Exam Digital Communications I

12th of March 2007, 9.00-14.00

Examiner: Catharina Carlemalm Logothetis (018 4717283)

Allowed material:

- Any calculator
- Mathematics handbook
- Swedish-English dictionary
- List of Formulas written by Sorour Falahati

Please write all your answers *neatly and clearly*. *Motivate* your answers thoroughly (except in Question1).

Good Luck!

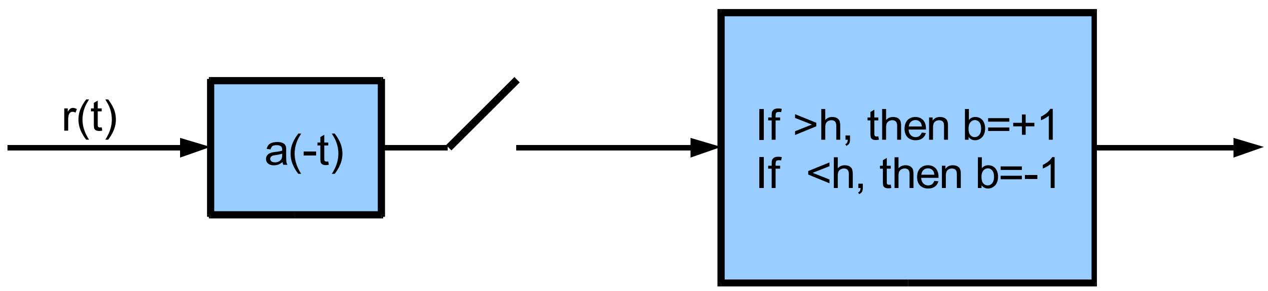
Question 1 *(10p)*

For each of the following sub-questions (a-g), you do not *need to provide motivation for your answers. Each of the sub-questions will be graded as indicated. An incorrect answer of each sub-problem gives 0 p.*

a) *(2p)* Determine *the minimum distance* of *the binary linear block code* defined by the following generator matrix

$$G=\begin{bmatrix}101100\\010110\\001011\end{bmatrix}$$

b) *(3p)* In this problem, we assume that the transmitted signal of a binary communication system is given by $s(t)=b\sqrt{E_b}a(t)$. Here, $b\in\{-1,+1\}$ is the transmitted bit, E_b is energy per bit and $a(t)$ is a unit-energy waveform. The signal is transmitted over an AWGN channel with noise power spectral density $N_0/2$. Assume $E_b=N_0=4*10^{-4}$. Let $p=Pr\{b=+1\}$.
In the figure below, the receiver structure is shown.



Determine the *threshold h* such that the receiver is a *ML* receiver. Compute the *probability of bit error* if *p* = 0.8.

c) *(2p)* Here, we consider a linear block code with parity check matrix given by

$$H=\begin{bmatrix}0001000\\1100100\\1010010\\0110001\end{bmatrix}.$$

The block code is used together with binary PAM modulation for transmission over an AWGN channel with noise power spectral density $N_0/2$. We assume that the PAM pulse shape has a root-raised cosine spectrum with roll-off α =0.1 and that the symbol rate is $1/T$ symbols/s.

Determine the error correcting capability of the code?

d) *(1p)* If X and Y are independent Gaussian variables, then $E\{XY\}=0$ always.
(Answer with TRUE or FALSE)

e) *(1p)* A Hamming code always has minimum distance 3.
(Answer with TRUE or FALSE)

f) *(1p)* A receiver that implements the ML decision rule is always optimal in the sense of minimum symbol error probability. *(Answer with TRUE or FALSE)*

Question 2

Here, we approximate a binary disc storage channel with a discrete-time additive Gaussian noise channel with input $x_n \in \{0,1\}$ and output $y_n = x_n + w_n$, where y_n is the decision variable in the disc-reading device at time *n*. It turns out that the noise variance depends on the value of x_n according to:

