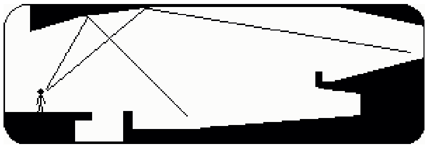
3. ROOM ACOUSTICS

Subjects of room acoustics (or architectural acoustics):

- how rooms affect sound
- help design a room for a certain purpose
- help improve sound performance of a room



Sound in rooms: Direct sound, & Reverberant sound

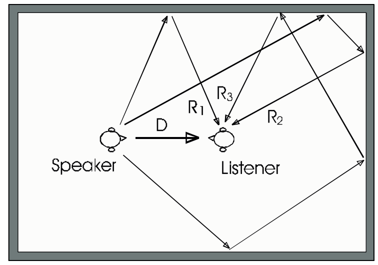


Fig. 3.1. A process of sound generation and perception in a reverberant room.

- *Direct sound*: the one directly from the source to the receiver (Ray D).

  Direct arrival $t = r/c$,
  r is distance from source to observation point,
  c is sound speed in air.

- *Reverberant sound*: the one undergoing one or more reflections (Rays R1, R2, R3).

Reverberant Consequence:

- a series of echoes from a single impulse source,
- masking effect in the far field

Impulse response of a rectangular room

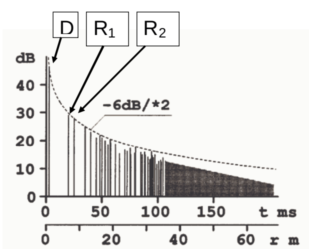


Fig. 3.2. Impulse response of a rectangular room.

1. ***Direct sound*:** 1st impulse;
2. ***Reverberant* sound:** 2nd, 3rd, & the others due to reflected sounds from the 1st and multiple reflections
3. ***Absorption*:** absorption by the room surface makes the reflected sounds become smaller and smaller.
4. ***Diffusion*:** the impulses come closer and closer as the time elapses, until they become diffused so that the sound throughout the room is well blended.

- Signal model of the sounds in the room with impulse response $h_s(n)$
  $x(n) = b(0)d(n) + b(1)d(n-1) + \ldots + b(q)d(n-q) + \ldots = h_s(n)*d(n)$

Anechoic chamber, reverberant chamber, reverberant (live) room

- *Anechoic **chamber***: an echo-free room in which the direct sound predominates everywhere, and the walls & floor are made of sound-absorbing wedges; used for testing the frequency responses of loudspeakers and microphones
- *Reverberant **chamber***: a room in which the reverberant wave predominates overwhelmingly. Absorption properties of the room surfaces known, used to measure the acoustic absorption of products: theatre seating, carpets, noise barriers and acoustic absorbers, etc.
- *Reverberant (live) **room***: the room that is neither anechoic nor reverberant chamber, but have a certain amount reverberant effects.

B. A simple model for the growth of sound in a live room

Sabine assumption:

- average acoustic energy density E is the same throughout the room,
- all directions of propagation are equally probable.

E in the room is the sum of the energy densities E_m of all the individual rays

$$\boxed{E = \sum_m E_m = \frac{\sum_m P_{em}^2}{\rho_0 c^2} = \frac{P_{er}^2}{\rho_0 c^2}}, \qquad (3.2)$$

E_m the mth ray's energy density,

P_{em} the mth ray's effective pressure

$P_{er}^2 = \sum_m P_{em}^2$ is the *effective pressure amplitude of the reverberant sound field*.

Governing equation for the growth of sound energy in a room:

Let W be the acoustic power produced by a sound source,
W_e be the rate at which sound energy increases in air throughout the room V,
W_a be the rate at which sound energy is absorbed by the surfaces.

Energy conservation says $\quad W_e + W_a = W$

Energy increasing rate: $\quad W_e = \frac{d(EV)}{dt} = V\frac{dE}{dt}$

Energy absorption rate: $\quad W_a = \frac{Ec}{4}A \qquad (3.3)$

$Ec/4$ is the rate at which the energy in the room impinges onto a unit area of the wall, and A represents the *total sound absorption* of the room.

The differential equation governing *the growth of sound energy* in a live room,

$$\boxed{V\frac{dE}{dt} + \frac{Ac}{4}E = W}, \qquad (3.4)$$

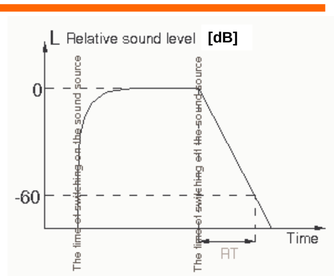
Solution of the governing equation:

If the source is switched on at t=0, solving the governing equation (noting $E = P_{er}^2/\rho_0 c^2$) yields

$$P_{er}^2(t) = \frac{4W\rho_0 c}{A}[1-\exp(-t/t_E)], \qquad (3.5)$$

where $t_E = \frac{4V}{Ac} \qquad (3.6)$

is the *time constant* governing the growth of acoustic energy in the room.



