sigma_{sat}^2 + \sigma_{lin}^2) = 23\text{ dB}$

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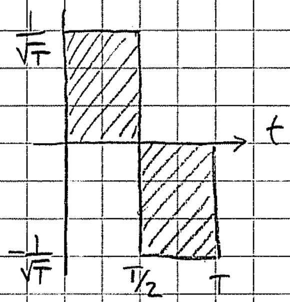
Tut 10. Ex 2

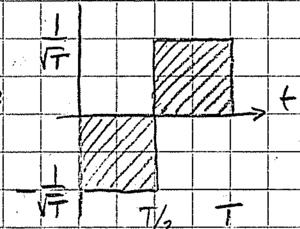
$s(t) = \pm A\,\psi(t)$, $\psi(t)$:



The matched filter $h(t)$ is $\frac{1}{\sqrt{\mathcal{E}_s}} s(T-t)$. $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$ so $\mathcal{E}_s = A^2$

A.

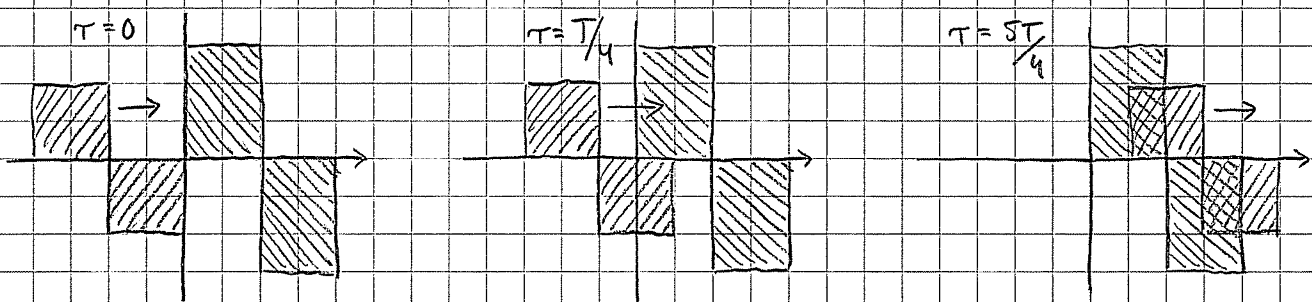
$h(t)$:



B.

The output of the matched filter is

$$z(\tau) = \int_{-\infty}^{\infty} s(t)\,h(\tau - t)\,dt$$



... and so on. Calculating the integral for each τ may be tedious, though. It is easier to look at each combination of boxes individually and then and then add the contributions.