- If $x=0$, then the variance is σ_0^2 .
- If $x=1$, then the variance is σ_1^2 .

where $\sigma_1^2 > \sigma_0^2$

We assume equally probable signals, that is $Pr(x_n=0)=Pr(x_n=1)=1/2$ for any *n*.

a) Determine the optimal (in the sense of minimizing $P_b = Pr(\hat{x}_n \neq x_n)$) detection rule (that is the decision boundary/threshold) to decide $\hat{x}_n \in \{0,1\}$ based on the value of y_n . You should formulate the detection rule in terms of σ_0^2 and σ_1^2 . *(6p)*

b) Find the corresponding bit error probability P_b . *(3p)*

c) What happens to the detector and P_b derived in a) and b) if $\sigma_0^2 \to 0$ and σ_1^2 remains constant? *(1p)*

Question 3

In this example, we will consider a system operating over a linear AWGN channel. We will use M-ary pulse amplitude modulation (PAM). Furthermore, we use square root raised cosine pulseshaping with roll-off factor factor α . We assume that baseband transmission is used.

The information bits are denoted by $b[n]$ and the channel noise has power spectral density $N_0/2$. Furthermore, the channel bandwidth is given by $W = 1000$ Hz.

a) *Choose modulation scheme (that is choose M in the M-ary PAM scheme)* such that

- the data rate is $R_b = 1900\, bits/s$ and
- the bit error probability is $P_b \leq 10^{-4}$.

Compute the required E_b/N_0 in dB (where E_b is the energy per information bit).

(4p)

b) *Repeat part a)* of this example but assume now that the data rate is $R_b = 5000\, bits/s$. *(6p)*

Hint: For a baseband system, we have $R_s/W = 2/(1+\alpha)$ where $R_b = \log_2(M) R_s$.

Question 4

In this problem, we study a rate 1/2 convolutional code with generator sequences given by $g_1 = (111)$ and $g_2 = (110)$. Furthermore, we assume that BPSK modulation is used to transmit the coded bits.

a) Draw *the shift register* for the encoder described. Furthermore, please draw *the state diagram* for this code. *(2p)*

b) Now, we are going to study so-called Viterbi *hard* decoding. We assume that the decoder always starts at the all-zero state and that it ends up at the all-zero state. *Draw a trellis* for a code terminated to a length of 10 bits. By inspection of the trellis, what could you say about the *free distance* d_{free} . Note that you are not asked to perform the decoding. *(4p)*

c) Here, instead we will perform Viterbi *soft* decoding. Just as in b), we assume that the decoder still starts from and ends up at the all-zero state.