Ultimate sound pressure (uniformly diffused): $P_{er}^2(\infty) = \frac{4W\rho_0 c}{A}, \qquad (3.7)$

Relative sound level: $L = 10\log\frac{P_{er}^2(t)}{P_{er}^2(\infty)} = 10\log\left[1-\exp\left(-\frac{t}{t_E}\right)\right] \qquad (3.8)$

C. Reverberation time (RT) (1)

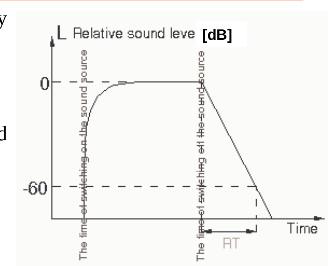
The source is turned off at t=0 for uniformly diffuse sound.
The pressure at t is

$$P_{er}^2(t) = P_{er}^2(0)\exp(-t/t_E). \qquad (3.9)$$

- *Reverberation time T*: the time required for the sound level to drop by 60 dB,

$$L = 10\log\frac{P_{er}^2(T)}{P_{er}^2(0)} = 10\log\left[\exp\left(-\frac{T}{t_E}\right)\right] = -60$$

$$\boxed{T = 13.8t_E = \frac{55.2V}{Ac}}$$



For air with c = 343 m/s ($20^\circ C$) $\boxed{T = \frac{0.161\ V}{A}}. \qquad (3.11)$

- The bigger the room V (or the smaller the absorption A) the larger the RT.

Average Sabine absorption $\bar{a}$:

$$\bar{a} = \frac{A}{S},$$

where S is the surface area of the room.

$$T = \frac{0.161\ V}{S\bar{a}}. \qquad (3.13)$$

The reverberation time is an important parameter determining the acoustic performance of a room!

Average Sabine absorptivity:

- *Total sound absorption* in a room with the materials of different absorptions:

$$A = \sum_n A_n = \sum_n S_n a_n \qquad (3.14)$$

A_n is the absorption of the nth material
a_n is the *Sabine absorptivity* of the nth surface S_n

- *Average Sabine absorptivity:* $\bar{a} = \frac{1}{S}\sum_n S_n a_n$ (3.15)

  where S is the total surface area and a_n is evaluated from standardized measurements on a sample of the material in a reverberant chamber.

- a_n depends on freq (representative ones: 125, 250, 500, 1000, 2000, 4000 Hz, covering the entire range important to speech and music).
- *Reverberation time T* is generally understood to refer to the freq. of 500 Hz.

Example of Average Sabine absorptivity: Table 3.1

Table 3.1 Representative absorption coefficients of surfaces.

Materials	Absorption coefficient a					
	125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz
Bricks, unglazed	0.03	0.03	0.03	0.04	0.05	0.07
Plaster, gypsum or lime, on brick	0.01	0.02	0.02	0.03	0.04	0.05
On concrete block	0.12	0.09	0.07	0.05	0.05	0.04
Concrete block, coarse	0.36	0.44	0.31	0.29	0.39	0.25
Painted	0.10	0.05	0.06	0.07	0.09	0.08
Plywood, 1-cm-thick paneling	0.28	0.22	0.17	0.09	0.10	0.11
Cork, 2.5 cm thick with airspace behind	0.14	0.25	0.40	0.25	0.34	0.21
Glass, typical window	0.35	0.25	0.18	0.12	0.07	0.04
Drapery, lightweight, flat On wall	0.03	0.04	0.11	0.17	0.24	0.35

Heavyweight, draped to half area	0.14	0.35	0.55	0.72	0.70	0.65
Floor, concrete	0.01	0.01	0.02	0.02	0.02	0.02
Linoleum on	0.02	0.03	0.03	0.03	0.03	0.02
Heavy carpet on	0.02	0.06	0.14	0.37	0.66	0.65
Floor, wood	0.15	0.11	0.10	0.07	0.06	0.07
Ceiling, gypsum board	0.29	0.10	0.05	0.04	0.07	0.09
Plastered	0.14	0.10	0.06	0.05	0.04	0.03
Plywood, 1 cm thick	0.28	0.22	0.17	0.09	0.10	0.11
Suspended acoustical tile, 2 cm thick	0.76	0.93	0.83	0.99	0.99	0.94
Gravel, loose and moist, 10 cm thick	0.25	0.60	0.65	0.70	0.75	0.80
Grass, 5 cm high	0.11	0.26	0.60	0.69	0.92	0.99
Rough soil	0.15	0.25	0.40	0.55	0.60	0.60
Water surface, as in a pool	0.01	0.01	0.01	0.02	0.02	0.03

Source: Allan D. Pierce, *Acoustics – an introduction to its physical principles and applications*, McGraw-Hill, 1981, p. 256.