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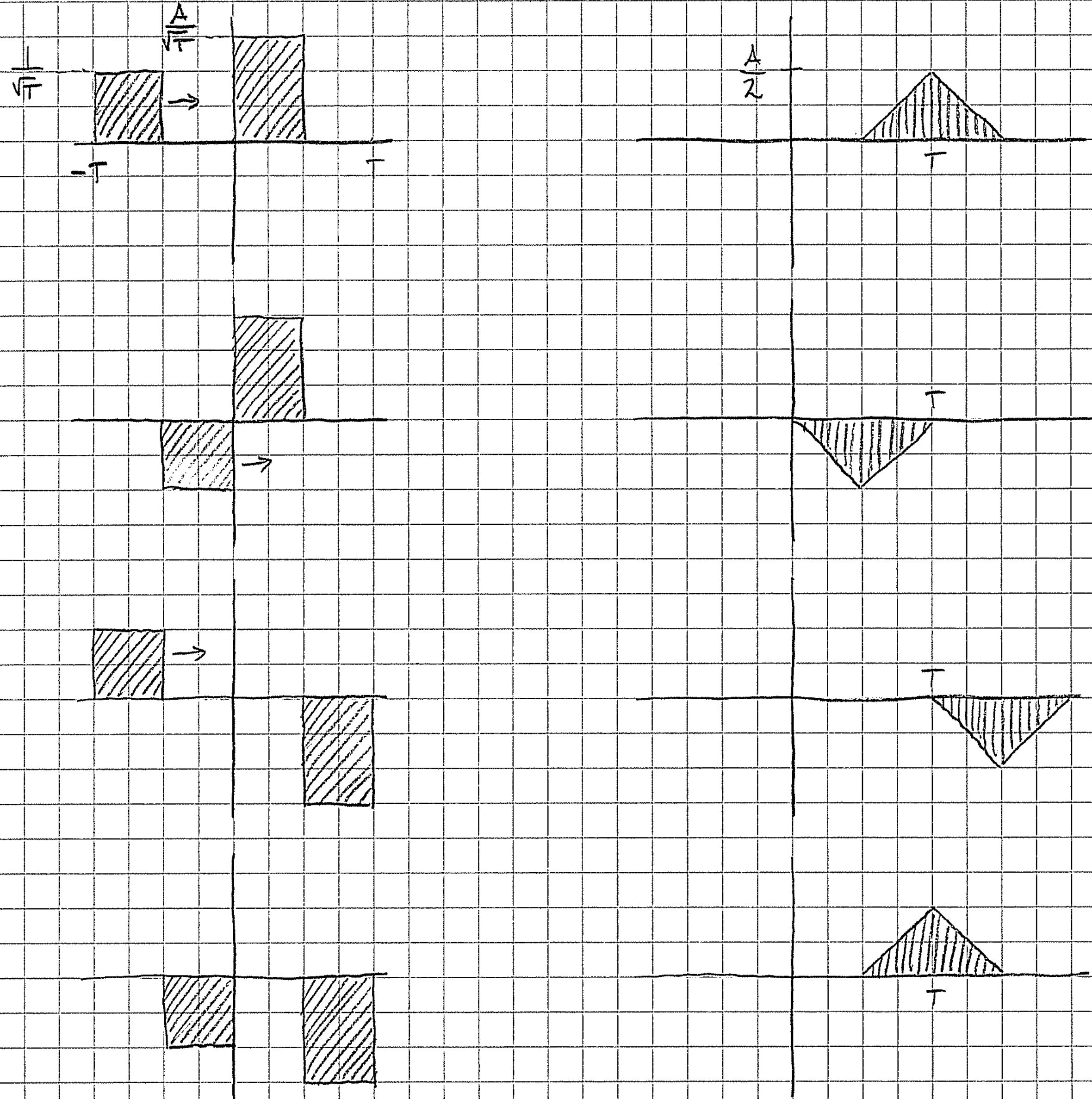
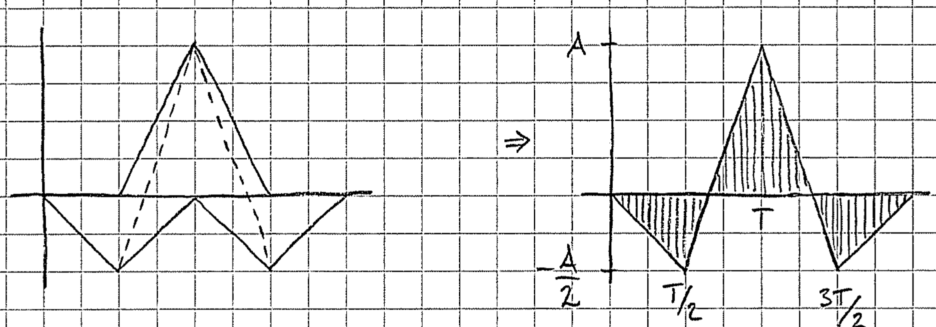
These boxes... ...integrate to...



The total $z(t)$ is (if the noise is zero)



So when we sample $z_k = z(kT)$, we sample at the peak of $z(t)$ and without ISI. If we send $-A\varphi(t)$ instead, then of course $z(t)$ is turned upside down.

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C.

The correlation metric (the decision variable), i.e the sampled output $z(kT)$ from the matched filter is

$$z(kT) = z_k = \pm A + n \quad \text{where } n \text{ is the noise.}$$

We know from previously that n is gaussian with variance $\frac{N_0}{2}$:

$$E(n) = E\int_{-\infty}^{\infty} n(t)\,h(\tau - t)\,dt = 0 \quad \text{since } E\,n(t) = 0$$

$$E(n^2) = E\left[\int_{-\infty}^{\infty} n(t_1)\,h(\tau-t_1)\,dt_1 \int_{-\infty}^{\infty} n(t_2)\,h(\tau - t_2)\,dt_2\right]$$

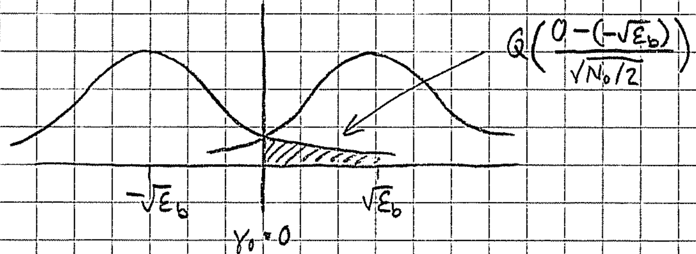
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{E\{n(t_1)\,n(t_2)\}}_{\frac{N_0}{2}\delta(t_1 - t_2)}\,h(\tau - t_1)\,h(\tau - t_2)\,dt_1\,dt_2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |h(\tau - t)|^2\,dt = \frac{N_0}{2}$$

An ML detector examines how the received data fits the different signals:

$$p(z|s_1) \underset{s_2}{\overset{s_1}{\gtrless}} p(z|s_2) \Rightarrow \frac{1}{\sqrt{\pi N_0}}\exp\left\{-\frac{(z-A)^2}{N_0}\right\} \underset{s_2}{\overset{s_1}{\gtrless}} \frac{1}{\sqrt{\pi N_0}}\exp\left\{-\frac{(z+A)^2}{N_0}\right\}$$

$$\Rightarrow 2zA/N_0 \underset{s_2}{\overset{s_1}{\gtrless}} -2zA/N_0 \Rightarrow z \underset{s_2}{\overset{s_1}{\gtrless}} \underbrace{0}_{\gamma_0}$$