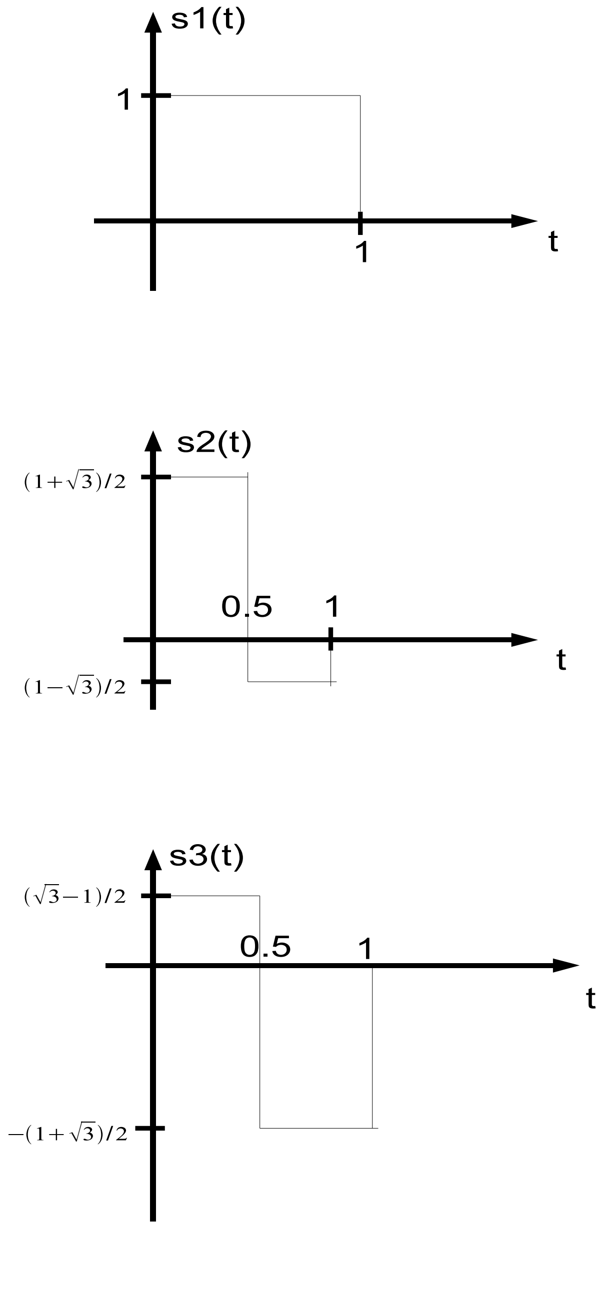
The received sequence r is *first fed to a 3 bits uniform quantizer*. The maximum reconstruction value of this quantizer is 1.75, while the minimum reconstruction value is -1.75.

The quantized sequence is *then fed into a Viterbi soft decoder*, which employs the *minimum Euclidean distance algorithm*. We assume that the received vector r is given by: $r = \{1.38, 0.68, 0.05, 1.04, -0.33, -0.81, 0.12, -0.41, -0.93, 1\}$.

Please estimate the information bits corresponding to r . *(6p)*

Question 5

In this problem, we consider the following six signal alternatives $s_1(t),\dots s_6(t)$. The signals $s_1(t), s_2(t), s_3(t)$ are described in the figure below.



Furthermore, $s_4(t)=-s_1(t), s_5(t)=-s_2(t), s_6(t)=-s_3(t)$.

These signals are used for transmitting equally probable symbols over an AWGN channel with noise spectral density $N_0/2$. We assume that an optimal (minimum symbol error probability) receiver is used.

Show that the following approximately holds: $Q\left(1/\sqrt{2N_0}\right)<P_e<2Q\left(1/\sqrt{2N_0}\right)$ with $P_e=Pr(symbol\,error)$. *(8p)*