Examples of reverberation time (1)

Example 1: Fig. 3.3 showing measurements of reverberation times at three different frequencies, 125, 500 and 2500 Hz.

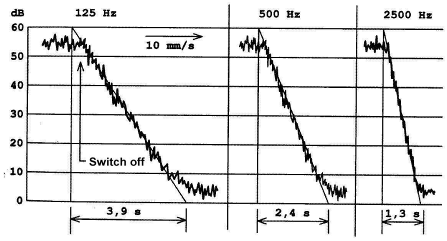


Fig. 3.3. Measurements of reverberation times at three different frequencies, 125, 500 and 2500 Hz.

Examples of reverberation time (2)

Example 2: The reverberation time plays a central role in the quantitative formulation of some of the criteria for what constitutes good acoustics for rooms intended for specified purposes.

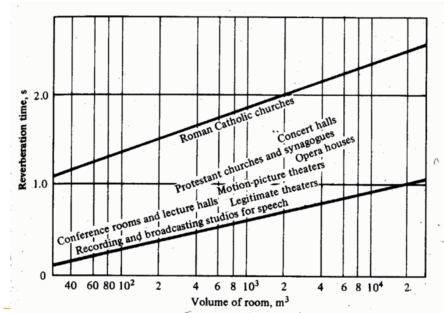


Fig. 3.4. Optimum midfrequency (500 to 1000 Hz) reverberation times for fully occupied rooms versus volume.

(From A. D. Pierce, *Acoustics – an introduction to its physical principles and applications*, McGraw-Hill, 1981, p. 271).

D. Radius of reverberation (1)

Assumption:
the reverberant field has no effect on direct sound field or source power.

For a point source radiating acoustic power W with effective amplitude P_{ed},

direct sound field: $$P_{ed}^2 = \frac{\rho_0 c W}{4\pi r^2} \quad (\because W = 4\pi r^2 \frac{P_{ed}^2}{\rho_0 c}) \quad (3.16)$$

reverberant field (when completely diffuse): $$P_{er}^2 = \frac{4W\rho_0 c}{A}$$

Total mean squared pressure: $$P^2 = \rho_0 c W\left(\frac{1}{4\pi r^2} + \frac{4}{A}\right). \quad (3.17)$$

D. Radius of reverberation (2)

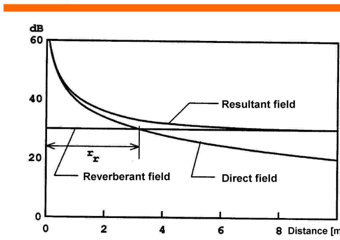


Fig. 3.5. Direct field, reverberant field & their superposition (the resultant field).

- *Radius of reverberation*, or *critical radius*: the distance at which $P_{ed}=P_{er}$,

$$r_r = \sqrt{\frac{A}{16\pi}} \quad (3.18)$$

In terms of T (=0.161V/A),

$$r_r = 0.056\sqrt{\frac{V}{T}}, \quad (3.19)$$

- Sound pressure level at r_r is 3 dB higher than expected from either alone.

The direct field is larger for $r < r_r$;
the reverberant is larger for $r > r_r$

3.2. Standing waves and normal modes in rooms

A. Normal modes and eigenfrequencies in a rectangular room

Normal modes

$$p_{lmn}(x,y,z,t) = A_{lmn}\cos\left(\frac{l}{L_x}\pi x\right)\cos\left(\frac{m}{L_y}\pi y\right)\cos\left(\frac{n}{L_z}\pi z\right)e^{j2\pi f_{lmn}t} \qquad (3.20)$$

Eigenfrequency: $$f_{lmn} = \frac{c}{2}\sqrt{\left(\frac{l}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2 + \left(\frac{n}{L_z}\right)^2} \qquad (3.21)$$

- f_{lmn} are completely determined by the nature of the room
- If two of integers (l,m,n) are zero, e.g., (1,0,0), the normal mode is termed *axial* since the propagation vector of the wave is parallel to one of the axes
- If one of the integers is zero, e.g., (1,2,0), the mode is termed *tangential* since the propagation vector of the wave is parallel to one pairs of surfaces
- If all the integers are not zero, e.g., (1,2,1), the mode is termed *oblique*

Some normal modes in a rectangular room 6x4 m

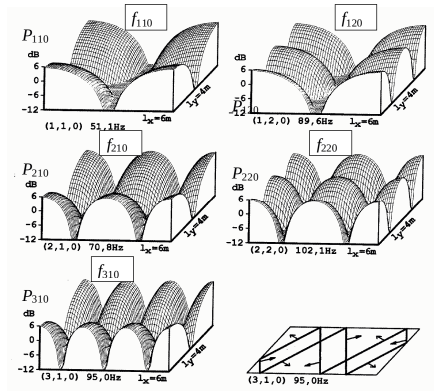


Fig. 3.6. Five different normal modes of pressure field in a rectangular room with dimension of 6x4 m^2.

