Exercises chapter 6 - Part I.

1. Exercise 6.2 on page 160. The problem involves much calculations using binomial coefficients and if you're, by any chance, not very comfortable with such calculations, the comments and hints below may help.

    - You will probably find the formulas in eq. (6.16) and (6.23) useful.
    - When computing the variance of R, it turns out that it can be easier to first compute the expectation $< R(R+1) >=< R^2 > + < R >$ instead of $< R^2 >$ and use this to get to the variance.
    - If I am not wrong, the "integral equation" in (6.11) (task (a)) is fulfilled but with the right hand side of the form $f(n,r)\binom{N-1}{n-1}$ rather than $f(n,r)\binom{N}{n}$. I haven't dwelled on it enough to see if this is an error/problem or not.

2. **Inferring a bit error probability.** In a packet transmission system, data is sent in blocks of K bits. These are encoded using a channel coding scheme in such a way that the decoder can correct up to t_0 erroneous bits. We say that the scheme has error correction capability of t_0. When the number of errors in the received packet, k, is larger than t_0, we detect the packet as erroneous. Let θ denote the error probability of a received bit. We here assume that this probability, which describes the quality of the radio channel, is the same for every received bit.

    We run an experiment where we transmit n packets and observe that r of them are detected wrongly.

    Based on the data, $D = (n, r)$, and prior information, I, calculate and plot $p(\theta|DI)$. Give also a numerical point estimate. (Choose a point estimate that you feel summarizes the information in $p(\theta|I)$ in a relevant way.). We have outsourced the assignment of the prior, $p(\theta|I)$, to an experienced colleague, see below. You can play around with different parameters, such as packet size, K, error correction capability, t_0, number of transmitted packets, n, and the number of packet detected erroneously, r. Do the results follow your intuition?

    Some hints and recommendations:

    - You colleague says that the following priors are reasonable: For an unknown channel, a uniform distribution $\theta \in [0.001, 0.5]$ is suitable. For a receiver located at a good spot, $\theta \in [0.001, 0.01]$ could be a reasonable choice.
    - To have a reference for choosing some interesting intervals for the data $D = (n, r)$, we suggest that you first prepare some plots of probability of packet error versus probability of bit error for different values of K and t_0 (for example, $K = \{10, 20, 100, 200, 500\}$ and $t_0 = \{1, 2, 3, 4, 5\}$).
    - Analytical solutions are complicated. Instead, approach the problem numerically by discretizing θ.