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
0.00	5.000E-01	0.76	2.236E-01	1.52	6.426E-02	2.28	1.130E-02	3.04	1.183E-03	3.80	7.235E-05	4.56	2.558E-06	5.32	5.188E-08
0.01	4.960E-01	0.77	2.206E-01	1.53	6.301E-02	2.29	1.101E-02	3.05	1.144E-03	3.81	6.948E-05	4.57	2.439E-06	5.33	4.911E-08
0.02	4.920E-01	0.78	2.177E-01	1.54	6.178E-02	2.30	1.072E-02	3.06	1.107E-03	3.82	6.673E-05	4.58	2.325E-06	5.34	4.647E-08
0.03	4.880E-01	0.79	2.148E-01	1.55	6.057E-02	2.31	1.044E-02	3.07	1.070E-03	3.83	6.407E-05	4.59	2.216E-06	5.35	4.398E-08
0.04	4.840E-01	0.80	2.119E-01	1.56	5.938E-02	2.32	1.017E-02	3.08	1.035E-03	3.84	6.152E-05	4.60	2.112E-06	5.36	4.161E-08
0.05	4.801E-01	0.81	2.090E-01	1.57	5.821E-02	2.33	9.903E-03	3.09	1.001E-03	3.85	5.906E-05	4.61	2.013E-06	5.37	3.937E-08
0.06	4.761E-01	0.82	2.061E-01	1.58	5.705E-02	2.34	9.642E-03	3.10	9.676E-04	3.86	5.669E-05	4.62	1.919E-06	5.38	3.724E-08
0.07	4.721E-01	0.83	2.033E-01	1.59	5.592E-02	2.35	9.387E-03	3.11	9.354E-04	3.87	5.442E-05	4.63	1.828E-06	5.39	3.523E-08
0.08	4.681E-01	0.84	2.005E-01	1.60	5.480E-02	2.36	9.137E-03	3.12	9.043E-04	3.88	5.223E-05	4.64	1.742E-06	5.40	3.332E-08
0.09	4.641E-01	0.85	1.977E-01	1.61	5.370E-02	2.37	8.894E-03	3.13	8.740E-04	3.89	5.012E-05	4.65	1.660E-06	5.41	3.151E-08
0.10	4.602E-01	0.86	1.949E-01	1.62	5.262E-02	2.38	8.656E-03	3.14	8.447E-04	3.90	4.810E-05	4.66	1.581E-06	5.42	2.980E-08
0.11	4.562E-01	0.87	1.922E-01	1.63	5.155E-02	2.39	8.424E-03	3.15	8.164E-04	3.91	4.615E-05	4.67	1.506E-06	5.43	2.818E-08
0.12	4.522E-01	0.88	1.894E-01	1.64	5.050E-02	2.40	8.198E-03	3.16	7.888E-04	3.92	4.427E-05	4.68	1.434E-06	5.44	2.664E-08
0.13	4.483E-01	0.89	1.867E-01	1.65	4.947E-02	2.41	7.976E-03	3.17	7.622E-04	3.93	4.247E-05	4.69	1.366E-06	5.45	2.518E-08
0.14	4.443E-01	0.90	1.841E-01	1.66	4.846E-02	2.42	7.760E-03	3.18	7.364E-04	3.94	4.074E-05	4.70	1.301E-06	5.46	2.381E-08
0.15	4.404E-01	0.91	1.814E-01	1.67	4.746E-02	2.43	7.549E-03	3.19	7.114E-04	3.95	3.908E-05	4.71	1.239E-06	5.47	2.250E-08
0.16	4.364E-01	0.92	1.788E-01	1.68	4.648E-02	2.44	7.344E-03	3.20	6.871E-04	3.96	3.747E-05	4.72	1.179E-06	5.48	2.127E-08
0.17	4.325E-01	0.93	1.762E-01	1.69	4.551E-02	2.45	7.143E-03	3.21	6.637E-04	3.97	3.594E-05	4.73	1.123E-06	5.49	2.010E-08
0.18	4.286E-01	0.94	1.736E-01	1.70	4.457E-02	2.46	6.947E-03	3.22	6.410E-04	3.98	3.446E-05	4.74	1.069E-06	5.50	1.899E-08
