Exam Digital Communications I

16th of March, 14:00–19:00

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Allowed material:

- The textbook Digital Communications by Bernard Sklar
- Any calculator
- Mathematical handbook, βeta
- Swedish-English dictionary
- List of formulas written by Sorour Falahati

Preliminary grading intervals: grade 3 $= (23 - 32)$, grade 4 $= (33 - 42)$, grade 5 $= (43 - 50)$

Please write all your answers neatly and clearly and motivate your answers thoroughly except in Question 1 where only true or false is required.

Good Luck!

Question 1: (10p)

For each of the following sub-questions answer by *true* or *false*. No motivation is required. An incorrect answer gives 0 p.

1. (2p) A signal can be strictly limited in time and have limited absolute bandwidth at the same time.
2. (2p) Non-coherent demodulation always requires an accurate phase reference
3. (2p) The rate 1/2 convolutional code with generator vectors $g_1 = (110)$ and $g_2 = (101)$ is catastrophic.
4. (2p) Let $E_b/N_0 = 10dB$ and assume an AWGN channel. The bit error rate for coherently detected BPSK is then less than 4×10^{-6}.
5. (2p) Consider binary signaling over an AWGN channel. The probability of bit error of antipodal signaling is always one half of the probability of error for orthogonal signaling

Question 2: (10p)

Find the bit error probability for a digital communication system with a bit rate of 1Mbit/s. The received waveforms

$$s_1 = Acos(\omega_0 t)\ ;\ s_2 = Asin(\omega_0 t)\ ;\ s_3 = -Acos(\omega_0 t)\ ;\ s_4 = -Asin(\omega_0 t)$$

are coherently detected with matched filters. The value of A is $8mV$ and the single sided noise power spectral density is $N_0 = 10^{-11} W/Hz$. Furthermore, the signal power and energy per bit are normalized relative to a 1Ω load. Assume equiprobable and Gray coded symbols.

Question 3: (10p)

Design an MPSK-modulation/coding scheme that fulfills the following system requirements:

- The bandwidth must not exceed 4000Hz
- The signal power to noise spectral density ratio $P/N_0 = 50dBHz$
- The bit rate is $R_b = 10kHz$
- The bit error rate $P_B \leq 10^{-5}$

Assume an AWGN channel, Nyquist signaling, and equiprobable Gray coded symbols. The symbol constellation consists of $M = 2^k$ symbols where k is positive integer. The symbols are assumed to be evenly spaced on a circle with radius $\sqrt{E_s}$ where E_s is the symbol energy.
Hint: The provided tables of $Q(x)$ and BCH-codes may be used.

Question 4: (10p)

A convolutional code has generator vectors: $g_1 = (1011)$; $g_2 = (1101)$; $g_3 = (1111)$.
Assume a binary symmetric channel with probability of error $p = 2 \times 10^{-2}$.

1. (4p) Draw a state diagram for this code.
2. (6p) Calculate an upper bound for the probability of (decoded) bit error.
   Hint: Use only dominating terms in the transfer function $T(D, N)$.

Question 5: (10p)

Let the sampled received signal in a digital communications channel subject to multi-path propagation be represented by

$$x_t = u_t + 0.7u_{t-1} + n_t$$

where the transmitted symbols $\{u_t\}$, and the noise $\{n_t\}$ are zero mean and mutually independent white sequences with $Eu_t^2 = \sigma_u^2$ and $En_t^2 = \sigma_n^2$.

1. (6p) Calculate the coefficients of the two tap linear equalizer

$$\hat{u}_t = c_0 x_t + c_1 x_{t-1}$$

   by minimizing the mean square error (MSE), $E\varepsilon_t^2$, where $\varepsilon_t = u_t - \hat{u}_t$. Assume $\sigma_u^2/\sigma_n^2 = 10dB$.

2. (4p) Calculate the MSE for the equalizer optimized in 1. above. Also find the optimal coefficients c_0 and c_1 for the case $\sigma_u^2/\sigma_n^2 = \infty$ dB (i.e. $\sigma_n^2 = 0$) and compare the two MSE results. What conclusions can you draw from this comparison? Would it be possible to construct an equalizer which attains a lower MSE? Motivate your answer clearly.