B-1. Frequency distribution of room resonances (1)

Table 3.2. Normal mode frequencies below 100 Hz for a rectangular room of 3.12×4.69×6.24 m with c = 345 m/s.

L	*m*	*n*	*f (Hz)*	*l*	*m*	*n*	*f (Hz)*
0	0	1	27.5	1	0	2	77.5
0	1	0	36.6	0	2	1	78.5
0	1	1	45.9	0	0	3	82.5
1	0	0	55.0	1	1	2	86.5
0	0	2	55.0	0	1	3	90.2
1	0	1	61.5	0	2	2	91.5
0	1	2	66.0	1	2	0	91.5
1	1	0	66.0	1	2	1	95.5
1	1	1	71.5	1	0	3	99.0
0	2	0	73.2				

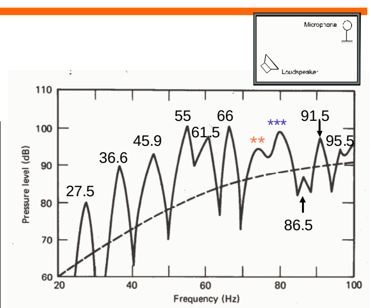


Fig. 3.7 Normal mode responses of a rectangular room 3.12×4.69 × 6.24 m at low frequencies from 20 to 100 Hz in an echo- free chamber (solid) and in a rectangular room (dashed), respectively.

B-2. HIGH FREQUENCY APPROXIMATION

Given a rectangular room $V=L_x \times L_y \times L_z$, *the number of room normal modes* whose eigenfrequencies f_{lmn} are less than a given value of f is,

$$N(f) \approx \frac{4\pi V}{3}\left(\frac{f}{c}\right)^3 + \frac{S}{4}\left(\frac{f}{c}\right)^2 + \frac{L}{8}\left(\frac{f}{c}\right) + \frac{1}{8}, \qquad (3.22)$$

$V=L_x \times L_y \times L_z$ is the volume,
$S = 4(L_x \times L_y \times + L_x \times L_z + L_y \times L_z)$ is the total surface area and
$L = 2(L_x + L_y + L_z)$ is the total length of all the edges in the room.

Modal density is defined as the number of room normal modes per unit frequency bandwidth, given by

$$\frac{dN(f)}{df} = \frac{4\pi V}{c^3} f^2 + \frac{S}{4c^2} f + \frac{L}{8c}. \qquad (3.23)$$

Average frequency spacing:

- The number of normal modes in a frequency bandwidth Δf and centered at frequency *f*,

$$\Delta N \approx \frac{dN(f)}{df}\Delta f .$$

- When the room dimensions are large compared with wavelength

$$\Delta N \approx \frac{dN(f)}{df}\Delta f = \frac{4\pi V}{c^3} f^2 \Delta f . \qquad (3.24)$$

- The *average frequency spacing* $\Delta \bar{f}_{\text{mode}}$ (in Hz) of eigenfrequencies in Δf:

$$\Delta \bar{f}_{\text{mode}} = \frac{\Delta f}{\Delta N} = \frac{c^3}{4\pi V f^2}. \qquad (3.25)$$

- The bandwidth of the resonance peak of the *n*th normal mode for the source driving frequency *f* is approximately

$$\Delta f_{res} = \frac{1}{2\pi t_E} = \frac{13.8}{2\pi T}. \qquad (3.26)$$

Schroeder cutoff frequency – The threshold to Statistical Room Acoustics:

Letting $\Delta \bar{f}_{\text{mode}} < \Delta f_{res}/2.5$, we have *Schroeder cutoff frequency*,

$$f_{Sch} = \sqrt{\frac{5c^3}{55.2}\frac{T}{V}}. \qquad (3.27)$$

In air: $f_{Sch} \approx 1900\sqrt{\frac{T}{V}}$. (3.28)

- The normal modes above f_{Sch} may be regarded as a smoothed-out continuum, and
- room acoustics in this case should be treated from the statistical aspects.
- Deviation of acoustic quantities from the averages predicted by the Sabine model are frequently given a statistical interpretation.