0.19	4.247E-01	0.95	1.711E-01	1.71	4.363E-02	2.47	6.756E-03	3.23	6.190E-04	3.99	3.304E-05	4.75	1.017E-06	5.51	1.794E-08
0.20	4.207E-01	0.96	1.685E-01	1.72	4.272E-02	2.48	6.569E-03	3.24	5.976E-04	4.00	3.167E-05	4.76	9.680E-07	5.52	1.695E-08
0.21	4.168E-01	0.97	1.660E-01	1.73	4.182E-02	2.49	6.387E-03	3.25	5.770E-04	4.01	3.036E-05	4.77	9.211E-07	5.53	1.601E-08
0.22	4.129E-01	0.98	1.635E-01	1.74	4.093E-02	2.50	6.210E-03	3.26	5.571E-04	4.02	2.910E-05	4.78	8.765E-07	5.54	1.512E-08
0.23	4.090E-01	0.99	1.611E-01	1.75	4.006E-02	2.51	6.037E-03	3.27	5.377E-04	4.03	2.789E-05	4.79	8.339E-07	5.55	1.428E-08
0.24	4.052E-01	1.00	1.587E-01	1.76	3.920E-02	2.52	5.868E-03	3.28	5.190E-04	4.04	2.673E-05	4.80	7.933E-07	5.56	1.349E-08
0.25	4.013E-01	1.01	1.562E-01	1.77	3.836E-02	2.53	5.703E-03	3.29	5.009E-04	4.05	2.561E-05	4.81	7.547E-07	5.57	1.274E-08
0.26	3.974E-01	1.02	1.539E-01	1.78	3.754E-02	2.54	5.543E-03	3.30	4.834E-04	4.06	2.454E-05	4.82	7.178E-07	5.58	1.203E-08
0.27	3.936E-01	1.03	1.515E-01	1.79	3.673E-02	2.55	5.386E-03	3.31	4.665E-04	4.07	2.351E-05	4.83	6.827E-07	5.59	1.135E-08
0.28	3.897E-01	1.04	1.492E-01	1.80	3.593E-02	2.56	5.234E-03	3.32	4.501E-04	4.08	2.252E-05	4.84	6.492E-07	5.60	1.072E-08
0.29	3.859E-01	1.05	1.469E-01	1.81	3.515E-02	2.57	5.085E-03	3.33	4.342E-04	4.09	2.157E-05	4.85	6.173E-07	5.61	1.012E-08
0.30	3.821E-01	1.06	1.446E-01	1.82	3.438E-02	2.58	4.940E-03	3.34	4.189E-04	4.10	2.066E-05	4.86	5.869E-07	5.62	9.548E-09
0.31	3.783E-01	1.07	1.423E-01	1.83	3.362E-02	2.59	4.799E-03	3.35	4.041E-04	4.11	1.978E-05	4.87	5.580E-07	5.63	9.010E-09
0.32	3.745E-01	1.08	1.401E-01	1.84	3.288E-02	2.60	4.661E-03	3.36	3.897E-04	4.12	1.894E-05	4.88	5.304E-07	5.64	8.503E-09
0.33	3.707E-01	1.09	1.379E-01	1.85	3.216E-02	2.61	4.527E-03	3.37	3.758E-04	4.13	1.814E-05	4.89	5.042E-07	5.65	8.022E-09
0.34	3.669E-01	1.10	1.357E-01	1.86	3.144E-02	2.62	4.396E-03	3.38	3.624E-04	4.14	1.737E-05	4.90	4.792E-07	5.66	7.569E-09
0.35	3.632E-01	1.11	1.335E-01	1.87	3.074E-02	2.63	4.269E-03	3.39	3.495E-04	4.15	1.662E-05	4.91	4.554E-07	5.67	7.140E-09
0.36	3.594E-01	1.12	1.314E-01	1.88	3.005E-02	2.64	4.145E-03	3.40	3.369E-04	4.16	1.591E-05	4.92	4.327E-07	5.68	6.735E-09
0.37	3.557E-01	1.13	1.292E-01	1.89	2.938E-02	2.65	4.025E-03	3.41	3.248E-04	4.17	1.523E-05	4.93	4.111E-07	5.69	6.352E-09