Remember that $A = \sqrt{E_s} = \sqrt{E_b}$ in this case



$$P_b = \frac{1}{3}Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \frac{2}{3}Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{which we already knew}$$

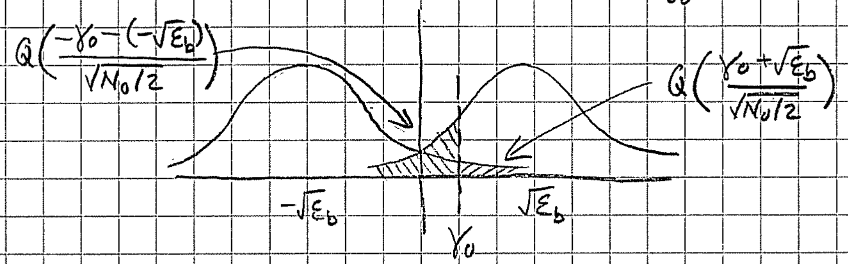
But this is not the BER for the optimum detector since the symbols aren't equiprobable.

D.

$$p(s_1|z) \underset{s_2}{\overset{s_1}{\gtrless}} p(s_2|z) \quad \Rightarrow \quad \frac{p(s_1)\,p(z|s_1)}{p(z)} \underset{s_2}{\overset{s_1}{\gtrless}} \frac{p(s_2)\,p(z|s_2)}{p(z)}$$

$$\Rightarrow \log\left(\frac{1}{3}\right) - \log\sqrt{\pi N_0} - \frac{z^2+\mathcal{E}_b-2z\sqrt{\mathcal{E}_b}}{N_0} \underset{s_2}{\overset{s_1}{\gtrless}} \log\left(\frac{2}{3}\right) - \log\sqrt{\pi N_0} - \frac{z^2+\mathcal{E}_b+2z\sqrt{\mathcal{E}_b}}{N_0}$$

$$\Rightarrow \frac{4\sqrt{\mathcal{E}_b}}{N_0} z \underset{s_2}{\overset{s_1}{\gtrless}} \log(2) \quad \Rightarrow \quad z \underset{s_2}{\overset{s_1}{\gtrless}} \underbrace{\frac{N_0}{4\sqrt{\mathcal{E}_b}} \log(2)}_{\gamma_0}$$



$$P_b = \frac{1}{3} Q\left(\frac{\sqrt{\mathcal{E}_b}-\gamma_0}{\sqrt{N_0/2}}\right) + \frac{2}{3} Q\left(\frac{\sqrt{\mathcal{E}_b}+\gamma_0}{\sqrt{N_0/2}}\right)$$

Tut 10, Ex 3.

A. The bandwidth efficiency is $\eta = \frac{R_b}{W}$

For a baseband system we have $W = \frac{1}{2}(1+\alpha)R_s$

Here we have 4-ary symbols and a rate $\frac{1}{2}$ code,

so $R_s = \frac{1}{2}R_c$ and $R_c = 2R_b$

$$\Rightarrow R_b/W = \frac{2}{1+\alpha} = 5/3$$

B. Remember that a codeword is the difference between two other codewords: $X = mG = (m_1 - m_2)G = X_1 - X_2$

m	X = mG
0 0 0	0 0 0 0 0 0
0 0 1	1 0 1 0 0 1
0 1 0	0 1 1 0 1 0
0 1 1	1 1 0 0 1 1
1 0 0	1 1 0 1 0 0
1 0 1	0 1 1 1 0 1
1 1 0	1 0 1 1 1 0
1 1 1	0 0 0 1 1 1

$\Rightarrow d_{min} = 3$

corrects 1, detects 2

C.

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

E	$S = EH^T$
0 0 0 0 0 1	1 0 1
0 0 0 0 1 0	0 1 1
0 0 0 1 0 0	1 1 0
0 0 1 0 0 0	0 0 1
0 1 0 0 0 0	0 1 0
1 0 0 0 0 0	1 0 0

$rH^T = 100 \Rightarrow E = 100000 \Rightarrow \hat{X} = 101110$

$\Rightarrow \hat{m} = 110$

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B.

We have to sample at 5 kHz.

Each sample is one of 32 levels

$\Rightarrow R_b = 5000 \cdot \log_2 32 = 25$ kbps

We send 3 bits per symbol

$\Rightarrow R_s = 8.3$ ksymbols/s

C.

Baseband signalling $\Rightarrow W_t = \frac{1+\alpha}{2} R_s = \frac{1+\alpha}{6} R_b$

$\Rightarrow R_b / W_t = \frac{6}{1+\alpha} = \frac{6}{1+0.2} = 5$ bps/Hz

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