0.38	3.520E-01	1.14	1.271E-01	1.90	2.872E-02	2.66	3.907E-03	3.42	3.131E-04	4.18	1.458E-05	4.94	3.906E-07	5.70	5.990E-09
0.39	3.483E-01	1.15	1.251E-01	1.91	2.807E-02	2.67	3.793E-03	3.43	3.018E-04	4.19	1.395E-05	4.95	3.711E-07	5.71	5.649E-09
0.40	3.446E-01	1.16	1.230E-01	1.92	2.743E-02	2.68	3.681E-03	3.44	2.909E-04	4.20	1.335E-05	4.96	3.525E-07	5.72	5.326E-09
0.41	3.409E-01	1.17	1.210E-01	1.93	2.680E-02	2.69	3.573E-03	3.45	2.803E-04	4.21	1.277E-05	4.97	3.348E-07	5.73	5.022E-09
0.42	3.372E-01	1.18	1.190E-01	1.94	2.619E-02	2.70	3.467E-03	3.46	2.701E-04	4.22	1.222E-05	4.98	3.179E-07	5.74	4.734E-09
0.43	3.336E-01	1.19	1.170E-01	1.95	2.559E-02	2.71	3.364E-03	3.47	2.602E-04	4.23	1.168E-05	4.99	3.019E-07	5.75	4.462E-09
0.44	3.300E-01	1.20	1.151E-01	1.96	2.500E-02	2.72	3.264E-03	3.48	2.507E-04	4.24	1.118E-05	5.00	2.867E-07	5.76	4.206E-09
0.45	3.264E-01	1.21	1.131E-01	1.97	2.442E-02	2.73	3.167E-03	3.49	2.415E-04	4.25	1.069E-05	5.01	2.722E-07	5.77	3.964E-09
0.46	3.228E-01	1.22	1.112E-01	1.98	2.385E-02	2.74	3.072E-03	3.50	2.326E-04	4.26	1.022E-05	5.02	2.584E-07	5.78	3.735E-09
0.47	3.192E-01	1.23	1.093E-01	1.99	2.330E-02	2.75	2.980E-03	3.51	2.241E-04	4.27	9.774E-06	5.03	2.452E-07	5.79	3.519E-09
0.48	3.156E-01	1.24	1.075E-01	2.00	2.275E-02	2.76	2.890E-03	3.52	2.158E-04	4.28	9.345E-06	5.04	2.328E-07	5.80	3.316E-09
0.49	3.121E-01	1.25	1.056E-01	2.01	2.222E-02	2.77	2.803E-03	3.53	2.078E-04	4.29	8.934E-06	5.05	2.209E-07	5.81	3.124E-09
0.50	3.085E-01	1.26	1.038E-01	2.02	2.169E-02	2.78	2.718E-03	3.54	2.001E-04	4.30	8.540E-06	5.06	2.096E-07	5.82	2.942E-09
0.51	3.050E-01	1.27	1.020E-01	2.03	2.118E-02	2.79	2.635E-03	3.55	1.926E-04	4.31	8.163E-06	5.07	1.989E-07	5.83	2.771E-09
0.52	3.015E-01	1.28	1.003E-01	2.04	2.068E-02	2.80	2.555E-03	3.56	1.854E-04	4.32	7.801E-06	5.08	1.887E-07	5.84	2.610E-09
0.53	2.981E-01	1.29	9.853E-02	2.05	2.018E-02	2.81	2.477E-03	3.57	1.785E-04	4.33	7.455E-06	5.09	1.790E-07	5.85	2.458E-09
0.54	2.946E-01	1.30	9.680E-02	2.06	1.970E-02	2.82	2.401E-03	3.58	1.718E-04	4.34	7.124E-06	5.10	1.698E-07	5.86	2.314E-09
0.55	2.912E-01	1.31	9.510E-02	2.07	1.923E-02	2.83	2.327E-03	3.59	1.653E-04	4.35	6.807E-06	5.11	1.611E-07	5.87	2.179E-09
0.56	2.877E-01	1.32	9.342E-02	2.08	1.876E-02	2.84	2.256E-03	3.60	1.591E-04	4.36	6.503E-06	5.12	1.528E-07	5.88	2.051E-09
0.57	2.843E-01	1.33	9.176E-02	2.09	1.831E-02	2.85	2.186E-03	3.61	1.531E-04	4.37	6.212E-06	5.13	1.449E-07	5.89	1.931E-09
0.58	2.810E-01	1.34	9.012E-02	2.10	1.786E-02	2.86	2.118E-03	3.62	1.473E-04	4.38	5.934E-06	5.14	1.374E-07	5.90	1.818E-09
0.59	2.776E-01	1.35	8.851E-02	2.11	1.743E-02	2.87	2.052E-03	3.63	1.417E-04	4.39	5.668E-06	5.15	1.302E-07	5.91	1.711E-09
0.60	2.743E-01	1.36	8.691E-02	2.12	1.700E-02	2.88	1.988E-03	3.64	1.363E-04	4.40	5.413E-06	5.16	1.235E-07	5.92	1.610E-09
0.61	2.709E-01	1.37	8.534E-02	2.13	1.659E-02	2.89	1.926E-03	3.65	1.311E-04	4.41	5.169E-06	5.17	1.170E-07	5.93	1.515E-09
0.62	2.676E-01	1.38	8.379E-02	2.14	1.618E-02	2.90	1.866E-03	3.66	1.261E-04	4.42	4.935E-06	5.18	1.109E-07	5.94	1.425E-09
0.63	2.643E-01	1.39	8.226E-02	2.15	1.578E-02	2.91	1.807E-03	3.67	1.213E-04	4.43	4.712E-06	5.19	1.051E-07	5.95	1.341E-09
0.64	2.611E-01	1.40	8.076E-02	2.16	1.539E-02	2.92	1.750E-03	3.68	1.166E-04	4.44	4.498E-06	5.20	9.964E-08	5.96	1.261E-09
0.65	2.578E-01	1.41	7.927E-02	2.17	1.500E-02	2.93	1.695E-03	3.69	1.121E-04	4.45	4.294E-06	5.21	9.442E-08	5.97	1.186E-09
0.66	2.546E-01	1.42	7.780E-02	2.18	1.463E-02	2.94	1.641E-03	3.70	1.078E-04	4.46	4.098E-06	5.22	8.946E-08	5.98	1.116E-09
0.67	2.514E-01	1.43	7.636E-02	2.19	1.426E-02	2.95	1.589E-03	3.71	1.036E-04	4.47	3.911E-06	5.23	8.476E-08	5.99	1.049E-09
0.68	2.483E-01	1.44	7.493E-02	2.20	1.390E-02	2.96	1.538E-03	3.72	9.961E-05	4.48	3.732E-06	5.24	8.029E-08	6.00	9.866E-10
0.69	2.451E-01	1.45	7.353E-02	2.21	1.355E-02	2.97	1.489E-03	3.73	9.574E-05	4.49	3.561E-06	5.25	7.605E-08	6.01	9.276E-10
0.70	2.420E-01	1.46	7.215E-02	2.22	1.321E-02	2.98	1.441E-03	3.74	9.201E-05	4.50	3.398E-06	5.26	7.203E-08	6.02	8.721E-10
0.71	2.389E-01	1.47	7.078E-02	2.23	1.287E-02	2.99	1.395E-03	3.75	8.842E-05	4.51	3.241E-06	5.27	6.821E-08	6.03	8.198E-10
0.72	2.358E-01	1.48	6.944E-02	2.24	1.255E-02	3.00	1.350E-03	3.76	8.496E-05	4.52	3.092E-06	5.28	6.459E-08	6.04	7.706E-10
0.73	2.327E-01	1.49	6.811E-02	2.25	1.222E-02	3.01	1.306E-03	3.77	8.162E-05	4.53	2.949E-06	5.29	6.116E-08	6.05	7.242E-10
0.74	2.296E-01	1.50	6.681E-02	2.26	1.191E-02	3.02	1.264E-03	3.78	7.841E-05	4.54	2.813E-06	5.30	5.790E-08	6.06	6.806E-10
0.75	2.266E-01	1.51	6.552E-02	2.27	1.160E-02	3.03	1.223E-03	3.79	7.532E-05	4.55	2.682E-06	5.31	5.481E-08	6.07	6.396E-10